The Story of 1,2,3,4 and the proportions of the John Dee Tower

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The Story of 1,2,3,4 and the Proportions of the John Dee Tower

by
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Cosmopolite Press
Newport, Rhode Island

"Citizen of the World"
(Cosmopolite, is a word coined by John Dee, from the Greek words cosmos meaning "world" and politês meaning "citizen")

Dedication

To Bill Penhallow who brilliantly recognized
the astronomy of the Tower,
and everyone at the
New England Antiquities Research Association
especially
Sue Carlson, Rob Carter, Alvin Holm, Duncan Laurie,
Dan Lorraine, Rick Lynch, James Mavor, Doug Schwartz,
Jeff Stevens, Ros Strong, Margaret Venator, Jim Whitall, and Don Winkler

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Pythagoras, Nicomachus, Boethius, Dee And the Story of "1,2,3,4"

This tale of "1, 2, 3, 4" spans over 1500 years. It connects Pythagoras (around 500 BC) to Nicomachus (around 100 AD) to Boethius (around 500 AD) to Dee (around 1550 AD).

Let's briefly review the lives of these wise philosophers of number.

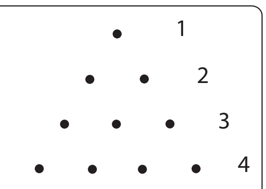
Pythagoras

Pythagoras (ca. 575 BC- ca. 495 BC), known as the "Father of Numbers," was born on the small island of Samos in the eastern Aegean. To escape the tyrannical government of Polycrates, he moved to the Greek colony of Croton, on the southern coast of Italy. According to later writers, the Greek astronomer Thales encouraged Pythagoras to travel to Egypt and Phoenicia to study with the wise priests. Upon returning to Croton he opened a school where both male and female students learned religion, philosophy, music and of course mathematics.

Pythagoras' philosophy of mathematics is summarized by his sacred **tetraktys**, an equilateral triangle consisting of ten points of four rows. It's even mentioned in the Pythagorean oath which has been poetically translated as:

"By that pure holy four lettered name on high,
Nature's eternal fountain and supply,
the parent of all souls that living be,
by him, with faith find oath, I swear to thee."

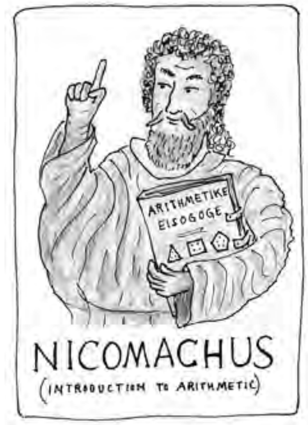
(Wikipedia, Pythagoras)



The Pythagorean Tetraktys is simply a triangle of ten dots

The core of Pythagoras' mathematical and musical cosmology can be seen by comparing the various rows. The first interval is 1:2, the next is 2:3 and the last is 3:4. Pythagoras could have arranged ten dots in the simpler pattern of two rows of 5, but it's these ratios that are important to him.

Nicomachus of Gerasa



Nicomachus of Gerasa, (ca. 60-ca.120) was born in Gerasa, (now Jarash, Jordan) about 50 miles northwest of Jerusalem. Not much is known of his life, but it is assumed he studied at Alexandria, Egypt, the hub for Neo-Pythagorean mathematicians.

He was so prolific, it's thought that he was a writer rather than a teacher. He wrote *Introduction to Arithmetic*, *Manual of Harmonics*, and *Introduction to Geometry*, and it's thought he filled out the quadrivium with an *Introduction to Astronomy*, but this work has not survived. He also wrote a book about his hero entitled *The Life of Pythagoras*.

Many of his' ideas about the mystical aspect of number were incorporated in the later *Theology of Arithmetic,* often credited to Iamblichus (ca. 245-ca. 325). Nicomachus was obviously famous in his time, for as around 150 AD, the Roman author Lucian had one of his characters compliment another by saying "You calculate like Nicomachus." (D'Ooge, introduction, p. 807-808)

Boethius

The next math text superstar is the Roman Anitius Boethius (ca. 480-523 AD). He was born to a powerful, aristocratic family, however, his father died when Boethius was young. He was adopted into the family of Symmachus. Not only was Symmachus his master, he also became his father-in-law, as Boethius later married Symmachus' daughter Rusticiana.

During Boethius' youth, the cultural heritage of Rome was waning as Theodoric the Great had captured and was ruling Rome. It's thought that Boethius might have studied in Athens or Alexandria because somehow he became an expert on Greek math and philosophy.

When Boethius weas only 20, his expertise came to the attention of Theodoric, who assigned him many projects, including designing a water clock and a sundial. When Boethius was 30, he became a consul to the Roman senate.

At 40, he became the "magister officiorum," the head of the court and all government services. Unfortunately, for reasons that are still unclear, Boethius was arrested for treason by Theodoric.

Perhaps Boethius had been negotiating with Theodoric's enemies in Byzantine Empire. Or, as Boethius maintained, he was slandered by political rivals who didn't like his tough stance on corruption. Whatever it was, Theodoric took away Boethius' wealth and titles, then threw him in jail. A year later, Boethius was executed. (Masi, p. 64-65)

However, during that year of imprisonment Boethius wrote *The Consolation of Philosophy*, "the work by which he is especially known." (Marebon, p. 10)

This classic Christian text deals with concepts like free-will, chance, fortune, fate, Providence, and moral character. It was a best-seller throughout the Middle Ages. King Alfred the Great, (849 - 899 AD) who saved Wesset (England) from being conquered by the Dane's, translated Boethius' *Consolation* into Anglo-Saxon. Geoffrey Chaucer (ca. 1342 - 1400) translated it into what is now called Early English. During the Renaissance, the learned Queen Elizabeth, who spoke 5 languages, also made an English translation from Boetheus' Latin. (Wikipedia, Boethius)

Consolation had an influence on the writings of Chaucer (particularly in *Troilus and Criseyde*), Dante, Sir Philip Sidney, Shakespeare and Dryden. (Masi, p. 45)

Boethius insists that the quadrivium (arithmetic, music, geometry and astronomy) must be studied to fully understand the nature of things. Nature's order, as found in these subjects, can help a man learn moral truths about life. As Boethius puts it in Book 2 of *Consolation*:

"… all this harmonious order of things is achived by love
which rules the earth and the seas, and commands the heavens.

But if love should slack the reins,
all that is now joined in mutual love would wage continual war,
and strive to tear apart the world, which is now sustained
in friendly concord by beautiful motion.

Love binds together people joined by a sacred bond;
love binds sacred marriages by chaste affections;
love makes the laws which join true friends.

O how happy the human race would be,
if that love which rules the heavens ruled also your souls."

(Translated by Richard Green, in Masi, p. 41)

To Boethius, the beauty found in number theory is found in the nature of human relations. Michael Masi, in *Boethian Number Theory* (1983) puts it this way:

"**As the planets, the seasons, the four elements, nights and day**
are all in proper order, held by the power of love,
so should relations between countries, individuals, and spouses be directed."

(Masi, p. 41).

This brings us to two of Boethius' other great works *De Institutione Arithmetica (Principles of Arithmetic* or as it is more commonly called, *Introduction to Arithmetic)* and *De Institutione Musica*.

Neither of these were as original as Boethius' *Consolation of Philosophy*. The *Introduction to Music* was probably based on Nicomachus' *Manual of Harmonics* and Ptolemy's *Harmonics*. The *Introduction to Arithmetic* is basically just a loose translation of Nicomachus' *Introduction to Arithmetic* (from Greek to Latin). The titles are the same, many chapter headings are the same, and even many of the sentences and illustrations are essentially the same.

However, Boethius must be given credit for spreading Nicomachus wisdom throughout Europe for centuries. From its publication around 500 AD, through the Dark Ages, the Middle Ages and into the Renaissance, *Introduction to Arithmetic* was **the** premier elementary math textbook. (A millennium is a long time to be on the best-seller list, but it helps if the book is "required reading" in schools.)

John Dee

The fourth member of this quaternary of mathematicians in this story of "1, 2, 3, 4," is John Dee. Among the books in Dee's library was Nicomachus' *Introduction to Arithmetic* which had been reprinted in Paris in 1538.

(Roberts and Watson, 450 and B260, and p. 211).

Dee not only owned 8 copies of Boethius' *Introduction to Arithmetic,* he also owned quite a few commentaries on it by numerous authors:

Roger Bacon

Bacon's Opus Maius (Major Opus) published around 1270 emphasized the Boethian idea that mathematics is essential to the understanding of natural and divine principles.

Luca Pacioli

Pacioli's *Summa de Arithmetica* actually duplicates parts of Boethius' *Introduction to Arithmetic*, only in Italian. Many of the of the chapter titles were even left in Boethius' original Latin words. Pacioli also wrote the *Divine Proportion* in which Leonarda da Vinci did the illustrations of the Platonic (and some of the Archimedian) Solids.

Pacioli recommended the study of mathematics to help understand "music, astrology, cosmography, architecture, law, and medicine." (Pacioli in Masi, p. 51).

Georgius Valla

Valla was a doctor who wrote a book called *De Arithmetica* in 1501 (published in Venice by the famed Aldus Manutius). To get a better understanding of physical health and moral philosophy, the doctor prescribed the study of numbers.

Hudalrich Regius

Regius, in his 1550 *Utriusque Arithmeticae Epitome,* saw the mathematical sciences in this order: arithmetic, optics, perspective and mechanics.

Nicomachus and Boethius texts don't really deal with practical, day-to-day mathematical calculations, which the Greeks referred to as *logismos* (reckoning or computation). Instead, they were concerned with number ratios, how geometric forms relate to number, and how certain numbers are derived from other numbers.

Likewise, Nicomachus and Boethius' musical texts, aren't about the performance of vocal or instrumental music. They are theoretical accounts of the mathematics of harmonic sound.

(Masi, pp 15-16)

Boethius' and Nicomachus' texts on Arithmetic deal with various aspects of number theory:

The interaction of odd numbers and even numbers,
The sieve of Eratosthenes and prime numbers,
Triangular and square numbers (as well as pentagonal, hexagonal, and heptagonal numbers.)
Arithmetic, geometric and harmonic ratios, as well as ratios in music.

I won't expound upon these topics here, but there are **3 important connections** I will point out that give insight into Dee's mathematical cosmology. One is a quote, the next is a chart and the third is a diagram showing various ratios.

1. In the *Preface to Euclid*, just before his mentioning the Exemplar Number, Dee quotes "**the great and Godly Philosopher Anitius Boethius:**"

" Omni quaecunq
a primeava rerum natura constructa sunt,
Numerorum videntur ratione formata.
Hoc enim fuit principale in animo
Conditioris Exemplar."

Which Dee translates as:

"All things
(which from the very first original being of things, have been framed and made)
do appear to be Formed by the reason of Numbers.
For this was the principal example or pattern in the mind of the Creator."

(Dee, *Preface*, p. j)

Michael Masi's 1983 translation of Boethius' Latin reads like this:

"Concerning the Substance of Number

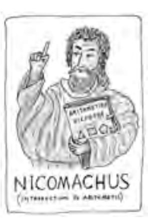
From the beginning, all things whatever which have been created
may be seen by the nature of things to be formed by reason of numbers.

Number was the principal exemplar in the mind of the creator.

From it was derived the multiplicity of the four elements,
from it were derived the changes of the seasons,
from it the movement of the stars and the turning of the heavens."

(Boethius, in Masi, p. 76).

It's pretty obvious Dee got the idea of using the word "**Exemplar**" from Boethius. And, as Boethius basically paraphrased Nicomachus' work, let's see how Nicomachus originally phrased it:



"CHAPTER VI
All that has by nature with systematic method been arranged in the universe
seems both in part and as a whole to have been
determined and ordered in accordance with number,
by the forethought and the mind of him that created all things;

For the pattern was fixed, like a preliminary sketch,
by the domination of number pre-existent in the mind of the world-creating God.

Number, conceptual only and immaterial in every way,
but at the same time, the true and the eternal essence,
so that with reference to it, as to an artistic plan, should be created all these
things, time, motion, the heavens, the stars, all sorts of revolutions."

(Nicomachus, D'Ooge, p. 814).

In Boethius' actual quote, he uses the Latin word "***Exemplar***." Dee translates this as "**example or pattern**" but then uses word in his expression, "**The Exemplar Number**." What word did Nicomachus originally use?

He used the Greek word "***paradeigmatos***" meaning "a pattern, a model, and example." Plato (in Timieus and Republic) used this word to describe a model that a sculptor or painter might use.

The word *paradeigma* comes from *paradeiknunai*, "to exhibit side by side" (*para* meaning "beside" and *deiknunai* meaning "to show"). As you might have guessed, this is where we get the English word paradigm, meaning a typical example or pattern of something. (The phrase "paradigm shift" was only coined in the 1970's by the scientific philosopher Thomas Kuhn).

The related Greek word *diegma* means a sample, pattern or proof. In Latin, this word became *documentum*, from which we get document and documentary. (Liddell/Scott, p. 595)

I'm not suggesting that Nicomachus or Boethius are making a cryptic reference to 12252240. They are saying that the Creator used all numbers as a pattern. In his explorations into how number worked, Dee came upon this number, saw how it agreed with this mathematical cosmology, and borrowed some of their Language to concisely describe it.

2. In Book 1, Chapter 19 of *Introduction to Arithmetic*, Nicomachus shows what might be the first multiplication table in a Greek text. It's hard to visualize it in Greek, so let's look at it with Arabic numerals.

It's not a 1-to-100 chart. Aside from 1, 2, 3, 5, and 7, all the numbers are composite numbers. Nicomachus provides a play-by play of what's important about his chart.

First, he joins the 1-10 row along the top with the 1-10 column along the left edge, making what he calls "the form of the letter Gamma" (an inverted L-shape).

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

1 to 10 form the Greek letter Gamma (Γ)

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

The "doubles" also form a Gamma (Γ)

Next, he combines the second row and the second column to make another "Gamma" shape. He calls these *diplasioi*, the "doubles." Reading left to right, they are the "doubles" of the numbers above them. Reading up and down, they are the doubles of the numbers to their left. Do you know why he **doesn't** include 2 in this chart of "doubles," even though 2 is obviously double of 1? (I'll give the answer in a moment.)

He calls the third row and the third column the *triada* or the "triples" or the "multiples of 3."

But this time, he refers to the shape formed by the row and column as being like the Greek letter Chi, which is the shape of an X. (Like Dee, Nicomachus is not bothered by the fact the X isn't equilateral or oriented like a cross.)

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

The "triples" form the Greek letter Chi (Χ)

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

The "quadruples " also
form the Greek letter Chi (Χ)

He makes another "Chi" from the rows and columns of the *tetraplasion,* the "quadruples," or multiples of 4.

But then he stops there. He doesn't point out the multiples of 5, 6, 7, 8, 9, or 10 even though they are quite apparent in the chart. (They continue to form chi shapes until the final 10-100, which form another Gamma shape.)

Nicomchus stops because he is primarily interested in what I call "Story of 1, 2, 3, 4."

Next, he identifies how to find the ratios 2:3 and 3:4 in his chart.

He points to the second and third rows (and the second and third columns as well), as I have highlighted here.

As Greeks expressed ratios by putting the larger number (*prologos*) before the smaller number (*upologos*), Nicomachus has us compare the "triples" in the third row with the "doubles" in the second row. The ratios 3:2, 6:4, 9:6, 12:8… are all examples of "hemiolion," or the 3:2 ratio."

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Ratios that are *hemiolios* (3:2)

Next, he compares the fourth row to the third row (and the fourth column to the third column), pointing out all the numbers that are in the ratio called "epitritos," or the 4:3 ratio. All these ratios, 8:6, 12:9, 16:12, 15:20… etc., are equivalent to the ratio 4:3.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Ratios that are *epitritos* (4:3)

Nicomachus does not point out the comparison between the second and first rows is the 2:1 ratio, because he has already explained that the second row is "doubles," which is the same thing as the 2:1 ratio.

To the Greeks, 2:1 wasn't really a ratio **because they didn't consider 1 to be a number.** This is why Nicomachus showed the "doubles" row as a Gamma shape and not a Chi shape.

Furthermore, he writes that "**By Divine nature, not by our convention or agreement…**" the ratios (like 3:2 and 4:3) are of "**later origin than the multiples**" ("the multiples" means the "doublings" row, the "triplings" row, ...). (Nicomachus in D'Ooge, p. 824).

This suggests that the 2:1 ratio is actually more important than the 3:2 and 4:3 ratios.

Nicomachus does find a common thread between the 3:2, and 4:3 ratios and the "doublings" (2:1 ratios) by pointing out that the "differences" between the members of all these various ratios progress the same way:

hemiolios	3:2	6:4	9:6	12:8	15:10
(differences)	3−2=**1**	6−4=**2**	9−6=**3**	12−8=**4**	15−10=**5**

These differences between the two terms of the *hemiolios* ratios...

epitritos	4:3	8:6	12:9	16:12	20:15
(differences)	4−3=**1**	8−6=**2**	12−9=**3**	16−12=**4**	20−15=**5**

...are the same as the differences in the terms of the *epitritos* ratios...

singles	1	2	3	4	5
doubles	2	4	6	8	10
(differences)	2−1=**1**	4−2=**2**	6−3=**3**	8−4=**4**	10−5=**5**

...and also the "differences" between the "doubles" and the "singles"

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

1, 10 and 100 are at the corners and this diagonal contains the squares of the members of the decad

Finally, Nicomachus explains a few details that are unrelated to analysis of the ratios.

He points out that 1, 10, 10, 100 are at the corners of the chart, and that the diagonal contains the squares of the members of the decad.

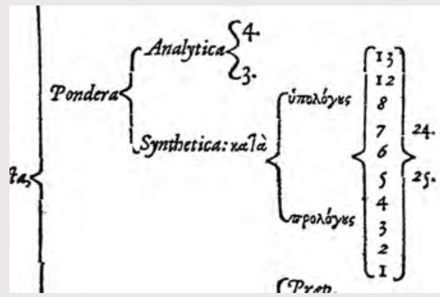
But essentially he was shown this “first Greek multiplication table” to help the student grasp the “doublings” (the ratio 2:1), hemiolion (the 3:2 ratio), and epitritos (the 4:3 ratio), that were so important in Pythagoras’ tetraktys.

There’s a big clue here in Nicomachus’ Chapter 19 that relates to the *Monas Hieroglyphica*. In discussing the 3:2 ratio, Nicomacus explains that all the ***prologous*** (the first, and larger number in a ratio) are multiples of 3 and all the ***upologous*** (the second, and smaller number in a ratio) are multiples of 2.

These are the exact same two Greek words that Dee uses in his Artificial Quaternary Chart!

In the category *Pondera* (weights), *Analytica* (Analysis) is the number 4 above the number 3.

But in *Synthetica* (Synthesis), *upologous* (the second, smaller number) is above *prologous* (the first, larger number). This suggests that Dee wants us to see 3:4 as well as 4:3.



The Greek words ***upologous*** and ***prologous*** are not very frequently used words in terms of Greek mathematics. The comprehensive *Liddell/Scott Greek Lexicon* defines ***upologos*** as “held accountable,” or “taking into account,” with no mention of its meaning in math. Likewise, it defines ***prologos*** as the “prologue of a play,” again with not a word about its use in mathematics.

Indeed, even though Nicomachus discusses a great deal more about ratios in his text, this reference in Chapter 19 seems to be the only reference to “prologous” and “upologous” in the entire book! D’Ooge translates these words as “**antecedents**” (*ante* means before) and “**consequents**” (*con* means “with” or “following”).

These terms, *upologous* and *prologous* are the **only** Greek words in his full-page Artificial Quaternary chart. The chart is all about **mathematics**. Nicomachus of Gerasa’s *Introduction to Arithmetic* was the **most famous** math book in history (Euclid’s *Elements* is mostly about geometry). Thus, Dee is dropping a fat clue that we should study the various ratios Nicomachus is highlighting in his well-known multiplication table.

Incidentally, Boetheus’ table (in Roman Numerals) is almost as confusing to the modern eye as Nicomachus’ original table is (which used Greek letters as numbers).

For the Greek terms *prologous* and *upologous*, Boethius uses the Latin terms *duces* and *comites,* which D’Ooge says literally mean “leaders” and “followers.”

I	II	III	IIII	V	VI	VII	VIII	VIIII	X
II	IIII	VI	VIII	X	XII	XIIII	XVI	XVIII	XX
III	VI	VIIII	XII	XV	XVIII	XXI	XXIV	XXVII	XXX
IIII	VIII	XII	XVI	XX	XXIIII	XXVIII	XXXII	XXXVI	XL
V	X	XV	XX	XXV	XXX	XXXV	XL	XLV	L
VI	XII	XVIII	XXIIII	XXX	XXXVI	XLII	XLVIII	LIIII	LX
VII	XIII	XXI	XXVIII	XXXV	XLII	XLVIIII	LVI	LXIII	LXX
VIII	XVI	XXIIII	XXXII	XL	XLVIII	LVI	LXIIII	LXXII	LXXX
VIIII	XVIII	XXVII	XXXVI	XLV	LIIII	LXIII	LXXII	LXXXI	XC
X	XX	XXX	XL	L	LX	LXX	LXXX	XC	C

Nicomachus’ chart in Boethius’ Latin translation which uses Roman numerals

3. *The very last Chapter in both Nicomahus' and Boethius' texts*

Besides the word "*Exemplar*" and the words "*prologous* and *upologous*" there is another important connection between Nicomachus' and Boethius' books and Dee's *Monas Hieroglyphica*. It's the final chapter in Nicomachus' text, which was paraphrased by Boetheus in the last chapter of his text. Nicomachus begins the chapter with this grand pronouncement:

"**It remains for me to discuss briefly the most perfect proportion, that which has three separate parts and embraces them all, and which is most useful for all progress in music and the theory of the nature of the universe.**"

What I have translated as "**the most perfect proportion**" is Nicomachus' Greek word *teleistatês*. The verb *teleioô* means "to make perfect, to make complete." (Which is very similar to the Latin word consummata).

Echoing Nicomachus, Boethius starts his final chapter with the dramatic heading:

"***De maxima et perfecta symphonia, quae tribus distenditur intervallis***."

which translates as:

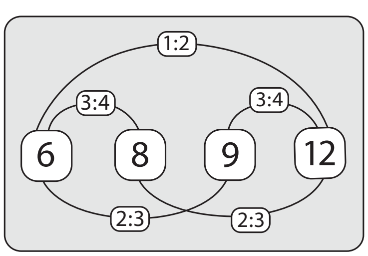
"**Of the greatest and most perfect harmony, which is stretched out across three intervals.**"

Boethius' first sentence is practically identical to Nicomachus' first sentence:

"**It remains now to discuss the greatest and most perfect harmony, which, made up of three intervals, holds great strength in the modulation and tempering of music and in speculation on natural questions.**"

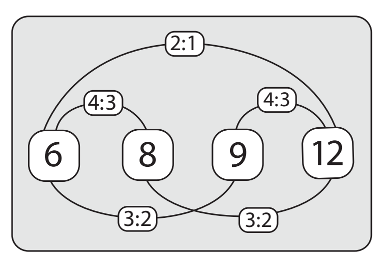
The Latin word "armonia" in Boethius' text, I have translated as "harmony." In the Chapter title, Boetheus actually uses the Latin word "symphonia."

Harmony, symphony, concord, agreement, all mean pretty much the same thing, but I'm hesitant to translate it as the "most perfect symphony" because to me this connotes a large group of musicians dressed in long, black dresses and tuxedos and playing Beethoven's Fifth. While Nicomachus certainly does connect this harmony to music, here in the final chapter of *Introduction to Arithmetic*, the emphasis is mostly on the numbers.

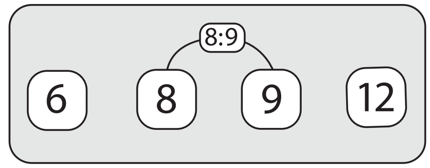


The 3 intervals are quite simple. They involve the numbers 6, 8, 9, and 12. Among the various pairings of these numbers, those ratios so important to Pythagoras (1:2, 2:3, and 3:4) can be found.

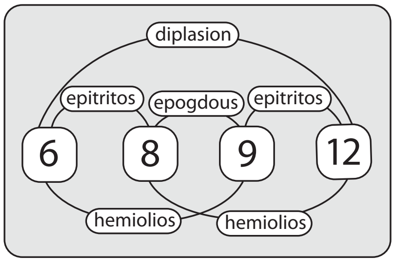
Pythagoras, Nicomachus, and Boethius, who always put their *prologous* in front of their *upologous*, actually would have expressed it this way:



But there is one more pairing which they considered important as well, the ratio 8:9 (or 9:8 as they would express it).



Nicomachus' Terms for these ratios.



Nicomachus' Greek words for these ratios sound pretty strange, but they are actually quite simple when broken down into parts:

2:1 Diplason

The prefix *di-* means "two," and *plason* means "to form," so *diplason* means "forming two wholes," or "double," or "two-fold" or "twice as much."

3:2 Hemiolios

The prefix *hemi-* means "half," and *olios* means "exceeding by," so *hemiolios* means "exceeding a whole by a half" or "containing one and a half" or "half as much again."

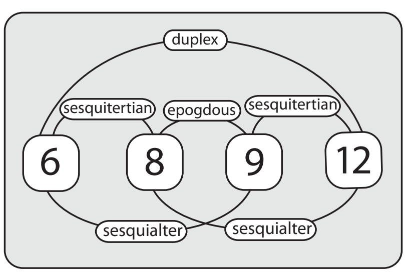
4:3 Epitritos

The prefix *epi-* means "upon," and *tritos* means "a third," so *epitritos* means "a third upon a whole" or "one more than three" or "one and a third."

9:8 Epogdous

The prefix *ep-* means upon, and *-ogdous* means an eighth, so *epogdous* means "an eighth upon a whole" or "containing a whole and an eighth."

Boethius' terms for these ratios



Boethius' terms are essentially the Latin words for Nicomachus' Greek terms.

2:1 Duplex

Duplex means "double," or "two-fold".

3:2 Sesquialter

The prefix *sesqui* is like the Greek prefix *epi* meaning "upon," and *alter* means "second," so *sesquialter* means "one more upon 2" or "the ratio of 3:2."

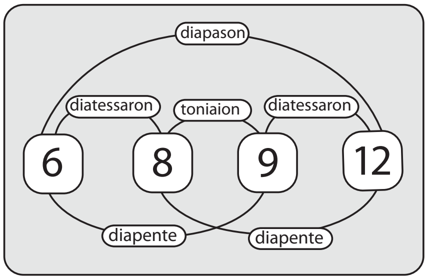
4:3 Sesquitertia

Like the Greek word *epitritos*, *sesqui + tertia* means "one more upon three," or the ratio of 4:3.

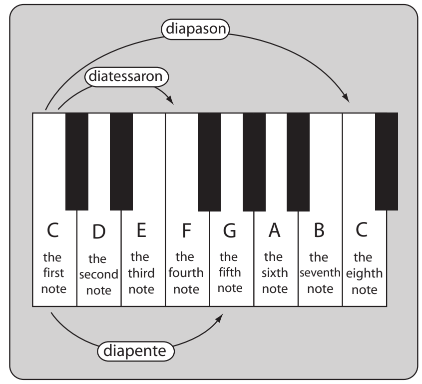
9:8 Epogdous

Boethius simply used Nicomachus' Greek word to describe this ratio of 9:8.

Nicomachus' and Boethius' names for these ratios in Music.



Nicomachus realized how important these ratios are in music, so he also provides they were called in Greek music.



The meaning of these musical terms can most easily be seen by looking at an octave on a keyboard.

2:1 Diapason

The prefix *dia*- means "across, through or between," and pason means "a whole," so *diapason* means "across a whole," or "a whole octave."

3:2 Diapente

Pente means "five," so *diapente* means "across five notes" or "a perfect fifth" or simply "a fifth," as it is called in music.

4:3 Diatesseron

Tesseron means "four," so *diatesseron* means "across 4 notes" or "a perfect fourth" or simply "a fourth," as it is called in music.

9:8 Toniaion

Tonos literally means "a stretching," but it also means the "measure or meter" of music. As Nicomachus explains, the *toniaion* is the common measure of all the ratios in music. The relationship of any of these piano keys to its next-door neighbor is a *toniaion*.

Boethius' used all of Nicomachus' Greek musical terms except he shortened *toniaion* to the Latin word *tonus,* from which we get the word "tone."

Nicomachus and Boethius both explain how the ideas of arithmetic proportion, geometric proportion, and harmonic proportion can be seen in the relationships between these four numbers, 6, 8, 9, and 12.

Pythagoras and the Blacksmith Shop

In Chapter 6 Nicomachus relates the story of Pythagoras hearing the sounds from the blacksmith beating out iron on the anvil with hammers of various weights.

When Pythagoras returned home he put a long piece of wood diagonally between two walls so it would be solid. Then he hung 4 weights each on strings of equal length. The weights were in the proportion 6, 8, 9, and 12. By plucking two weighted strings, simultaneously he found the various consonances or harmonic relationships in sound.

Nicomachus was the first person to write about Pythagoras' "blacksmith sounds" story. It has been passed down to us through later writers like Iamblicus, Boethius and Isidore of Seville. Even Handel's *Harpsichord Suite No. V.* is known as "*The Harmonious Blacksmith*."

However, there are two major problems with Nicomachus' account.

First, differently weighted hammers do not make different sounds when smacked on the same anvil. The sounds are the same. Percussion depends on the object being stuck (the anvil size) not the strikers (the hammers).

However, Nicomachus does say that Pythagoras did more tests on "percussion on plates,"and variously-sized plates, pans, cymbals, or bells **will** give varied sounds. One of Pythagoras' followers, Hippasus of Metapontum, apparently did use 4 discs in his experiments. Their diameters were the same, but their thicknesses had proportions of 2:1, 3:2, and 4:3.

The second flaw is in the lengths of the plucked strings strings that Pythagoras was purported to have used to make various sounds. When Ptolemy (around 150 AD) tried to recreate Pythagoras' experiment, he found the sounds from the various plucked strings differed, but not in the ratio of the weights. As the French scholar Théodore Martin discovered in the 1800's, tension must be squared to double frequency. In other words, to raise the pitch of the 6 weight, another string of equal length must have a 36 weight on it (not "double" or a 12 weight). (Levin, p. 93).

Despite the fact that Pythagoras' two tests are acoustically inaccurate, they demonstrate a key Pythagorean concept: numbers were conceived as having material substance. Pythagoras even used the term *ongkos* which means "mass, bulk or volume" to describe numerical units. (An *ongkolothos* is a "large block of stone."). (Levin, P. 94).

Indeed, even Nicomachus used words denoting weight in his treatment of Pythagoras' blacksmith story:

bare – to weigh down
brithos – to be heavy
holke – weight, pull (a Greek cargo ship was called a holka," from which we get our moniker "The Incredible Hulk.")
stathman – weight
sekoma – lifting or raising
(Levin, p. 93 and *Liddell/Scott Greek Lexicon*)

This does not mean that Pythagoras was a hoaxer or Nicomacus was a liar. Nine hundred years had passed between their two lives, and communication transfer was not what it is these days. Nicomachus was probably relating "the folk tale" exactly as it had reached him. Indeed, even Iamblichus and Boethius later ignore these inconsistencies as they retell Nicomachus' version of the tale.

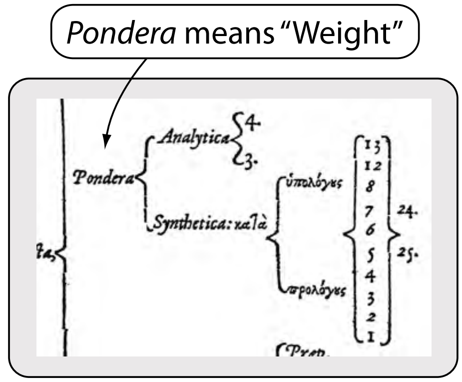
Despite some incorrect details, the basic concept the "musical" ratios of weights is correct. If Pythagoras used various bells or metal discs or even 4 different anvils, the "musical" ratios of weights would be correct. If Pythagoras used various string lengths (instead of weights on the same length strings) the "musical" ratios of weight would be correct.

As J. Burnet puts it in Early Greek Philosophy, "**they are not stories which any Greek mathematician could possibly have invented, but popular tales bearing witness to the existence of a real tradition that Pythagoras was the author of this momentous discovery.**"
(Burnett, *Early Greek Philosophy*, p. 107 in Levin p. 87 and note 5, p. 95).

John Dee and Pythagoras' musical ratios

Dee doesn't overtly discuss music in the *Monas*, but he hints about Pythagoras' "weights" in his Artifical Quaternary chart. That category where he shows the ratio 4:3, and suggests the ratio 3:4 (with the words *upologous* and *prologous*) is called **Pondera**, meaning weight.

As we've just seen, Nicomachus treatment of Pythagoras' blacksmith story is filled with references to weight.

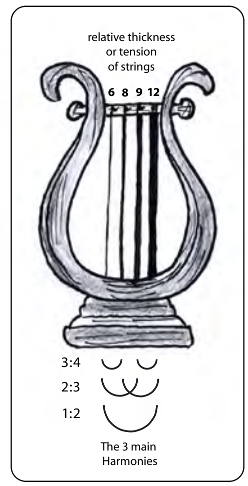


Dee was keenly aware the connection between musical harmony and mathematical harmony, as is evident in Aphorism 11 of his *Propaedeumata Aphoristica*:

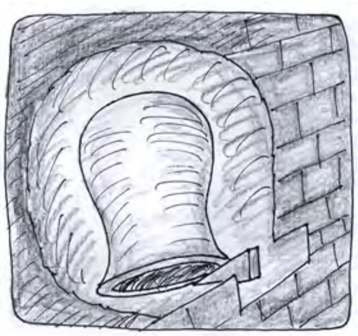
"**The entire universe is like a lyre tuned by some excellent artificer, whose strings are separate species of the universal whole. Anyone who knew how to touch these dexterously and make them vibrate would draw forth marvelous harmonies.**"

(Schumaker, p. 127)

A simple way to visualize the "most perfect harmonies" might be with 4 strings if various thicknesses.



Also, in describing the "Arte of Architecture" in the *Preface to Euclid*, Dee says the "**Brass Vessels**" distributed throughout theaters for acoustical purposes are arragned according to:

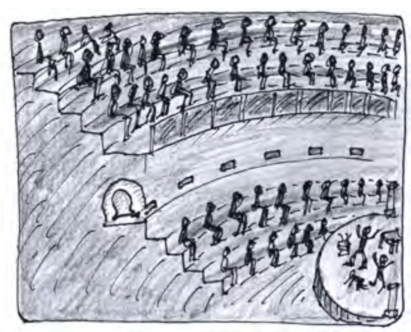


Greek "sounding vessel", as described by Vitruvius

"**Musical Symphonics and Harmonies, being distributed in the Circuits by Diatessaron, Diapente and Diapason.**"

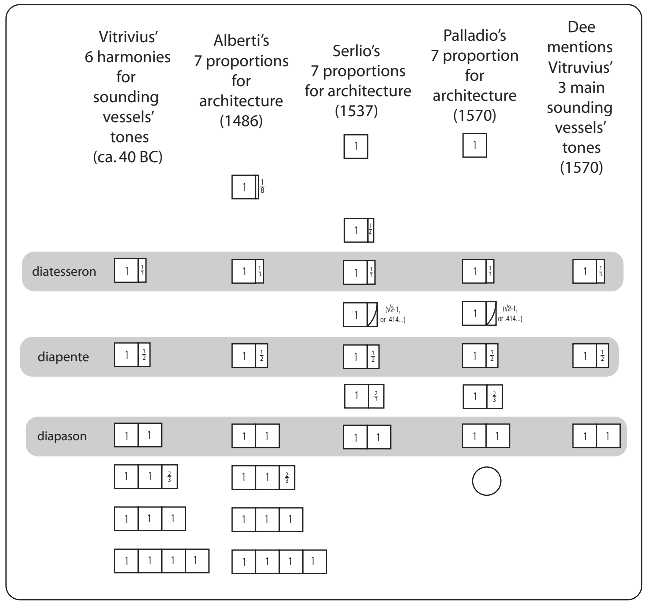
(Dee, *Preface*, p. d.iij verso)

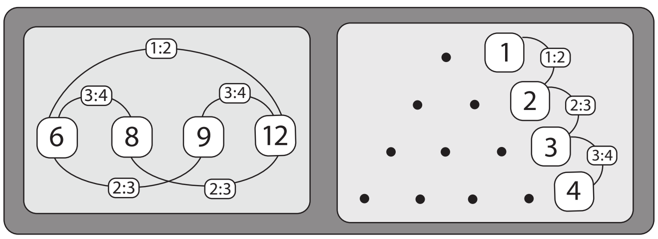
These are among there commendations for brass acoustic vessels that Vitruvius made in his book *On Architecture* (ca. 40 BC). Vessels that amplified various tones were placed in small cave-like chambers in appropriate places in a theater, in accordace to these 3 main musical ratios.



The "sounding vessels" were placed in small chambers scattered throughout the theater to improve acoustics.

Dee was hardly alone in his enthusiasm for the ratios 1:2, 2:3, and 3:4. The famed architects Leon Battista Alberti, Sebastiano Serlio and Andrea Palladio all described them as among the most pleasing ratios for the dimensions of rooms.





To summarize, the numbes 6, 8, 9, and 12 incorporate the 3 key ratios, but the essence of the ratios is best expressed by simply comparing the rows of the tetraktys.

As the philosopher and historian Empiricus Sextus, (who lived around 225 AD) puts it:

"The Pythogoreans are accustomed to say
'All things are like numbers'
and sometimes to swear this most potent oath:

'Nay by him that gave to us the Tetraktys,
which contains the fount and root of ever-flowing nature'…
as the whole universe is arranged according to attunement...
a system of three concords the fourth, the fifth, and the octave
and of these proportions are found the four numbers just mentioned –
in one, two, three and four."

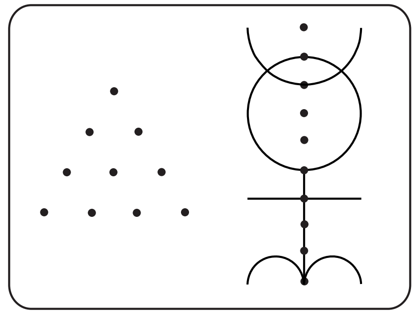
(Sextus Empiricus, Advanced Mathematics, VII p. 94-5;
in Kirk, Raven, Scholfield, *The Presocratic Philosophers* p. 233)
(Dee owned Sextus book, in 1569)

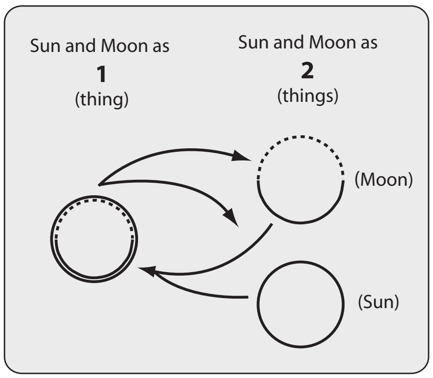
Sextus' actual Greek words describing the Tetraktys are: *pêgên genaou physeos rizôma + exousan,* meaning "which contains the **fount and root** of ever-flowing nature."

The word "pêgên" means a spring where water gushes forth. The word "rizôma" (from which we get the word rhyizome) means root or source of origin, like the root of a tree or a hair or a fingernail

The related Latin expression “fons et origio” (fount and origin) has also been used to describe “one,” or the “unit” or the “Monad.” But this is not contradictory, because Dee saw 10 as a return to 1.

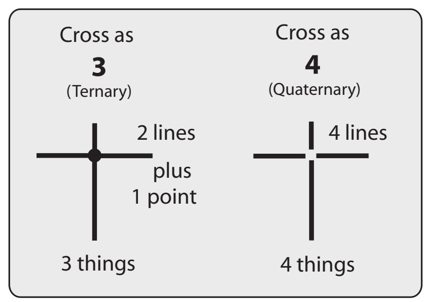
Like the tetraktys, the Monas symbol emphasizes 10 points (on its spine). Thus, we should expect the Monas symbol to express the 3 key harmonies as well.



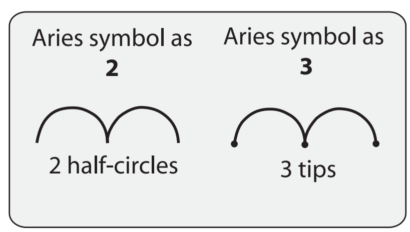


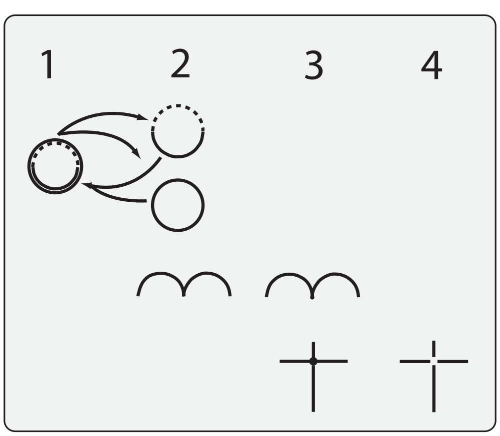
The ratio of 1:2 is like the Sun and the Moon, two things being transformed into one.

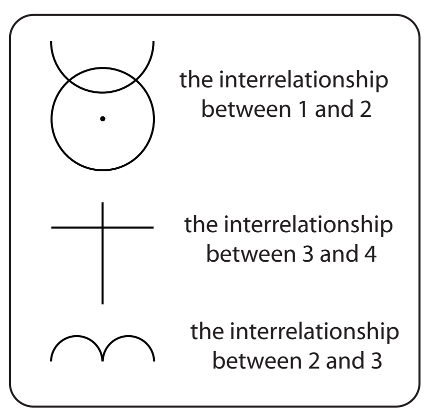
In Theorems 6 and 20, Dee makes a big deal about the Cross being either Ternary or Quaternary, so it might be seen as the ratio 3:4.



And the Aries symbol is made from two half circles, but in Theorem 21, Dee emphasizes its “3 tips.” Thus, it might be seen as the ratio 2:3.

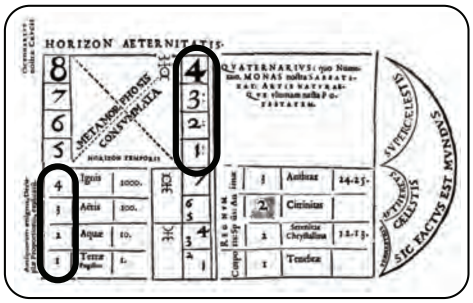






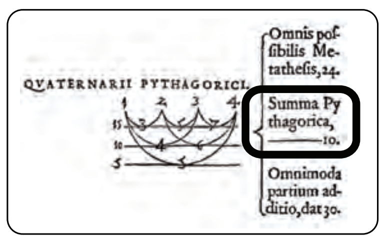
Can you find the 3 key harmonies hidden in the “Thus the World Was Created” chart?

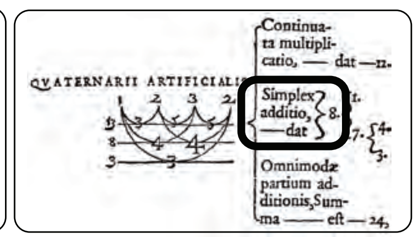
We have investigated most of Dee’s “Thus the World Was Created” chart, but there are some small clues yet to be explored. The 1, 2, 3, 4 in the “Below” half of the chart clearly refer to the Pythagorean quaternary. The 1, 2, 3, 4 in the “Above” half of the chart are larger, they are engraved (as opposed to typeset) and they comprise the first half of the octave. But the 4 digits are in the **exact same size boxes** as those in the “Below” half.



The engraved digits 1:, 2:, and 3: have colons next to them, but the 4 does not. To me this is Dee’s way of visually suggesting the series of proportions, 1:2, 2:3, and 3:4.

Another small clue can be seen by comparing the additive results of the Pythagorean and Artificial Quaternaries in Theorem 23.

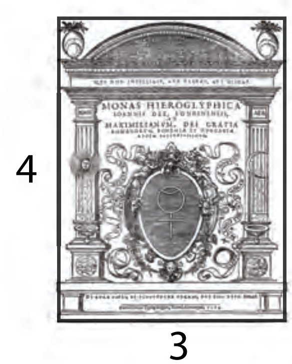




In Dee’s Artificial Quaternary he writes “Simple addition yields 8,” but in the Pythagorean Quaternary he writes “The Pythagorean Sum 10,” hinting at the Pythagorean tetraktys.

Knowing how Dee felt about Number and Geometry being sisters, I measured the various illustrations proportions of the illustrations in the *Monas* in search of these 3 key harmonies.

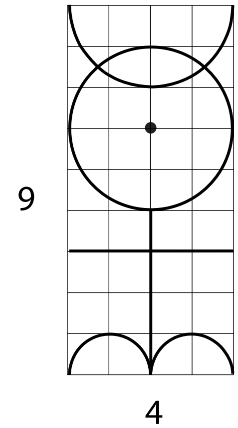
The architecture on the Title Page, measured approximately 7-1/8" tall by 5-3/8" wide, which is the ratio of **4:3**. Dee had put a geometrical expression of “Quaternary rests in the Ternary” right in front of the reader’s nose!



The “Inferior Astronomy” diagram (of Theorem 13) and the “36 Boxes” chart (of Theorem 22) were both squares (or the 1:1 ratio). The “Vessels of the Holy Art” diagram (of Theorem 22) was in the proportion of 5:4. Not much luck there.

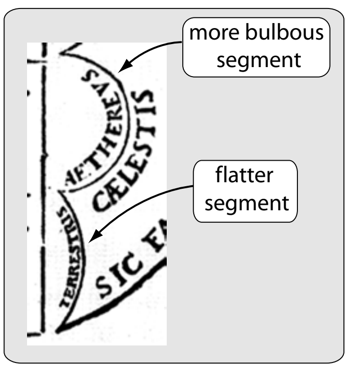
The “Thus the World Was Created” chart has a curved right edge, but the rectangular part of the chart measured approximately 2-5/8" wide by 3-15/16" tall. It was in the **2:3** ratio!

I searched to find that one missing ratio, 2:1. The two rectangular diagrams in Theorem 12 that explains Lunar Mercury were each too long and narrow. The height : width ratio of the Monas symbol was close to 2:1, but actuality its in the 9:4 ratio. (And Dee was a precise geometer.)



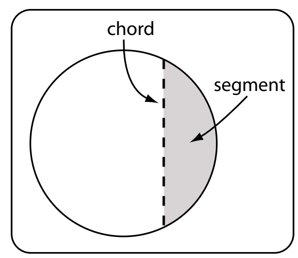
Where could that 2:1 ratio be hiding?
This was an important ratio to Dee.
It was the Sun and the Moon (at full moon).
It was simply two tangent circles.
But yet he doesn't appear to have illustrated
it with a 2:1 geometric rectangle.

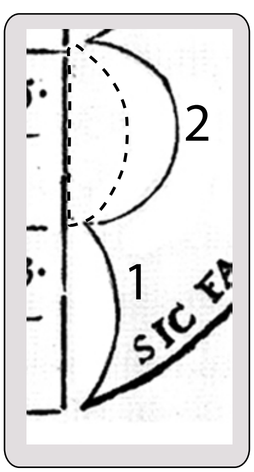
In my searching, I decided to complete the curved brackets in the "Thus The World Was Created" chart. Unfortunately, this didn't seem to lead anywhere.



I had long wondered why the "Terrestrial" bracket was "flatter" than the "Aetheric Celestial" bracket. Then it occurred to me that geometer Dee might be comparing the area **inside** the curved brackets.

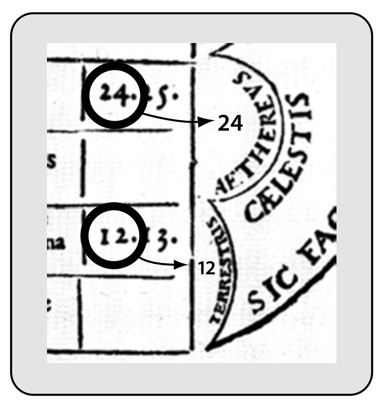
Geometers call this area a circle segment. It is cut off from the rest of the circle by a straight line called a chord.





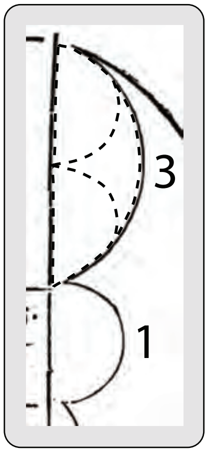
I drew the "Terrestrial" circle segment inside the Aetheric Celestial circle segment. Just eyeballing it, it looked as though two Terrestrial segments would fit nicely.

This 2:1 ratio was echoed by numbers 24 and 12, just to the left.



12
12 x 2 = 24
24 x 3 = 72

If Dee wanted the reader to see the Terrestrial segment as 12 and the Aetheric Celestial segment as 24, the Metamorphosis-minded Dee would no doubt make the Supercelestial segment 72.

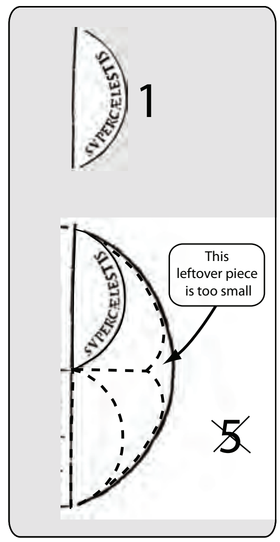


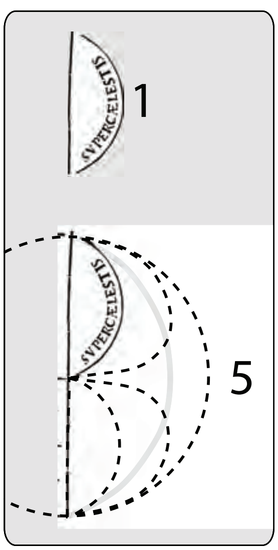
I drew two "Aetheric Celestial segments" in the Supercelestial area. Just eyeballing it, it was pretty obvious that a third "Aetheric Celestial segment" would fit in the remaining area.

Using geometrical area Dee was expressing the numerical Metamorphosis sequence!

I immediately tried to see how if 5 Supercelestial segments would fit in the whole segment labeled "Sic Factus est Mundi."

Unfortunately, four of them would easily fit, but the remainder was just a tiny area, not even close to making a fifth segment.





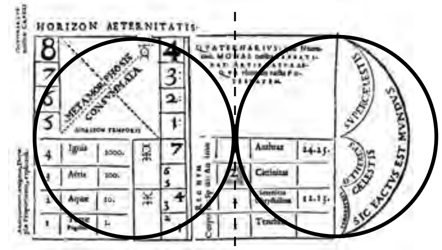
Perhaps Dee was urging the reader to creatively expand this area, so I drew in a semi-circle whose center point was on the right edge of the rectangular part of the chart. Now 5 supercelestials seemed to fit perfectly!

"Ballooning" Dee's illustration this way It seemed strange, but I really was only drawing one of Dee's half-circle Moons.

With a compass I made the half circle Moon into a full circle Moon, then added a "Sun circle"of the same size next to it. Two circles fit perfectly!

Not only that, but the line tangent to the point where the circles touched ran right smack dab through Dee's cherished Artificial Quaternary! Even more special was that it ran right through that Engraved 2.

As that Engraved 2 is what boosts 6126120 to become122252240, these circles might be seen as the 2 wings of what Marshall calls the "Even Greater Eagle"!



The "Even Greater Eagle"
12252240

There is a clue that seems to confirm that Dee intended the reader to "balloon" the large segment to make it a half circle. It would be instantly recognizable to anyone who contemplates Metamorphosis numbers.

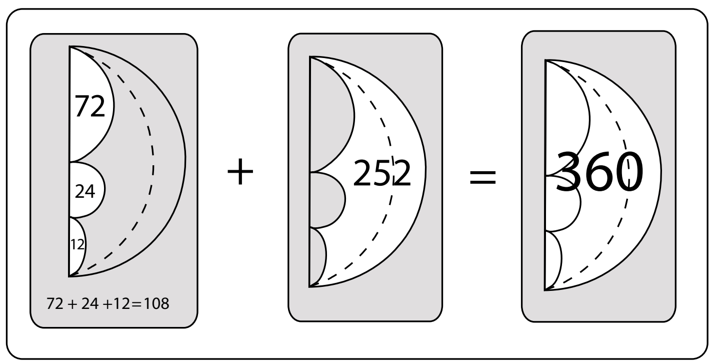
Metamorphosis numbers 12 and 24 have a curious relationship with 72. When added, they sum to 36, which is half of 72.

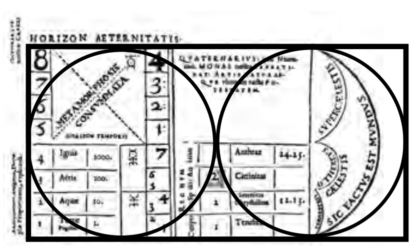
When 12, 24, and 72 are all added, the sum is that special number 108.

And what must be added to 108 in order to reach 360?

252, Dee's Magistral number!

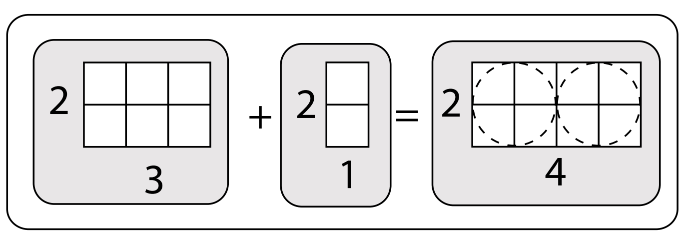
Dee's diagram (once it is "restored") depicts this. The area in the "ballooned" bracket that is not included in the other three smaller brackets is 252!



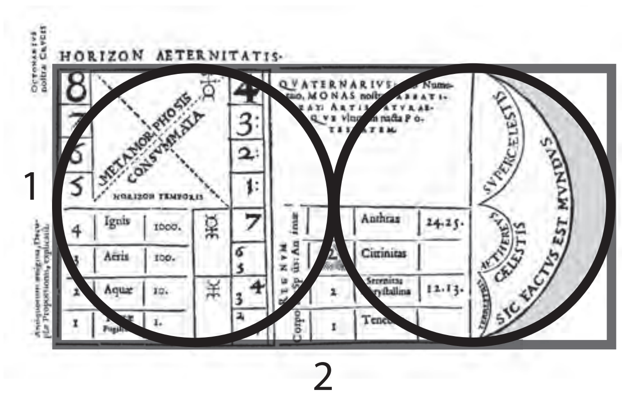


To summarize, if a rectangle is drawn to enclose these two circles, it's obviously in 1:2 ratio (one diameter high by two diameters wide).

Essentially what Dee has done is to add a 2:3 section plus a 2:1 section resulting in aa 2:4 rectangle into which two circles fit perfectly.



The "Ballooned 360 chart"



One might think I have unjustifiably altered Dee's chart to "fit" my thesis. But once its understood how the Metamorphosis numbers are important not only in Dee's math, but in his whole cosmology, you will see that this was his intent.

12 and 24 are "Earthly" numbers

The numbers 12 and 24 bracket the "Below" half of the chart because they are associated with "Earthly" things. The Sun-and-Earth-dance makes for 12 hours of daylight and 12 hours of darkness (totaling 24) on the First of Aries.

72 is a "Heavenly" number

Dee also has a good reason for associating Metamorphosis number 72 with the "Above "half of the chart. For centuries, many theological philosophers have asserted there are 72 Angels in the "Supercelestial" realm.

Medieval Kabbalists found that three important verses in the Book of Exodus (in the Hebrew *Torah)* each contained 72 letters (lines 19, 20, and 21). By putting these three lines next to each other (and reversing the sequence of the second line) they formed the Names of the 72 Angels (or the 72 Names of God.)

The whole assembly of 72 letters x 3 = 216 letters is called the *Shemhamphorasch* (*Shem-ha-Mephorash* means "interpreted name.")

Here are the three verses in English:

“And the angel of God, which went before the camp of Israel,
removed and went behind them; and the pillar of the cloud
went from before their face, and stood behind them:

And it came between the camp of the Egyptians and the camp of Israel;
and it was a cloud and darkness [to them]**,**
but it gave light by night [to these]**:**
so that the one came not near the other all the night.

And Moses stretched out his hand over the sea;
and the LORD caused the sea to go [back] **by a strong east wind all that night,**
and made the sea dry [land]**, and the waters were divided**.”

This chart shows the Latin versions of the Hebrew letters and an English transliteration to show how each name is pronounced. (from Tyson, *Agrippa*, p. 769-81)

The names of the 72 angels in the Shemhamphorasch

Fire Trine		Water Trine		Air Trine		Earth Trine	
1.	VHV : Vehuiah	19.	LVV : Levoiah	37.	ANI : Aniel	55.	MBH : Mabehiah
2.	ILI : Yeliel	20.	PHL : Paheliah	38.	ChAaM : Chaumiah	56.	PVI : Poïel
3.	SIT : Sitael	21.	NLK : Nelakel	39.	RHAa : Rehauel	57.	NMM : Nememiah
4.	AaLM : Aulemiah	22.	III : Yiaiel	40.	IIZ : Yeizel	58.	IIL : Yeilel
5.	MHSh : Mahasiah	23.	MLH : Melahel	41.	HHH : Hahahel	59.	HRCh : Harachel
6.	LLH : Lelahel	24.	ChHV : Chahuiah	42.	MIK : Mikael	60.	MTzR : Metzerel
7.	AKA : Akaiah	25.	NThH : Nethahiah	43.	VVL : Vevaliah	61.	VMB : Umabel
8.	KHTh : Kahathel	26.	HAA : Haaiah	44.	YLH : Yelahiah	62.	IHH : Yehahel
9.	HZI : Heziel	27.	IRTh : Yerathel	45.	SAL : Saeliah	63.	AaNV : Aunuel
10.	ALD : Eladiah	28.	ShAH : Sheahiah	46.	AaRI : Auriel	64.	MChI : Mechiel
11.	LAV : Laviah	29.	RII : Riyiel	47.	AaShL : Aushaliah	65.	DMB : Damebiah
12.	HHAa : Hahauah	30.	AVM : Aumel	48.	MIH : Miahel	66.	MNQ : Menaqel
13.	IZL : Yezalel	31.	LKB : Lekabel	49.	VHV : Vehuel	67.	AIAa : Aiauel
14.	MBH : Mebahel	32.	VShR : Vesheriah	50.	DNI : Daniel	68.	ChBV : Chebuiah
15.	HRI : Hariel	33.	IchV : Yechoiah	51.	HChSh : Hachashiah	69.	RAH : Raahel
16.	HQM : Haqemiah	34.	LHCh : Lehachiah	52.	AaMM : Aumemiah	70.	IBM : Yebemiah
17.	LAV : Leviah	35.	KVQ : Keveqiah	53.	NNA : Nanael	71.	HII : Haïaiel
18.	KLI : Keliel	36.	MND : Menadel	54.	NITh : Neithel	72.	MVM : Moumiah

360 is a “Creation” number

Dee has good reason to use the number 360 to bracket “everything” in his “Thus the World Was Created” chart. It’s the number of degrees in a circle. It’s the number the ancients used for the number of days in a year.

252 is closely related to 2520

The transpalindromic mate of 2520 is 0252, or Dee’s Magistral number. The number 2520 is not only Metamorphosis number, it’s a special one, as it’s the lowest number divisible by all the single digits. It’s Dee’s “Sabbatizat,” as 7 years times 360 days=2520 days.

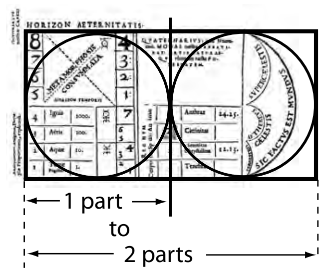
(A tantalizing clue: I wll show later that Dee has supporting authority about the importance of 360 from the man he calls “the greatest philosopher.”)

To summarize, 12, 24, 72, 360, and 2520 are key numbers, not only because they are Metamorphosis numbers, but because they are part of Dee’s view of the Universe.

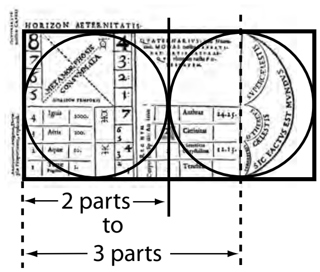
The 3 harmonies in the "Ballooned 360" chart

Having found that the Artificial Quaternary is the midline of the "Ballooned 360" chart, all 3 of the key harmonies can be found by simply by comparing various widths:

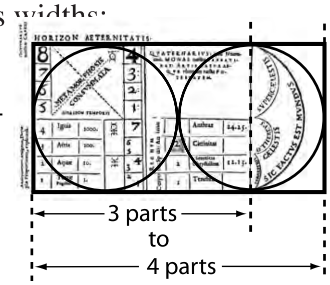
1:2



2:3



3:4



Now that we have the 3 ratios so special to Pythagoras, Nicomachus, and Boethius, the question becomes: What does this have to do with the Tower?

To get a better grasp of Dee's intent, let's first look at what he wrote about ratios in his other books.

Dee was the First Mathematician to use the colon (:) *to represent proportion*

Besides adding his own theorems, lemmas, and corollaries throughout this first English translation of Euclid's *Elements*, it appears as though Dee wrote all the introductions and Definitions for all the chapters (called "books") as well. Most of the introductions are rather short, but those for *Book 5* and *Book 10* are a little longer than the rest.

In *Book 10*, the discussion of "points" and "units" is clearly Dee's writing (and not that of Henry Billingsley). He describes the difference between numbers and lines using the ideas of the "unit" and the "point." Basically Dee asserts there are a **finite** amount of "unities" (ones) contained in a number, but there are an **infinite** amount of points in a line.

We've seen that Dee quoted the "great and Godly Philosopher" Boethius in the *Preface to Euclid,* where he makes reference to the Exemplar Number. He also cites him in he introduction to *Book 5* of Euclid.

"This fifth book of Euclid is of very great commoditie and use in all Geometry.

Of all the books, it should be thoroughly and most perfectly and readily known.
For nothing in the books following can be understood without it,
the Knowledge of them all depend on it.

And not only they[meaning "the books following"] **and other writings on Geometry,**
but all other Sciences and Arts,
like Music, Astronomy, Perspective, Arithmetic,
the art of accounting and reckoning, and others.

Therefore, this book is a chief treasure
and a peculiar jewel much to be accounted of.

**It deals with proportion and Analogy or proportionality,
that pertains not only to lines, figures and bodies of Geometry,
but also of sounds and voices of Music as explained
by Boethius and others who write about Music.**

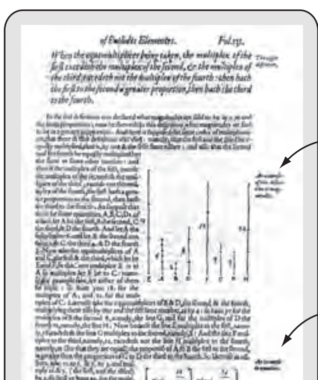
**Also the whole art of Astronomy teaches how to measure proportions of times and motions.
Archimedes and Jordanus** [Jordanus Nemoriarius, ca. 1225-1260] **and others
who write about weights affirm that there is proportion
between weight and weight, and also between place and place.**

**You can thus see how large is the use of this Fifth Book.
Its definitions are common, but here Euclid only applies them to Geometry.
The first author of this book was, as many claim, Eudoxus
who was a student of Plato, but it was later organized by Euclid."**

(Dee, Euclid, Book 5, folio 125 verso)

The third "definition" explains that a ratio is a kind of a size relationship between two magnitudes of the same kind. Dee calls the first "Term" of a ratio the "**antecedent**" and the second "Term" the "**consequent**." But he cites Boethius' and others' use of the terms ***Dux*** and ***Comes***.

Recall that Boethius uses these words in the chapter where he explains Nicomachus' first Greek chart of multiples (up to 100) that formed the various "Gamma and Chi" shapes. They are Boethius' Latin translations of Nicomachus' Greek words *prologous* and *upologous*, which Dee used in his Artificial Quaternary chart.



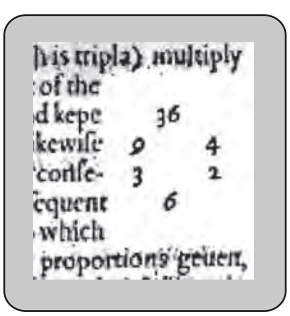
Dee has written in the margin: "An example of this definition in magnitude."

Here he has written: "an example in number"

Throughout the 14 pages of Dee's explanation of Book Five's "Definitions," Dee intersperses illustrated examples of various geometrical proportions (comparing lines of various lengths) with illustrated examples of number proportions. (In the marginalia, Dee writes "***An example in magnitudes.***" or "***An example in number.***" over 20 times. He certainly likes that word "example.")

I've enlarged Dee's illustration of how to **multiply** the ratio 9:3 times 4:2, resulting in 36:6. Here's how we would write it today:

$$\frac{9}{3} \times \frac{4}{2} = \frac{36}{6} = \frac{6}{1}$$

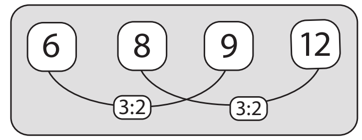


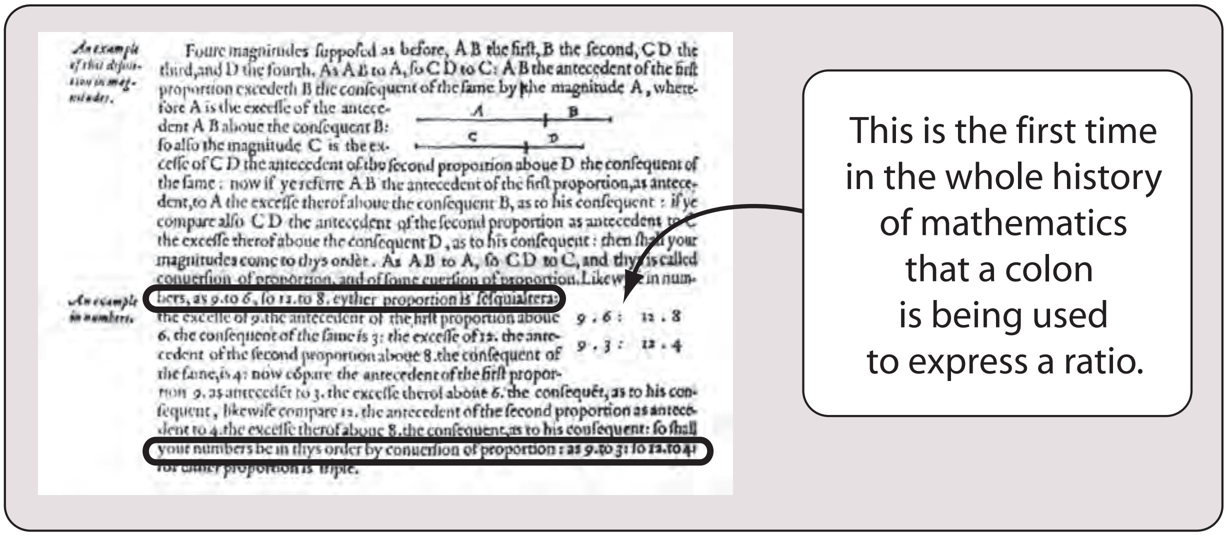
Dee calls this example "bringing together" the proportions of "tripla"and "dupla" to make "sextupla." It's a very basic example how to multiply fractions (numerator times numerator over denominator times denominator).

Dee informs the reader that the usefulness of this procedure is more apparent when multiplying complex fractions or when multiplying three or more fractions.

He also illustrates two examples of **equivalent ratios**.
For the top line, he writes… "as 9. to 6, so 12. to 8."
For the bottom line he writes as … "as 9. to 3, so 12. to 4"

This illustration is revealing for two reasons. First, beause Dee is demonstrating two ratios (9:6 and 12:8) that are in that special proportion of 3:2 (hemiolios or sesquialter or diapente or the width:height of "rectangular part of his "Thus the World Was Created" chart.)




An example of this definition in magnitudes.

Foure magnitudes supposed as before, A B the first, B the second, C D the third, and D the fourth. As A B to A, so C D to C: A B the antecedent of the first proportion exceedeth B the consequent of the same by the magnitude A, wherefore A is the excesse of the antecedent A B aboue the consequent B: so also the magnitude C is the excesse of C D the antecedent of the second proportion aboue D the consequent of the same: now if ye referre A B the antecedent of the first proportion, as antecedent, to A the excesse therof aboue the consequent B, as to his consequent: if ye compare also C D the antecedent of the second proportion as antecedent to C the excesse therof aboue the consequent D, as to his consequent: then shall your magnitudes come to thys order. As A B to A, so C D to C, and thys is called conuersion of proportion, and of some euersion of proportion. Likewise in numbers, as 9. to 6, so 12. to 8. eyther proportion is sesquialtera: the excesse of 9. the antecedent of the first proportion aboue 6. the consequent of the same is 3: the excesse of 12. the antecedent of the second proportion aboue 8. the consequent of the same, is 4: now compare the antecedent of the first proportion 9. as antecedent to 3. the excesse therof aboue 6. the consequent, as to his consequent, likewise compare 12. the antecedent of the second proportion as antecedent to 4. the excesse therof aboue 8. the consequent, as to his consequent: so shall your numbers be in thys order by conuersion of proportion: as 9. to 3: so 12. to 4: for either proportion is triple.

An example in numbers.

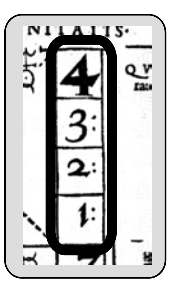
9 . 6 : 12 . 8
9 . 3 : 12 . 4

But just as significant is Dee's use of the colon to compare these two ratios. Florian Cajori in *A History of Mathematical Notation*s explains that William Oughtred (1575-1660) was the first to use a double colon (: :) in his 1631 *Clavis mathematicae* (*Key to Mathematics*), but he adds **"It is possible that Oughtred took the symbols from Dee."**

Cajori adds that this example in *Book 5* **"indicates the origin of these symbols. They are simply the rhetorical marks used in the text."** Cajori pointing specifically to the colons used in Dee's second example shown in the above illustration.

"… this order by conversion of proportion:
as 9. to 3 : so 12. to 4:
for either proportion is triple."
(Florian, p. 168)

Dee doesn't use colons the way exactly the way Oughtred used them or the way mathematicians use them today (for example, 9:6 : : 12:8), but remember, Dee was writing a full half century before Oughtred.



The point here is that Dee appears to be expressing 1:2, 2:3, and 3:4 in his summarizing chart.

This is actually easier for us to see today than it would have been for Dee's contemporaries, as using the colon to express "ratio"was something that Dee himself had devised.

The main point here is: Dee loved ratios.

The "greatest and most perfect harmony" in the Renaissance musical texts of Gaffurio and Zarlino

Dee was not alone in his fascination with the "greatest and most perfect harmony" of Pythagoras, Nicomuchus, and Boethius. Renaissance musicians embraced these proportions in their books on Music Theory.

In this 1518 woodcut from Franchino Gaffurio's (1451-1522) book on Musical Harmony, the ratios made between 6, 8, 9, and 12 have been simplified by the sequence 3, 4, 6 (which are half of 6, 8, and 12).

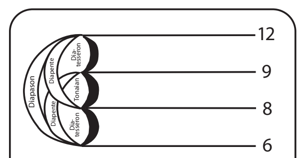
Franchino is lecturing to his students about the three organ pipes on the left.
The first two pipes are in the 3:4 ratio (*diatesseron* or a musical, or perfect fourth).
The last two pipes are in the 4:6, or 2:3 ratio (*diapente* or a musical, perfect fifth).
The first and last are in a 3:6, or 1:2 ratio (*diapason* or a musical octave).
To the right of Franchino is a geometer's compass,
which is associated with three lines of length 3, 4, and 6.


70 Seconda

LIRA DI MERCVRIO

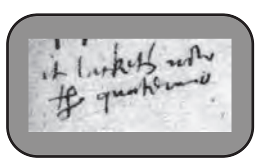
Gioseffo Zarlino (1517-1590) in his *De institutioni harmonice* call the intervals among 6, 8, 9, and 12 the LIRA DI MERCURIO (The LYRE of MERCURY).

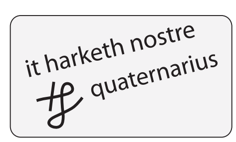
Dee owned 2 copies of Zarlino's book, which was printed in 1571 in Venice.
(Roberts and Watson, 73 and 2116)



My simplification of Zarlino's "Lyre of Mercury"

Even though this was published 7 years after the *Monas*, there appears to be an important clue about it in Dee's 1583 Library Catalog. In the margin next to Zarlino's title Dee wrote "it harketh nostre quaternario." (a curious mix of Englih and Latin that means "it brings to mind our quaternary")





Dee also added a curious symbol that combines a cross and a flowing S shape, which just might be just Dee's personal code abbreviation for the Monas. (He writes something similar 68 entries later, next to Christoff Rudolff's 1571 *Book on Algebra*, which is illustrated with various examples by the numerologist Michael Stifel.)

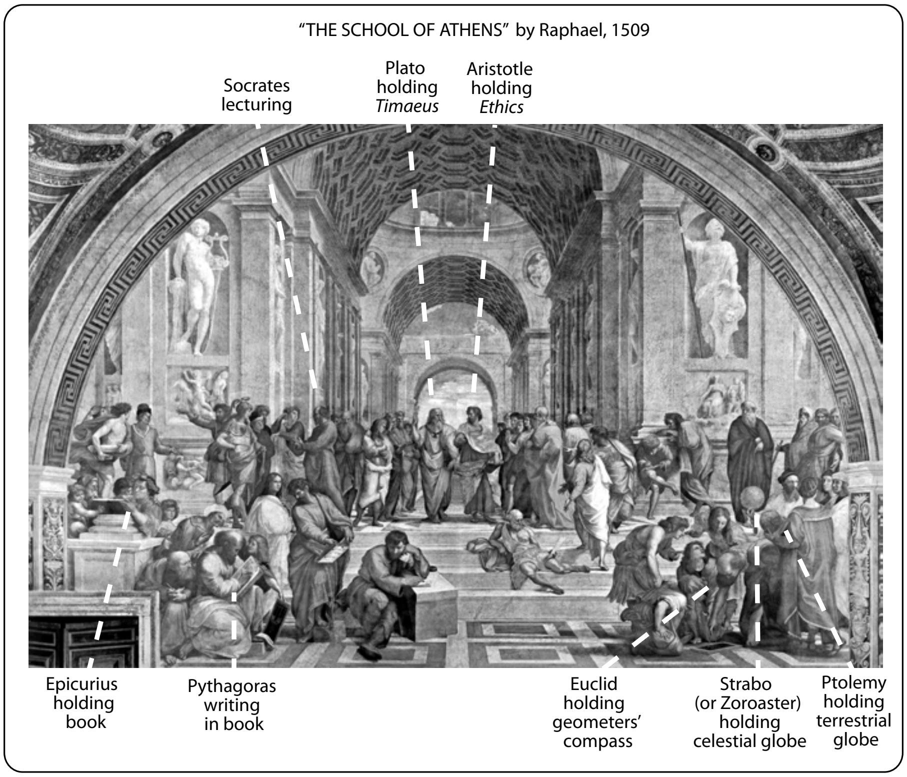
The greatest and most perfect harmony in Raphael's painting, "The School of Athens"

Perhaps the most famous depictions of these 3 key harmonies can be found in a giant fresco painted by Raphael (Sanzio) in the "Room of the Segnatura" in the Vatican. This 26-foot-wide by almost 19-foot-high fresco was originally called *Causarum Cognito* (*Knowledge of the Causes*) but in the 1600's it became known as *The School of Athens*.

Pope Julius II's architect Donato Bramante recommended the talented 27-year-old Raphael, who was from Bramante's hometown of Urbino. The pope was so pleased with the work, he commissioned Raphael to paint the whole papal suite.

Framed by a semicircular arch, the midst of a grand architectural interior, are over 50 robed people in various groupings. Giant marble sculptures of Athena (upper right) and Apollo (holding a lyre in the upper left) hover over the busy scene.

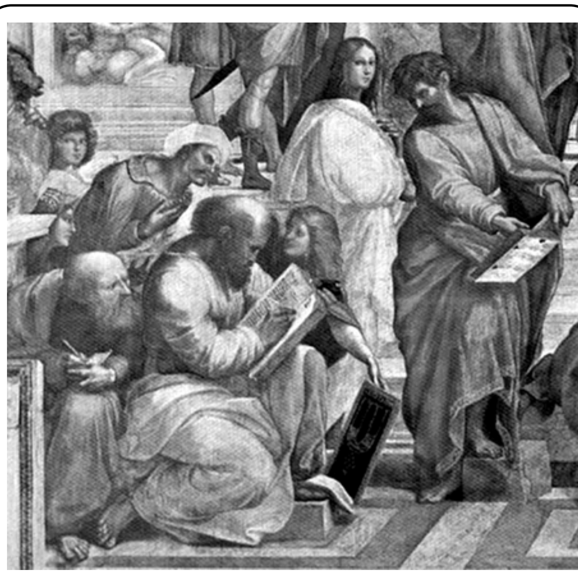
Raphael didn't leave a chart of "who-is-who," but he did leave quite a few props as clues. Many identities are still debated, but most historians agree upon who the central characters of the various groupings are. (See Raphael's "School or Athens" edited by Marcia Hall, Cambridge University Press, 1997).



Starting on the left, the figure holding a book on top of a tall pedestal is considered to be **Epicurius**. Above him and to the right, **Socrates** lectures to a group of men. To his right, the two central figures are **Plato**, (holding his book *Timaeus*) and **Aristotle** (holding his book *Ethics*). In the lower right hand corner, **Strabo** (or perhaps Zoroaster) holds a celestial sphere and **Ptolemy** holds a terrestrial sphere. To their left, **Euclid** (bent over) is drawing a geometric shape with his large metal compass.

Euclid and his group of geometry students in the front right is balanced by a group of arithmeticians in the front left. Let's zoom in for a close-up look.

The central figure here is the seated **Pythagoras**, writing in a large book. He is surrounded by 4 main characters. Another seated old man peers around Pythagoras' elbow apparently copying Pythagoras' work. For the moment, let's call him "Man A."



Above him is a turbaned Arab thought to be Averroës, (1126-1198 AD) an important translator and commentator on Greek Wisdom.

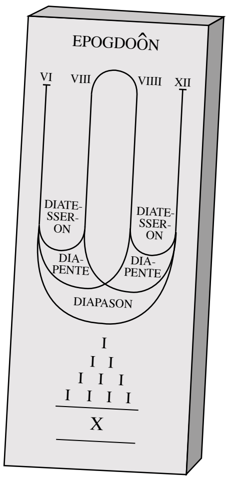
To his right is the great woman geometress, Hypatia of Alexandria, (ca. 360-415 AD), whose father was the noted mathematician Theon. She studied in Athens and later became head of the Platonist school in Alexandria.

To her right is a bearded man pointing to an open book, which he balances on his thigh. He is clearly younger than the balding Pythagoras and the bald "Man A." Let's call him "Man B" for a moment.

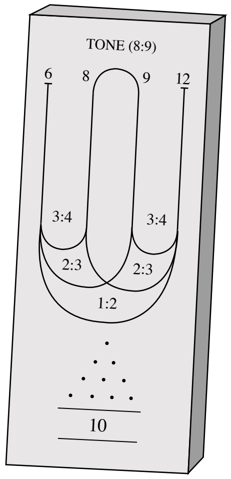
In the midst of these mathematicians is a youth propping up a tablet that rests on the floor.

Zooming in even closer on the tablet, we can see that Raphael has succinctly summarized Nicomachus' and Boethius' "greatest and most perfect harmony," and put Pythagoras' tetraktys underneath it.

close up view of
"the greatest and
most perfect harmony"



(My transcription)



(My simplification)

This is my transcription of the tablet and a simplification using modern numerical terms.

To me it's obvious that the peering old "Man A" is Nicomachus, who compiled what was known about Pythagorean number and music theory in his *Introduction to Arithmetic* (ca. 125 AD).

And the standing, younger, "Man B" is Boethius who translated Nicomachus' work into Latin in his famed *Introduction to Arithmetic* (ca. 525 AD).

The idea that "Man A" is old and "Man B" is young seems to be Raphael's way of showing the 400 year difference between Nicomachus and Boethius. But there's another subtle clue.

Averroes
(Arab translator
of Greek works,
ca.1175 AD)

Hypatia of
Alexandria
(famous woman
mathmetician,
ca. 400 AD)

Boethius
holding his
Introduction to Arithmetic
with 3 book
clasps visible
on the
back cover
(ca. 525 AD)

Nicomachus
peering
(ca. 125 AD)

Pythagoras
writing
(ca. 500 BC)

diagram of
"the greatest
and most perfect
harmony"

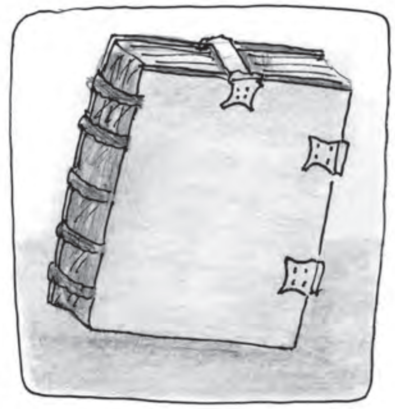
It was Boethius' Latin version, not Nicomachus' Greek version that became the primary mathematical text through the Dark ages, through the Medieval era and into the Renaissance.

If you look closely at Boethius' book you can see the parts of three book clasps on its back cover.

thin rope woven through the holes binds the wooden covers

In Medieval Times, the pages of parchment didn't lay as flat as modern book pages. To remedy this tendency to warp, they were sandwiched between 2 stiff wooden boards that were laced together with cords or thongs.

After these cords were sewn into holes drilled in the wood, everything was covered with leather. These cords are the bumps that stick out on the spine of old books. (Though many more recent books have faux bumps to make them seem old.)



all this was wrapped in leather and clasps were added

The wooden covers were the exact same size as the pages, like our modern paperback books, not larger like they are in modern hardbound books). **Metal clasps**, custom made for the thickness of the book, kept the wooden covers closed and the pages flat.

In the 1100's and 1200's, books were wrapped several times around with a long strap. In the 1300's, these straps were replaced by these hinged clasps and catches attached to the edge of the book.

In the 1400's and 1500's you could tell a book's country of origin from how the clasps were arranged. In England and France, the hinged clasps were on the front of the book with the catches on the book cover.

In Germany and the Netherlands, the hinged clasps were attached to the rear cover with the catches on the front cover.

Italian bindings were like the English and French (hinged clasps on front), but they often used as many as 4 clasps (one on the top, two along the right edge and one on the bottom).

In the early 1500's, when wooden covers were replaced by pasteboard, the clasps could no longer be attached and were slowly phased out. Besides, the smaller books with better paper no longer required mechanical clasps.

(However, the tradition of using clasps for Bibles continued until around 1700. Brass clasps made a brief comeback in the 1800's on Bibles, prayer books, diaries and photograph albums.)

The three catches for clasps on the back of Boethius' book suggest it was made in Italy in the 1400's or 1500's. This is not the type of detail Raphael would put in the hands of an Ancient Greek character. Even though Boethius lived long before the era of clasped books, his text was still a best-seller in Medieval days.

Another clue involves the other text for which Boethius was perhaps even more well-known *Consolation of Philosophy*. This popular theosophical guidebook was often reproduced as a small book with a "girdle" binding. In addition to the clasps to hold it tightly shut, brass mounts attached the wooden boards to the inside of a leather pouch.

The bottom of the pouch had a long tail that terminated in a large "Turk's head" knot (a bulbous knot that looked like a turban). The knot was slipped under one's belt (thus the term girdle) so the book could be easily carried. Whether one was walking on a long journey or even riding a horse, the girdle binding allowed for quick access to inspiration.

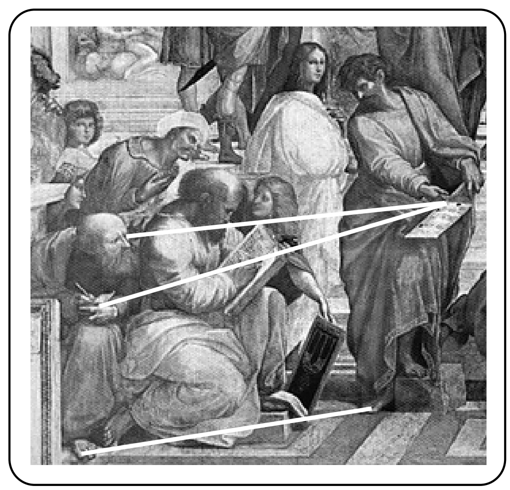
Boethius'
Consolation of Philosophy
with "girdle" binding

To me, this clasp clue confirms that the standing "Man B" is Boethius. But there's another clue that seems to connect the Nicomachus, Pythagoras, and Boethius.

A straight line connects the peering eyes of Nicomachus, the mouth of Pythagoras (he left no written works), and the pointing hand of Boethius.

Another line connects the point where Nicomachus' quill-tip touches his notebook, Pythagoras' quill-tip and the Boethius' hidden fingertip, suggesting they have all written the same thing.

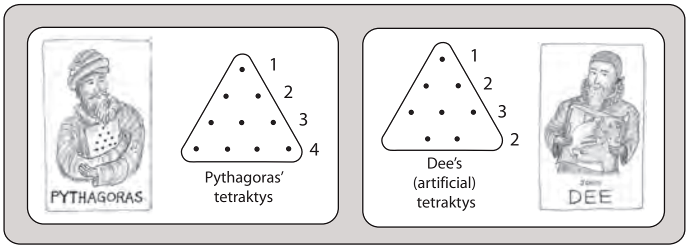
Yet another line connects the prominent big toes of Nicomachus, Pythagoras, and Boethius. (This might seem insignificant, but remember, each of these three men had a decad of toes [toetraktys?]) This line also intersects the corner of the black tablet with the "perfect harmony" written on it.



At the bottom of the tablet is where Raphael put his depiction of the tetraktys along with a giant X. Neither Pythagoras nor Nicomachus would have used an X to depict the number 10. That's a Roman numeral depiction. Among the three, only Boethius would have used an X.

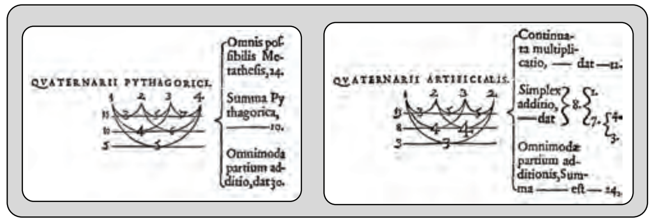
To summarize, not only was the "greatest and most perfect harmony" known to Renaissance scholars, it was revered as a mathematical depiction of Nature. It's so important, it would be surpirising if Dee had **not** included it in his *Monas Hieroglyphica* cosmology. Certainly Dee believed it to be an intrinsic part of how "… the World was Created."

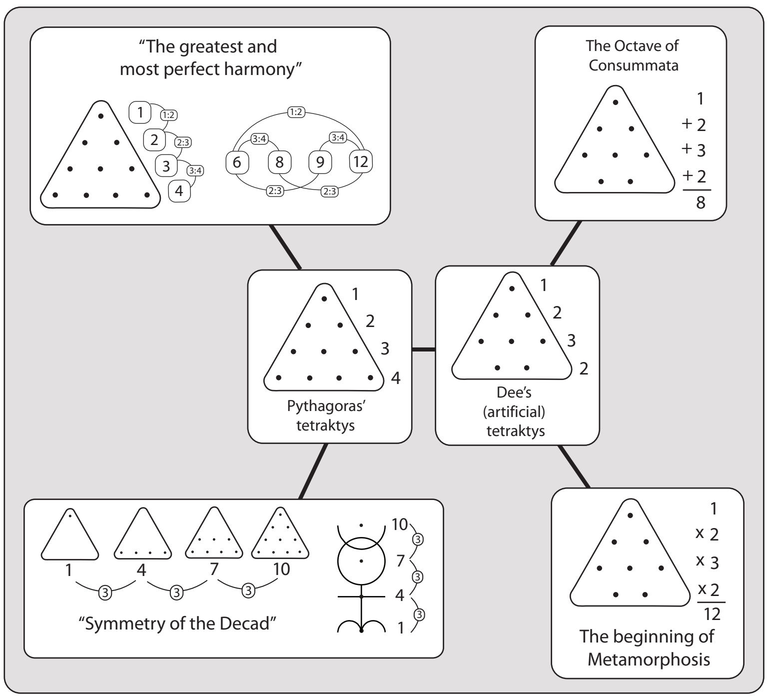
Pythagoras' tetraktys and Dee's tetraktys



Just as Pythagoras envisioned his tetraktys (meaning "fourfold") graphically as an arrangement of dots, it helps to visualize Dee's Artificial Quaternary as an arrangement of dots ("Dee's tetraktys", if you will).

As Dee presents both quaternaries in his Monas Hieroglyphica, he felt they were both important and interrelated.

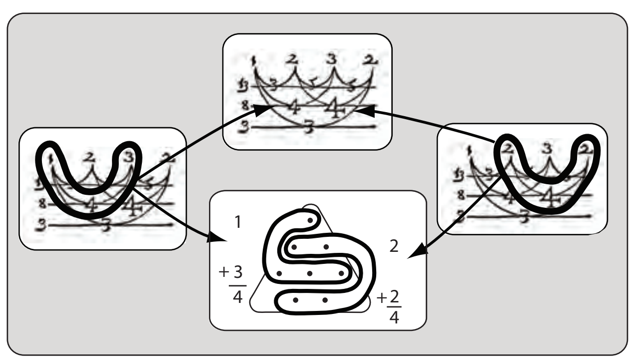


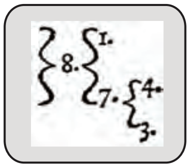


The arrangement of the 10 dots in Pythagoras' tetraktys expresses the "greatest and most perfect harmony" and the Symmetry of the Decad (which the Monas symbol also expresses).

The 8 dots of "Dee's tetraktys" express the octave of Conummata, Or, when the rows are multiplied, they express 12, the first member of Metamorphosis.

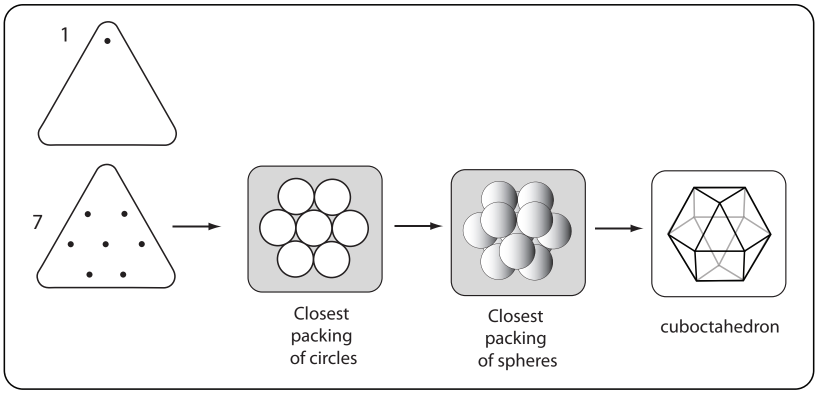
By adding the various rows of "Dee's tetraktys," the "+4, –4" nature of the octave of Consummata can be seen. Dee seems to be making a reference to this by enlarging the "two 4's" in his exposition of the Artificial Quaternary.





In his Artificial Quaternary, Dee divides his result of 8 into 7+1. (This is like Dee's adding the "planets"(7) and a "sharp point"(1) to make Mercurius(8) in the maxim of the flowing ribbons on the Title page.)

Doing the same with the "Dee's tetraktys" leads to a depiction of the closest-packing-of-circles. This natural 2-D arrangement relates to the 3-D closest-packing-of-spheres arrangement, which is a cuboctahedral in shape.



Did Dee really have what I call the "Dee tetraktys" in mind?

Admittedly, Dee does not depicted what I call the "Dee's tetraktys" in the *Monas Hieroglyphica*. But he never depicts Pythagoras' tetraktys either.

In the "Thus the World Was Created" chart, Dee alters the sequence from (1, 2, 3, 2) to (1, 2, 2, 3). However, this was done to bring attention to a different clue, the Engraved 2.

Drawing Dee's two "Quaternaries" as simple dots makes it easier to understand how the various aspects of his cosmology are all woven together. But it's not a great leap for someone who contemplates Dee's assertion that Arithmetic and Geometry are sisters. And all this provides insights into what he is trying to express in the architecture of the John Dee Tower, because in its own way, the Tower tells the Story of "1, 2, 3, 4."

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THE STORY OF 1,2,3,4 IN APHORISM 18 (IN DEE'S PROPAEDEUMATA APHORISTICA)

Aphorism 18 of the *Propaedeumata Aphoristica* nicely
encapsulates the heart of Dee's mathematical cosmology.
It incorporates 2-D and 3-D Geometry with Number
(and even with Latin alphabet letters).

Shumaker called Aphorism 18
"the most inscrutable of all the aphorisms."

(Shumaker, p. 210)

Without a clue as to what Dee is referring to, the Aphorism does indeed sound non-sensical.
But once you understand what his metaphors refer to,
it proves to be a clear, logical, and exciting cosmological assertion.
For purposes of analysis, I've divided my translation of the Aphorism into seven sentences.

**"In each of the four separate, great Wombs of the Larger World
[Majoris Mundi magnus Matricibus] are three different parts.**

**However, at the same time, these parts take form
and are equitably shaped by their own considerations.**

They may be called, by Notaraical design, ȦȮṠ or ȮṠȦ or ṠȮȦ**.
(Pyrologians will understand what I mean)**

**Learn as precisely as possible the natural properties
of these Three and what they produce naturally.**

Learn not only the primary, but also the secondary and tertiary productions.

**And also learn the way of restoring
the tertiary to the secondary, and the secondary to the primary.**

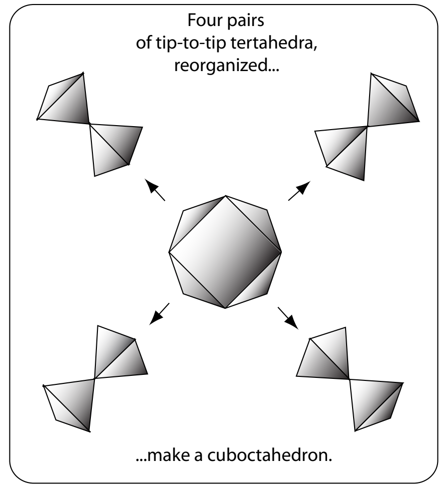
**In the same way, you should give the greatest consideration to why this very same
part may be the cause of not only differing effects, but sometimes of opposing effects."**

(Dee, Aphorism 18, my translation)

Dee starts off with an alliterative assertion:

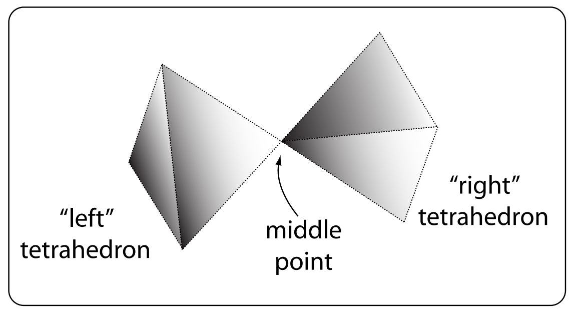
"**In each of the four separate,**
great Wombs of the Larger World
(Majoris Mundi magnis Matricibus)
are three different parts."

Envision each of these "wombs"
as a pair tip to tip tetrahedra.
Four of them assemble into a cuboctahedron.



Let's take one of these 4 Bucky Bowtie
"wombs" and identify its 3 parts.

Here, I have oriented one of them
to show a "left" tetrahedron,
a "middle" point and a "right" tetrahedron.



"However, at the same time, these parts take form
and are equitably shaped by their own considerations."

Dee is hinting that he sees these 3 parts as a representing
something other than simply 2 tetrahedra and their common tip.

"**They may be called, by Notariacal design**
ȦȮṠ or ȮṠȦ or ṠȮȦ
(Pyrologians will understand what I mean)"

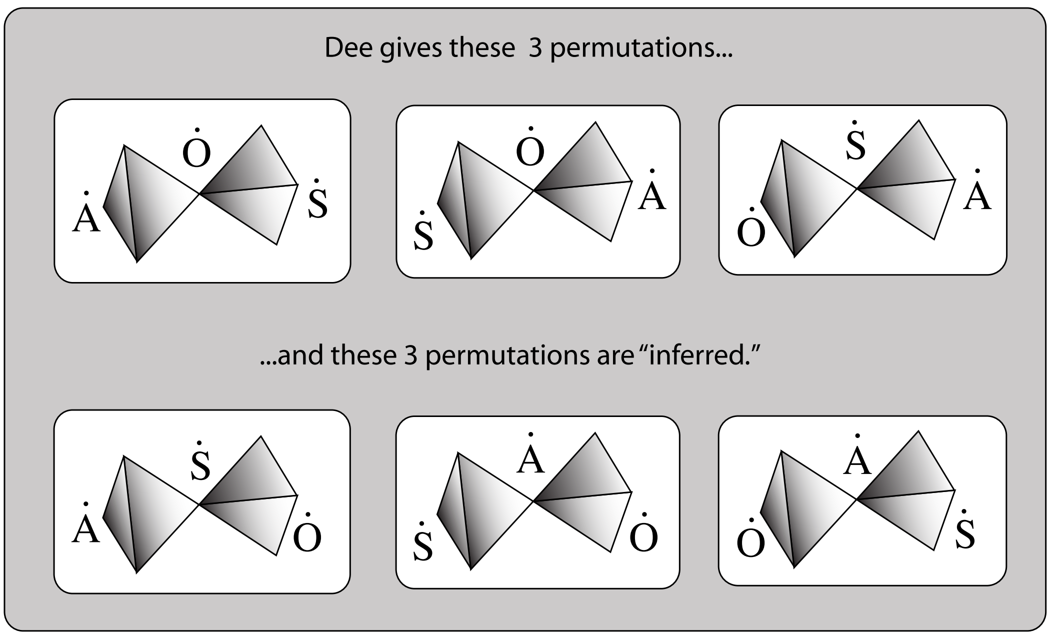
Pyrologians are scholars who are familiar with tetrahedra.
Plato equates a tetrahedron with the element of fire.
In fact, we get our word pyramid from the Greek word "pyr" which means "fire."

Notariacal design means "shorthand" or using one thing to refer to something else
(in this case it is alphabet letters).

An astute mathematician might see something strange
about these three combinations of letters
Ạ, Ọ, and Ṣ
(besides the strange dots on the tops of their heads).

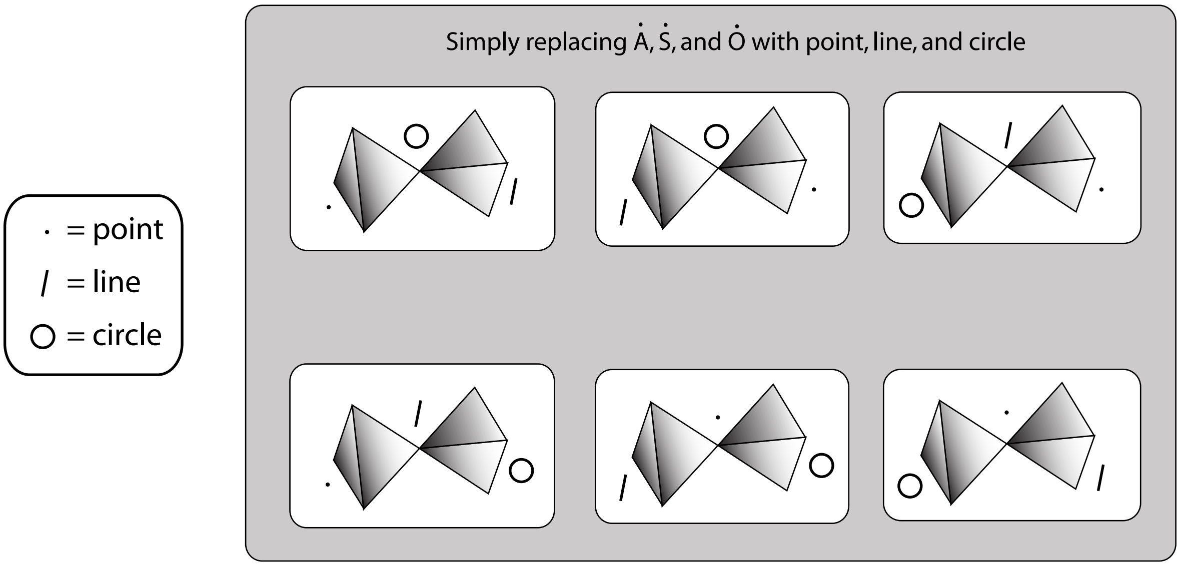
Shumaker noticed it and wrote:
"**any set of three letters yields six permutations, not three.**"
(Shumaker and Heilbron, *John Dee on Astronomy*, p. 212)

The first 3 permutations on the following chart are the ones Dee provides.
The last 3, he doesn't give, but they help fill in the picture of what's going on.



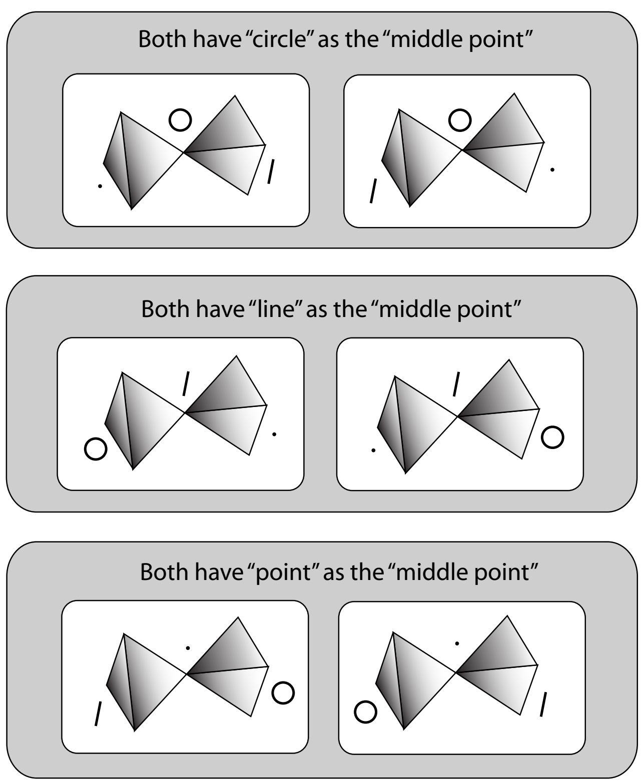
(Don't be confused. I'm not suggesting that now there are
6 "major Wombs of the Larger World",
I am merely analyzing various aspects of one "Womb")

Having previously concluded that Ȧ, Ṡ, and Ȯ stand for point, line, and circle, respectively, the six permutations can be seen this way:

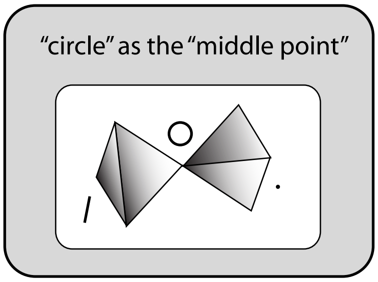


(Perhaps the dot above the letters Ȧ, Ȯ and Ṡ is there to suggest a relationship with "the point"; after all, circles, lines and points are all made from points.)

Notice how these 6 permutations can be seen as three pairs of "reflections".

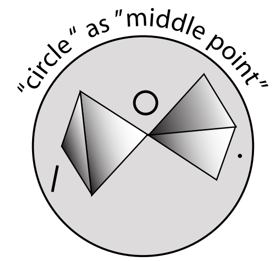


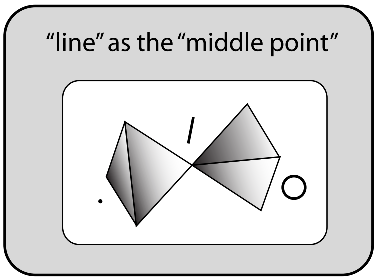
To simplify our analysis, let's take only one representative (just the left ones) from each of these 3 pairs.



This depiction expresses the idea that "the point" and"the line" are opposites, connected via "the circle."

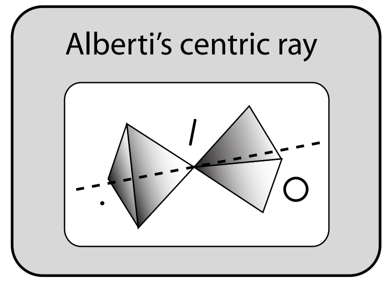
Having the circle in the center position suggests "Forma Circulata," or a "wholeness" that relates point and line.

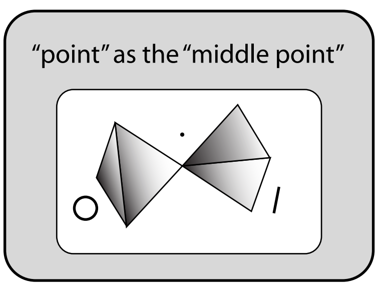




In this representation from the next pair, "the point and "the circle" are opposites and are connected with "the line."

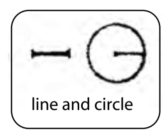
This line might be seen as Alberti's "centric ray" ("the prince of all rays"), which runs as straight through the center of the two tetrahedra as well as the "tip" they have in common.



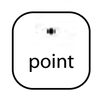


And finally, the most convincing display has "the line" and "the circle" as opposites, connected by "the point."

This corresponds with what Dee says in Theorems 1 and 2 of the *Monas*:

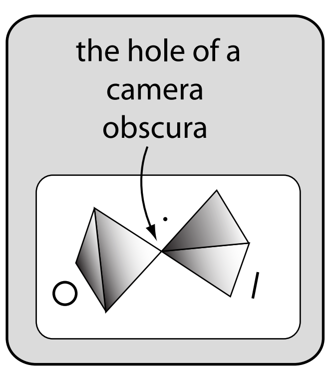


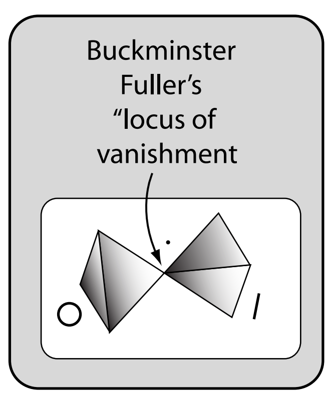
The line and the circle are the "first representations of things"...



...but they "depend" upon the point for their existence.

The idea that the point is in the center of this arrangement corresponds with the idea of hole in a camera obscura or Bucky's "locus of vanishment," the centerpoint of the 4 pairs of tetrahedra that form a cuboctahedron.

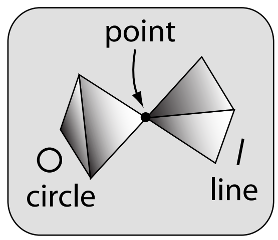




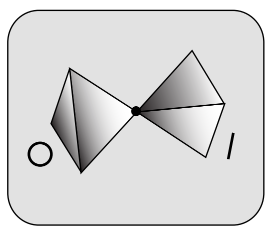
Putting "the point" at the center of the diagram
is the ***most meaningful*** way to see "point, line and circle,"
yet ***it is not*** one of the 3 sample ways that Dee actually gives in Aphorism 18.
In the three permutations he provides (ȦȮṠ or ȮṠȦ or ṠȮȦ),
none of them have an A as the middle letter.
I think Dee was simply being very cryptic here.
It's certainly implied.
(Dee makes a big deal about permutations in Axioms 107 and 108,
as well as in his explanation the Pythagorean Quaternary in Theorem 23)
(Dee frequently uses this literary and graphic technique of
"showing something by not showing it."
He has the clever ability to hide something "just below the surface"
so it's intuitable, yet, to the casual reader, it's invisible.)

You might be wondering how "the circle" and "the line" might be considered "opposites."
Well, he has included another layer of Notariacal design in his work, and it's fairly obvious.

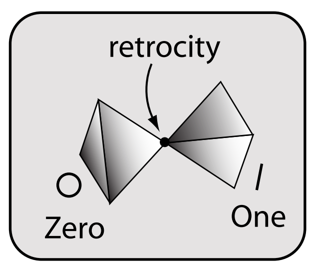
What does this
representation
remind you of?



Hint: here it is
with no labels:



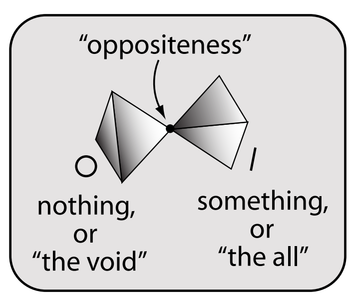
It's a demonstration
of the "zero–one-retrocity"
which Bob Marshall calls the
"Prenumerical Tertiary Singularity,"
the trinity of concepts that combine
to jumpstart the number realm.



"The circle" is the shape of the digit 0.

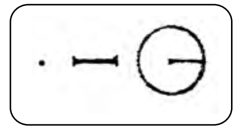
"The line" is the shape of the digit 1.

And "the point" is the idea of "oppositenesss,"
the mathematical function called retrocity!

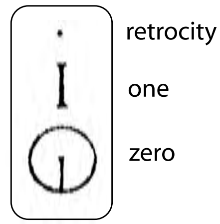


To summarize, this (hidden) ȮȦṠ, by “Notariacal design”
means “circle-point-line,”
which means “zero-retrocity-one.”

This relationship can be seen in Dee’s illustration accompanying Theorem 2 in the *Monas*.



It looks like a simple depiction of a “point, a “line”, and a “circle” (with a radius and a centerpoint).

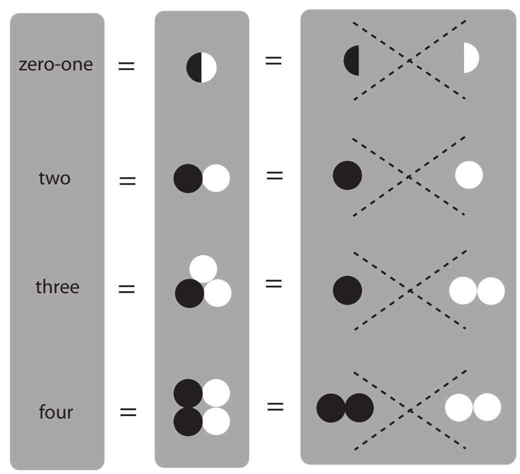


But, rotate it by 90 degrees, and the expressions of zero, one, and retrocity become much more apparent!

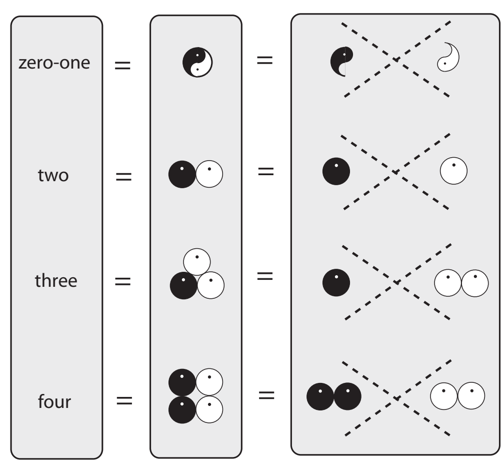
Next, Dee writes:
“**learn as precisely as possible the natural properties of these Three and what they produce naturally**.”

Let’s visually translate “zero-retrocity-one” in terms of black and white discs (or spheres), and review how 2, 3, and 4 get “**produced naturally**.”

(The graphic depiction of “3 discs” here might not seem like a very good representation of retrocity, but there must be retrocity in “asymmetrical three” for it to be able to energize into “symmetrical four.”)



As explained earlier, perhaps it’s more appropriate to show “zero-retrocity-one” as the interswirling halves of the yin-yang symbol, compete with their complementary dots.



Then, Dee recommends:
"**learn not only the primary, but also the secondary and tertiary productions**."

Here, he is referring to "productions" or "yieldings,"
which indicates that there is some sort of "activity" or "process" going on.

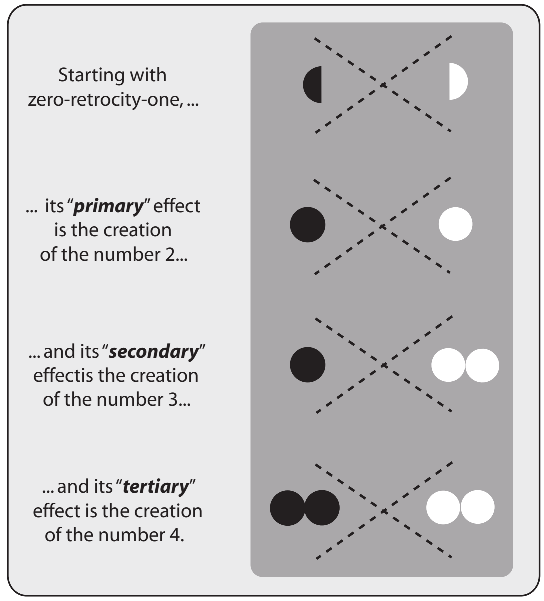
The "primary" effect of "zero–retrocity–one" is to create 2.

("primary", means "first" or "chief")

It might seem absurd to suggest that the word "primary" pertains to "2," but this is exactly what Bucky said: "Unity is plural and at minimum 2."

Dee didn't consider "one" to be a number, but its primary effect (or result or yielding), is the first real number, 2.

(This is related to the idea that 2 is considered the first prime number, despite the fact that it is even, and all the other primes, up into foreversville, are odd.)



The "secondary effect" of "zero–retrocity–one" is to create 3.

The "tertiary effect" of "zero–retrocity–one" is to create 4.

(Note especially that Dee stops at 4.)

Dee uses the Latin words "principales," "secundarios," and "tertios." Note that he does not say "binary, ternary, and quaternary."

These terms mean something quite different to Dee.

1	"One"	"Monas"	"zero-retrocity-one"
2	"Two"	"Binary"	"primary production"
3	"Three"	"Ternary"	"secondary production"
4	"Four"	"Quaternary"	"tertiary production"

Next, Dee writes:
"and also learn the way of restoring the tertiary to the secondary, and the secondary to the primary."

1	"One"	"Monas"	"zero-retrocity-one"
2	"Two"	"Binary"	"primary production"
3	"Three"	"Ternary"	"secondary production"
4	"Four"	"Quaternary"	"tertiary production"

$\frac{2}{3}$ (or $\frac{3}{2}$)

$\frac{3}{4}$ (or $\frac{4}{3}$)

...and the secondary to the primary.

And also learn a way of restoring the tertiary to the secondary...

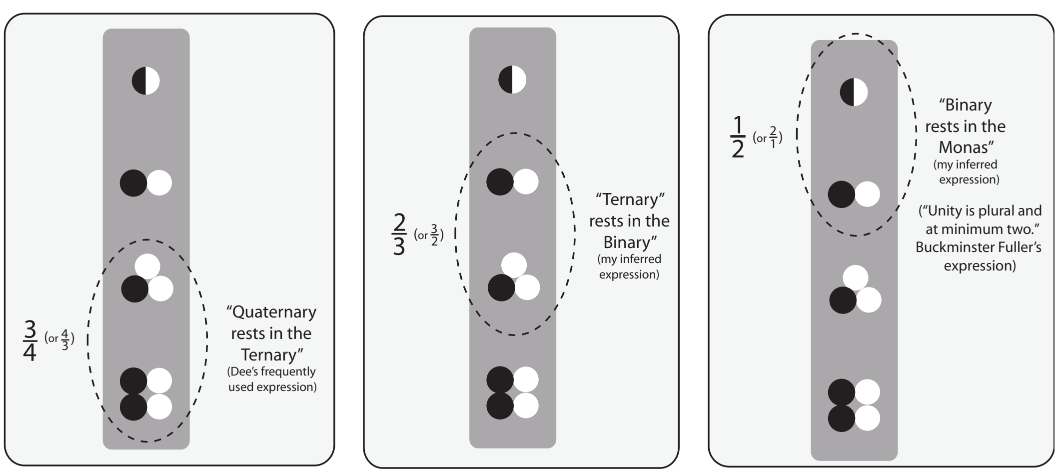
This provides a real clue that we're on the right track.

"Restoring the tertiary to the secondary"here means "restoring" 4 to 3, which is another way of phrasing Dee's ubiquitous declaration "Quaternary rests in the Ternary"!!

His other advice is to "restore" the "secondary to the primary," which means restoring the Ternary (3) back into the Binary (2).

Seen arithmetically this yields those key harmonies of 3:4 (or 4: 3 or diatesseron, or 3/4 or 4/3) and 2: 3 (or 3:2 or diapente, or 2/3, or 3/2).

And the "primary effect" of zero-retrocity-one, is Twoness in itself. This interaction expression of that third great harmony 1: 2 (or 2:1 or diapason, or 1/2 or 2/1).



These "three harmonious" interrelationships among 1, 2, 3,and 4 (that is, 1:2, 2:3, and 3:4) are as vitally important to Dee as they were to the Pythagoreans, the Neoplatonists, Boethius, and many more wise philosophers and mathematicians (and musicians) throughout the centuries.

Dee's final sentence in this Aphorism reads:

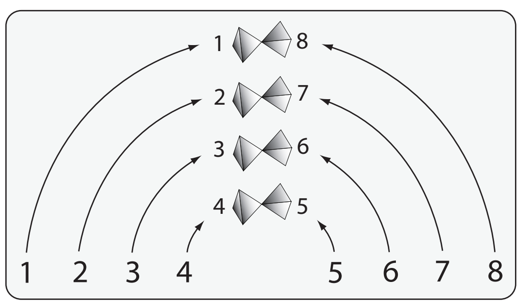
"**In the same way, you should give the greatest consideration to why this very same part may be the cause of not only differing effects, but sometimes of opposing effects.**."

(Dee changed the word "quality" to "part" in his 1568 second edition.
As explained elsewhere, he was trying to eliminate the word "quality" which
had become such an important clue in this 1564 *Monas Hieroglyphica*.)

This "quality" that he is asking us to ponder is the idea of "zero-retrocitiy-one."
He has explained (cryptically) how its energy marches up through 2, 3, and 4.
But, the added power it has accumulated in these brief, yet important steps,
cycles its way through the rest of the realm of numbers in what he calls "Consummata."
Bucky simply called it the "+4, -4, octave; null 9" nature of Number.
Marshall calls it the "Cycloflex."
I call it the "9 Wave/11 Wave, 99 Wave, 1089 Wave…"

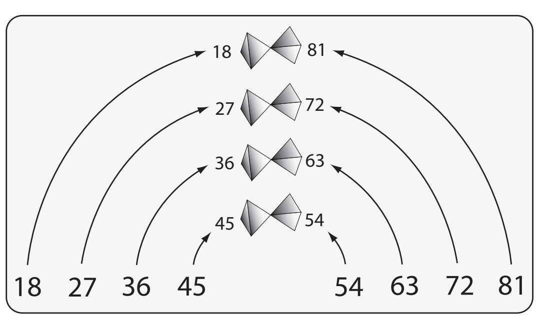
Actually we have now come back around "full Circle"
to the "4 great Wombs of the Larger World)
he mentions at the very beginning of the Aphorism.

If we assign 1, 2, 3, and 4 to the "left tetrahedron
and their counterparts 8, 7, 6, and 5 (respectively)
to the "right" tetrahedron,
we can see retrocity in action!

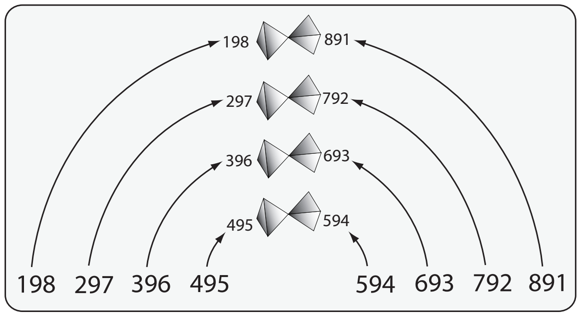


While it's true that these various pairs
like "1 and 8" or "2 and 7" are not transpalindromic mates,
they are essentially "opposites" within the octave of the single digits.

This becomes evident when we see
the energy of retrocity (+4, -4, octave; null 9)
in the "sngle and double-digit" realm of number.
(For example, the 1 and 8 now can be seen in 18 and 81)

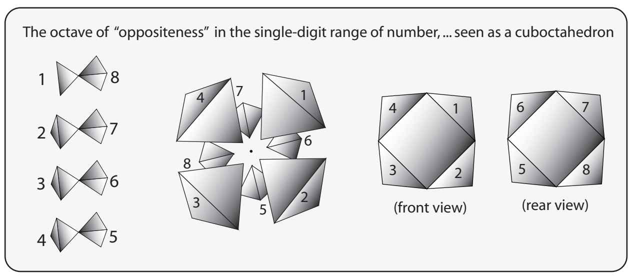


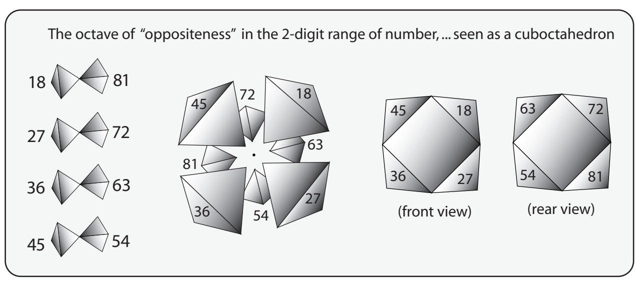
The “oppositeness” can also be seen in the
“+4, -4, octave; null 9” rhythm of the 3-digit range of number.
(Simply take out the “middle nine” of these numbers and you’ll see what I mean)

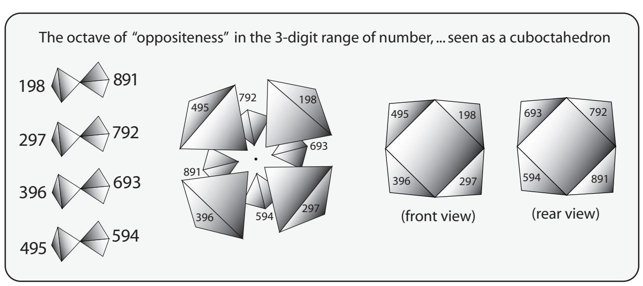


These 4 “characters” infused with “retrocity-power”
are “self-reflective,” thus making an octave, (followed by a null nine).
This energy pattern cycles its way through number, continuously reflecting
back on itself even as it proceeds further onwards (1089 Wave, 10890 Wave, ...).

Any of the octaves just described could be seen as 4 tip to tip tetrahedra,
or, when appropriately joined, a cuboctahedron.

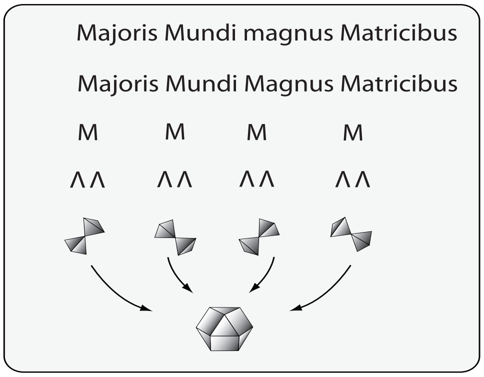






This display of “Consummata” is what he wants us to recognize after
“utmost thoughtful pondering” about the idea of “zero–retrocity-one” being
the “cause” of “oppositeness” that he mentions in the final sentence of the aphorism.

Its reflective nature permeates the realm of of numbers
starting with 2 (primary), 3 (secondary), 4 (tertiary).



Dees alliterative phrase
"**Majoris Mundi magnis Matricibus**"
the "great Wombs of the Larger World"
seems to be a confirming clue here.

The fact that Dee did not capitalize the word *magnis* (great) seems to be to a red herring. Perhaps the word means of the "greatness" implies capitalization.

If this ***was*** a capital M, we might envision "4 capital M's" as "8 inverted V's," representing the "4 pairs of tip to tip tetrahedron" of a cuboctahedron.

This might seem imaginative, but remember, Dee uses a similar cryptic, graphic technique with the alphabet letters in the "36 boxes" chart of Theorem 22.

(Two of those boxes read "Crux," or "Cross," just above a box containing the word Vivificans, with its 2 V's.)

Aphorism 18 is a tasty stew
with a wide variety
of ingredients:

Latin Alphabet Letters
(A,S,O)

2-D Geometry
(point, line, circle)

3-D Geometry
(tetrahedron, cuboctahedron)

Number
(2, 3, 4)

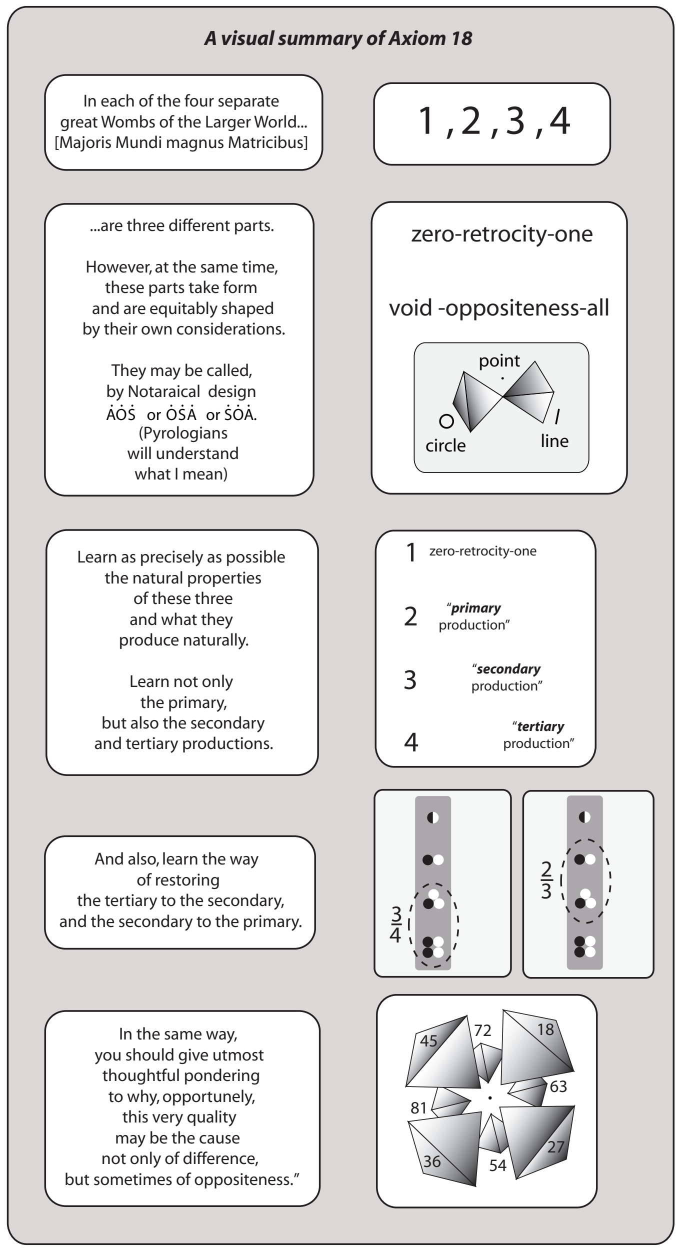
Consummata of Number
(+4, +4, octave; null 9)

Harmonies
(1:2, 2:3, 3:4)

Dee mixes them altogether
skillfully and concisely.

Aphorism 18 is like a mini-version
of the *Monas Hieroglyphica,*
in which all of these concepts are
explained more thoroughly
(yet just as cryptically).

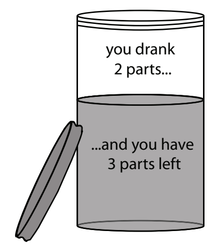
(It's noteworthy tat Dee did not use the term
Majoris Mundi magnis Matricibus in his 1558 edition.
He added it in the 1568 edition)



Fractions and Ratios

(A fraction is a special kind of a ratio)

Imagine you are riding a bike on a hot summer day.
You stop to quench your thirst and guzzle down
2/5 of the water in your bottle.

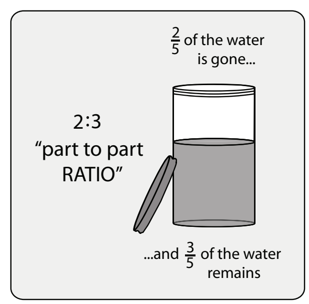


Looking at the side of the bottle,
there are 2 parts empty and 3 parts of water left.
Thus, your empty-to-full water supply is in a 2:3 ratio.

This seems to imply you have 2/3 of your water left.
But this is not the case.

You actually have 3/5 of the water left.
What's going on here?

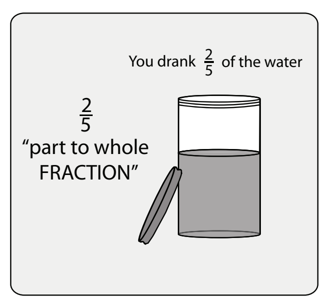
The main kind of ratio I call a
"part to part" ratio.
You drank "2 parts" and there are "3 parts" remaining.



(The "part to part" ratio 2:3 actually implies
a comparison of two fractions, 2/5 and 3/5).

$2 + 3 = 5$

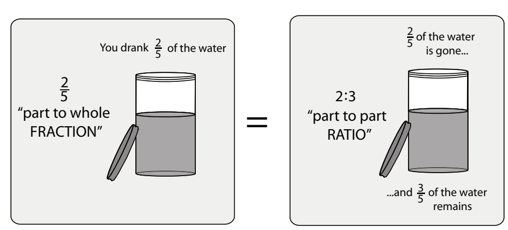
$\frac{2}{5}:\frac{3}{5}$ is the implied relationship of the 2:3 "part to part" ratio



A fraction is a special kind of a ratio
which compares "a part to the whole."
You drank "2 parts" of the "whole"(5 parts), or 2/5 of the bottle.

For clarity, I call this kind of ratio a
"part to whole" fraction.

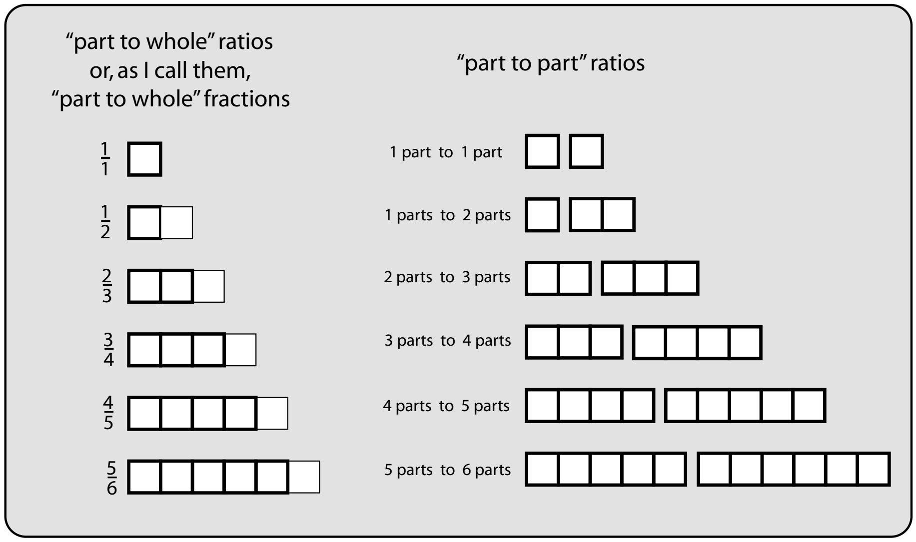
Thus you could express the same quanity of water
in two different ways, both of which are correct.



Remember, a fraction is still *a kind* of a “ratio”, so 2/5 can also be expressed as 2:5.
Sometimes I might refer to a “**part to whole” fraction** as a “ratio.”
But with the expression **“part to part” ratio**, I am never implying a fraction.

As another example, here are some common “ratios, which are quite different animals when seen as **“part to whole” fractions** versus **“part to part” ratios**.

Its ambiguous as to what “kind” of ratios I am referring to here:
1:1
1:2
2:3
3:4
4:5
5:6



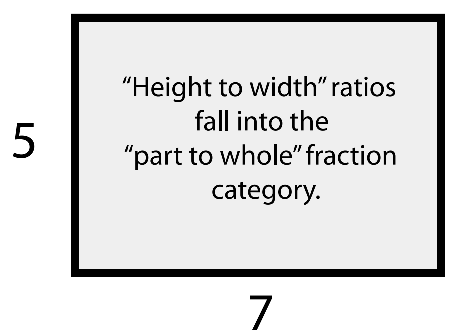
In which of these 2 categories is the “height to width” comparison of a rectangle?

Looking at a 5 x 7 photo,
it seems like the 5-inch dimension (vertical height) is one “***part***”
and the 7-inch dimension (horizontal width) is another “***part***,”
thus falling into the category of a **“part to part” ratio.**

But this is not so!
The 5-inch dimension is being ***compared***
to the 7-inch dimension,
which acts as a “***whole***.”

In other words,
we’re not comparing 5/12 and 7/12 here,
but 5 inches to a “whole” of 7 inches.

Thus, rectangular dimensions fall in the
“part to whole” fraction category.



What about the word proportion?

A proportion as a statement of equality between two ratios.
If two “part to whole” fractions are in “proportion,
they are essentially equal.

$$\frac{2}{5} = \frac{4}{10}$$

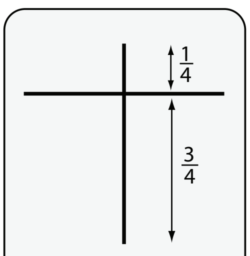
For example, two rectangles expressed as
“height to width” might be in proportion.
A “5 x 7 photo” is in the **same proportion** as a “10 x 14 photo.”

But “part to part” ratios can *also* be in proportion.
A (5 parts scotch : 7 parts soda) drink
is in the same proportion as a
(10 parts scotch : 14 parts soda) drink

$$\frac{5}{12} : \frac{7}{12} = \frac{10}{24} : \frac{14}{24}$$

When Dee says “Quaternary rests in the Ternary,” which of these two categories of “ratios” does he have in mind?

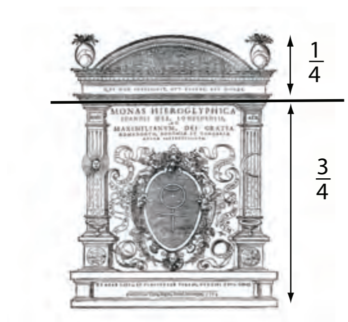
The answer seems to be: **BOTH!**
Here are some ways he expresses th 3:4 “part to part fraction”:



Dee’s Cross of the Elements

The horizontal line of
Dee’s offset Cross of the Elements
intersects the vertical spine
at 3/4 of its height.

A horizontal line
3/4 of the way up the Title Page
coincides perfectly with the
top of the capital of the 2 columns
(or the bottom of the entablature
that rests on the columns).



In fact, the whole Title Page
has a height : width
ratio (the fraction kind) of 4:3.

These measurements suggest that Dee saw “Quaternary rests in the Ternary”
in the “part to whole” fraction category.
But not so fast.
In many ***other*** ways,
Dee used this maxim to express a “part to part” ratio.
In Theorem 6, when he describes the Cross as “ternary” or “quaternary,”
he says they “manifest a remarkable septenary.”

In the Artificial Quaternary of Theorem 23,
Dee breaks the 7 into 4 and 3,
without explaining why.

$$8\begin{cases}1\\7\begin{cases}4\\3\end{cases}\end{cases}$$

Dee’s unexplained division
of the result “8”
in his Artificial Quaternary.

In his 1570 *Preface to Euclid*, Dee explains how specifically the 3:4
“part to part”ratio is useful in the field of law:

**“Wonderful many places, in the Civil law,
require an expert Arithmetician, in order to understand
the deep Judgment and Just determination
of the Ancient Roman Laws.”**

He adds:
“**the Ancient Roman Laws, cannot be perceived
without good Knowledge of Number’s art.
Nor is Justice (in infinite cases) able to be executed
without due proportion (narrowly considered).**”
(My transcription of Dee, *Preface* p. a.j. verso).

(Dee’s parenthetical expression “narrowly considered”
appears to refer to the “due proportion” resulting from what I call a “part to part” ratio.)
He cites the ancient Roman inheritance law, the ***Lex Falcidia.***
(*Falcidia* seems related to *falcifer*, meaning “scythe-bearing,” like Saturn, Father Time.)

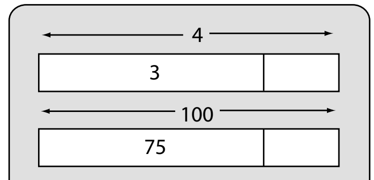
This law, instituted in 40 BC decreed that a Roman could only give away 3/4 of his estate.
The other 1/4 (“*quarta Falcidia*”) was guaranteed to his heirs.
(There are other rules that make the system a mathematical challenge,
for example, if there are multiple heirs, they are each entitled to a “quarta Falcidia”.)

To demonstrate, Dee gives the example of three heirs (a wife, a son, and a daughter)
who each get “30” of something (probably aurei, gold coins).

At the time of death, each heir gets 4/7 of their portion (or “17 1/7” of the “30”).
Ten months later (to ensure that another heir is not born during that time),
the other 3/7 (or “12 6/7” of the “30”) is distributed the heirs.

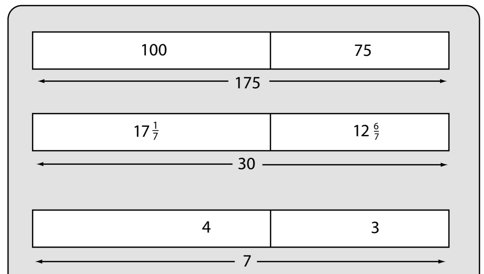
Dee clarifies:
"**For, what proportion, 100 hath to 75:**
the same hath 17 1/7 to 12 6/7:
Which is Sesquitertia:
that is, as 4, to 3, which makes 7."
(Dee *Preface*, p. a.j. verso)

At first glance it seems like Dee is referring to 3/4 or 75% of 100 (that is, a "part to whole" fraction).



"three quarters" or 75 %

But closer inspection shows he's really talking about "part to part" ratios.
Here are the three equivalent proportions he mentions:



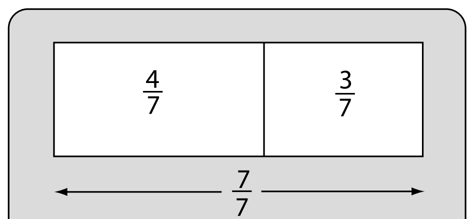
The 4 to 3 "part to part" ratio

Now its easy to see how Dee came up with those strange numbers, 17 1/7 and 12 16/17.

These "part to part" ratios imply the comparison of two fractions.

The keys are these fractions 4/7 and 3/7, which are divisions of 7/7, a whole.

In short, this is clearly a reference to "Quaternary rests in the Ternary" as a "part to part" ratio.



The 4 to 3 "part to part" ratio

(A little more about Lex Falcidia)

Dee explains that six of the greatest legal minds of the Middle Ages and early Renaissance (Accursius, Baldus, Bartolus, Jason, Alexander and Alciatus) were confounded by the mathematics involved in the Lex Falcidia inheritance laws.

Accursius (1182–1263) was an Italian jurist who compiled the "Great Gloss," containing over 100,000 "glosses." (a gloss is a translation or an interpretation of a phrase).

According to Dee, when Bartolus (1313–1357) tried to understand the Lex Falcidias's math, even as explained to Accursius' gloss, he declared:

"In the whole book,
there is no Gloss harder than this,
whose account or reckoning,
neither the Scholars, nor the Doctors understand."
(Dee, *Preface*, p. a.j. verso).

Dee was able to understand the mathematics of the Roman's Lex Falcidia by studying the works of Al-Farghani. Dee owned at least 8 treatises written by this noted Arab astronomer **Al-Farghani** (ca. 815–ca. 861) was born in present-day Uzbekistan, but died in Egypt. His most important work is *Elements of Astronomy* (a summary of Ptolemaic astronomy), but he also wrote on the use of astrolabes and sundials. These texts were translated into Latin in the 1100's and were widely circulated up into the 1600's.

Al-Farghani's Latinized name was **Alfraganus**, which Dee uses in all the entries in his Library Catalogs of 1557 and 1583, but in the Preface to Euclid he refers to him as "Africanus."(Roberts and Watson, p. 208)

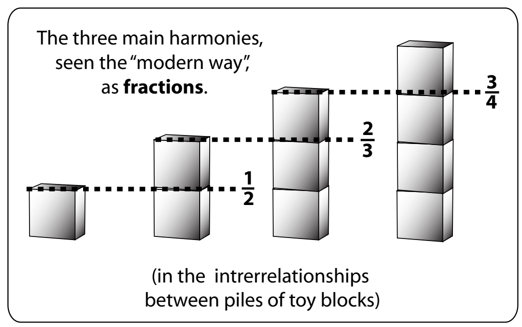
Modern "fractions" are simply upside down Greek "fractions."

Even with terms clearly defined, discussing "part to whole" fractions and "part to whole" ratios together still can get confusing.
So, let's concentrate first on "part to whole" fractions
(which includes the subcategory of "height-to-width" comparisons).

Let's start with the 3 main Harmonies (1/2, 2/3, and 3/4), seen in what I call the "modern way," (that is, as fractions).

By making 4 stacks of blocks, (a unit, 2 units, 3 units, 4 units) we can get a visual depiction of how 1/2, 2/3, and 3/4 integrate with 1, 2, 3, 4.

As each pile is a one-unit "step-up" from its predecessor, the fractions proceed 1/2, 2/3, 3/4
(and might continue on as 4/5 5/6 6/7 7/8…)

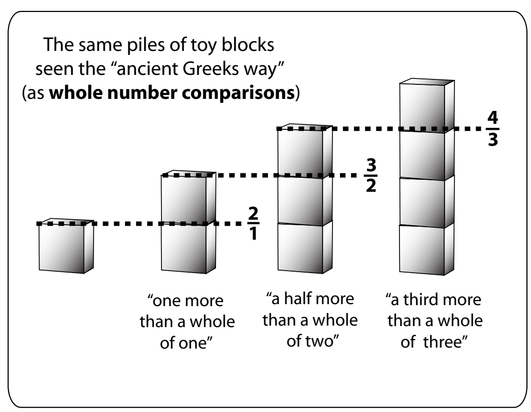


I call this the "modern way" because the Greeks expressed the same relationships in a different way.

They didn't like fractions.
They preferred to deal in whole numbers.

They would express (1/2, 2/3, and 3/4) as (2/1, 3/2, and 4/3).

They simply flipped the numerator and denominator from the way we are used to seeing it.



I'm not suggesting that 2/3 = 3/2,
as 66 1/3% and 150% are clearly different things.
But, we could say: (modern 2/3 = the Greek 3/2).

The Greeks used letters for numbers, so for “4 over 3”
they would use lowercase delta (δ , the fourth letter) and a lowercase gamma (γ , the third letter).
But they didn’t use the “dividing line” which we conventionally use in fractions.
In text, they would sometimes write the numerator first, followed
by an accent mark, then the denominator, written twice,
each time with two accent marks (δ’ γ” γ”).
(James Gow, *A Short History of Greek Mathematics*, p. 48).

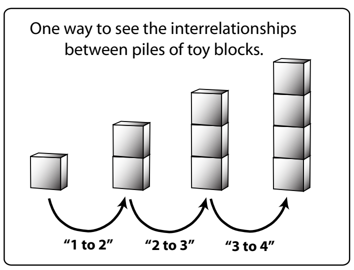
As we’ve seen, Dee uses the Greek term ***prologous***
for that larger, first term (4 or delta with one accent mark, δ’)
and ***upologous*** for that smaller, second term (3 or gamma written twice, each with two accent marks γ”γ”).
(Dee, Artifical Quaternary chart, *Monas,* p. 26 verso).

Our modern 3/4, which the Greeks saw as 4/3, they called ***epitritos***.

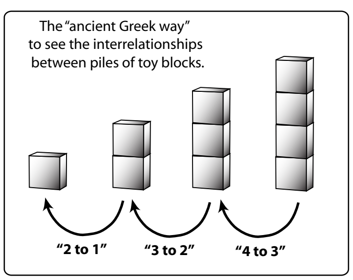
Epi means “upon” and ***tritos*** means “a third.”

They saw 4/3 as “a third part upon a whole.”

Child’s play: expressing the 3 main harmonies as toy blocks or as rectangles (horizontal or vertical).

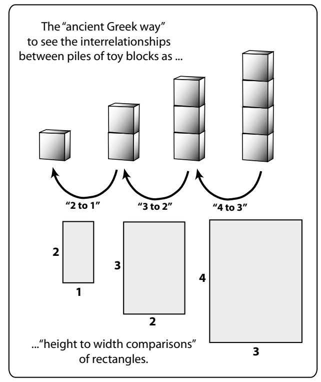


A simple way to see
the “3 main harmonies” (1/2, 2/3, and 3/4)
is in the interrelationships
between these piles of children’s blocks.

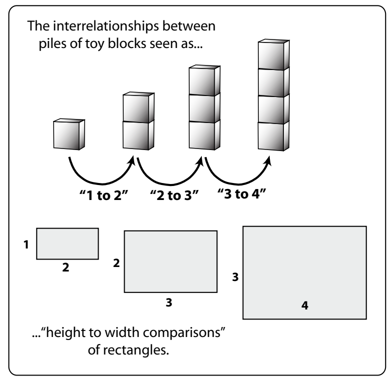


Using the same piles and interrelationships,
the Greeks would have seen it this way.

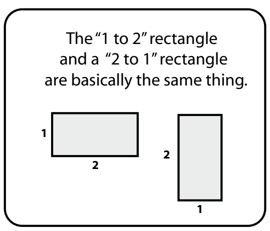
Next, let's make these interrelationships more graphic by applying them to the "height to width" of rectangles.

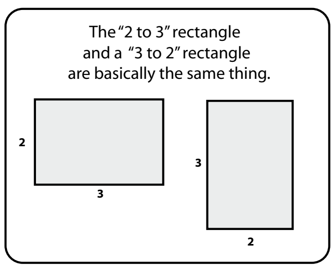


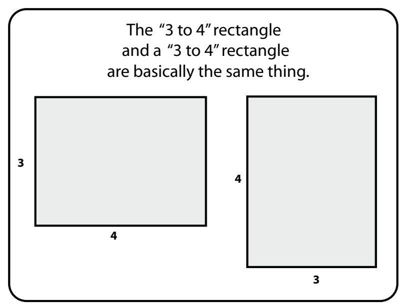
Applying the "Greek way" of seeing it simply makes vertical rectangles with the exact same proportions.



See how the "Modern" results
are essentially the same thing
as the "Greek" results
(only with a 90-degree rotation).



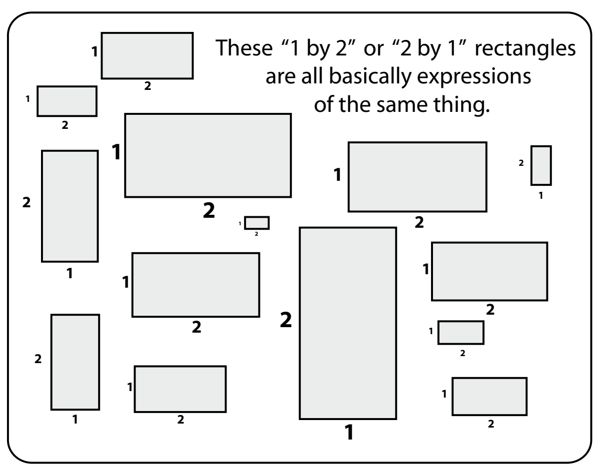


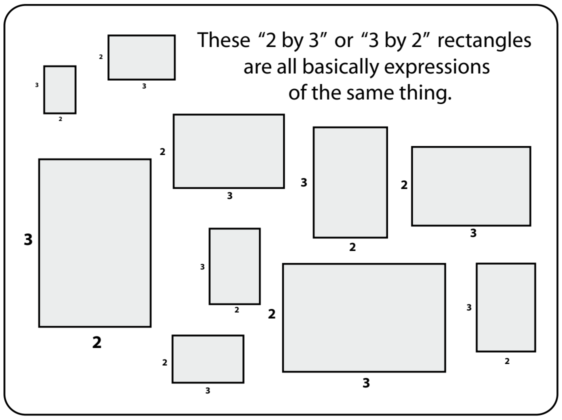


Here is how
it might be said that
2/3 =3/2

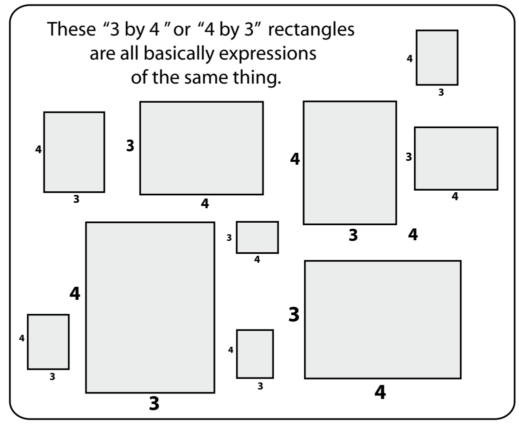
Thus, all shapes of the same proportion,
regardless of scale (size) or orientation
are all basically expressions
of the same thing.

For example, all these "1 by 2" rectangles
or "2 by 1" rectangles
are essentially expressions of the same thing.





As are any
“2 by 3” or “3 by 2” rectangles,
regardless of scale or orientation.

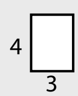


And
“3 by 4” or “4 by 3” rectangles
as well.

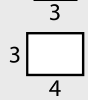
To summarize, we shall see how Dee plays with these 4 ideas,
which are all essentially the same thing.

$\frac{3}{4}$ “Part to whole” fractions (Modern, smaller number on top).

$\frac{4}{3}$ “Whole to part” Greek “fractions” (larger number on top).



“Height to width” making horizontal shapes.



“Height to width” making vertical shapes.

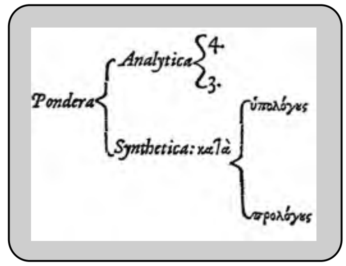
The preceeding discourse might seem overly simplistic,
but it provides an essential foundation from which
we can see the geometric secrets hidden
within Dee’s illustrations in the *Monas*,
(and in the design of the John Dee Tower).

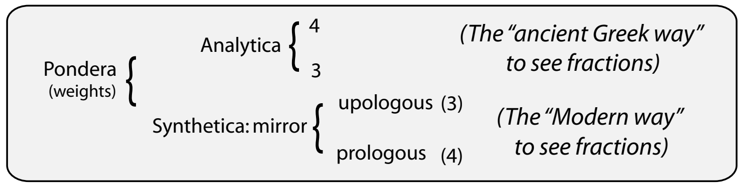
A mirror in the middle of the Artificial Quaternary chart.

In this category of *Pondera* (weights) are two categories:
Analytica (analysis, or breaking a whole into parts) and *Synthetica* (Synthesis or using parts to make a whole).
Analysis and Synthesis are opposites.

Dee describes Analysis as 4: 3.

In Synthesis, if Dee wants us to put the upologous (second, smaller term in a ratio) before the prologous (first, larger term in a ratio), that would be 3:4





Is Dee trying to say the fraction 4/3 is the same as the fraction 3/4 ?
No.
He's saying the Greek ratio of 4: 3 is the same as the modern ratio 3: 4.
In terms of a rectangle, a 3-inch by 4-inch horizontal photo is 4 inches by 3 inches if it's held vertically.

Thus 3:4 and 4:3 are a mirror of each other.
They appear to be the reverse of each other, but they're essentially the same thing.

The confirming clue here is the word ***kata***,
(which follows the word Synthesis).

Kata is a preposition meaning "down or downwards,"
as in our word **cataract**, a "down-rushing" waterfall
or catastrophe, a sudden "downturn" of events.
(*kata* means "down" +*strephein* means "to turn")

The Greek expression *omosai kata tinos* (vow + down + pay)
means to "vow or swear by something" because one
calls "down" the vengeance of the gods upon it.

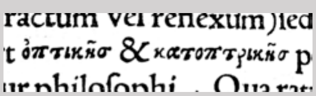
In mortal affairs, *kata* means "against,"
like giving a speech "against" an opponent or
like a judge imposing the sentence "against" a criminal.
From this "against" sense we get the word "**catapult**"
(kata means "against" + pallein means to hurl or cast an object"

Also, is this "against" sense that is found in the word *katoptrikê* or "**catoptrics.**"
There are three parts to the word:
kata means "against"
Op means "see"
trikê (from tron) means "instrument"
Thus, *katoptrike* is an "instrument for seeing against" an apt description of a mirror.

While the word catatropic was popular with the ancient Greeks, Dee apparently introduced into the English language. The Oxford English Dictionary cites Dees 1570 *Preface to Euclid* as the first time "catatropic" was used.
(Dee, *Preface*, p. 20)

But he also used the word *katoptrik*ê (twice in Greek and once in Latin) in his 1558 *Propaedeumata Aphoristica*. Dee expert Nicholas Clulee explains "as with [Roger] Bacon, optics plays a crucial role in Dee's magic as well as in his astrology because of the conformity of natural causes to the laws of optics."
Dee discusses optics and catoptrics in Aphorisms 45, 48, 52, and 99.

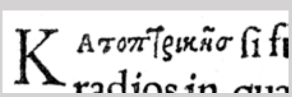
In Aphorism 48 he uses the words optikês and catoptrikês.
Also,in Aphorisms 45 and 99 he writes about using this art to focus rays from celestial objects.


ὀπτικῆς & κατοπτρικῆς

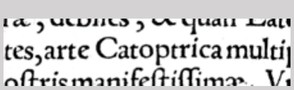
optikês and katoptrikês
in Theorem 48

48
...This happens (as I said) not through any principal ray
(meaning direct, refracted or reflected)
but through what philosophers skilled in
optics and catoptrics call Reflections of Reflections...

Aphorism 52 begins with the word Katoptrikês, Dee explains that the art of of Catoptrics goes way back in history and that he has incorporated it in his Monas symbol
(using the symbols of the various planets).



Katoptrikês in Theorem 52


tes,arte Catoptrica

Catoptrica in Theorem 52

52
If you are skilled in Catoptrics you will be able to artfully impress the rays
of any Star much more strongly upon any given material than Nature does by itself.
Indeed, this was by far the greatest part of the Natural Philosophy of the Ancient Wise Men.

And this Secret is no less dignified than the most distinguished
ASTRONOMY of the philosophers commonly called INFERIOR.
The symbols used in Inferior Astronomy are incorporated in a certain MONAD
which is derived from our Theories and which we send along with this little book.

Obscure, weak and (as it were) Hidden Virtues of things, when
strengthened by the Catoptric art, can become more apparent to our senses.
The diligent Investigator of Secret has this great assistance available to him
when examining the particular powers, not only of stars, but of other things
that the stars affect with their perceivable rays.

He also mentions the use of mirrors in his advice to Opticians in his *Letter to Maximilian*
"And won't the Optician condemn the Senselessness of his ingenious work,
laboring in all sorts of ways to make a mirror…"
(when he might learn even more by exploring by camera obscura.)

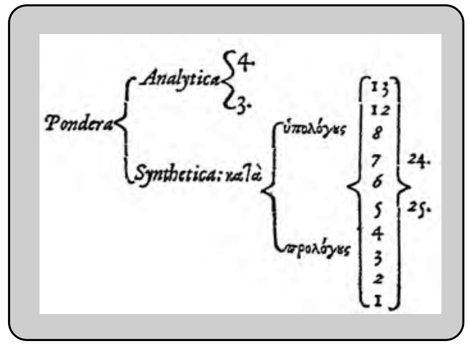
(Dee, *Monas*, p.6)

Dee wants us to see 4:3 as a mirror of 3:4. He wants us to see
that the "4:3-proportioned-upright-Title page" (height: width)
can be turned 90 degrees to become 3:4 proportion (height:width),
and **both** are still the same Title page.
So here in the midst of the Artificial Quaternary chart
this tiny word *kata* seems to refer to Katoptrike, or Catoptric, a mirror.

Beyond this, Dee also wants us to see the cuboctahedron
as exemplifying this "oppositeness."

As shown previously, the digits
(1 to 8), (12 and 13), (24 and 25),
all relate to the cuboctahedron.

And the idea of 4:3 or (8 square faces: 6 triangular faces)
and 3:4 (3-sided triangles: 4-sided squares)
are key components of the cuboctahedron.



Besides Dees references to Catoptrics in his 3 main mathematical works,
he wrote several texts that focused specifically on optics.
In 1557, he wrote *On Burning Mirrors* (how to focus the sun's rays using parabolic mirrors).
Also in 1557, he wrote *On Perspective* (for painters).
In 1559, he wrote 3 books on the
Third and Most Excellent Part of Perspective,
on the Refraction of Rays.

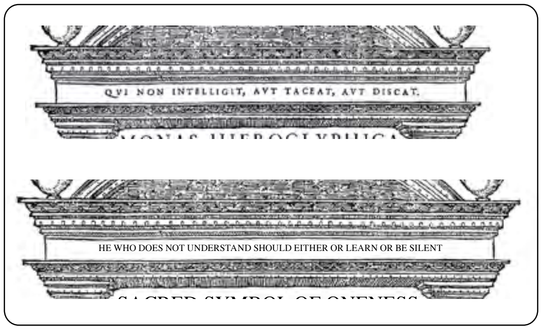
Aside from simply coining the word
Dee was an expert on catoptrics.
Dee saw Nature's characteristic of reflectiveness in many things,
from optics to number to geometry and more.

He saw reflectivity…
..in a mirror (an object and an image of itself)
...in a camera obscura (inside and outside)
...in ratios (Greek way and modern way)
...in Consummata (the transpalindromic 9 wave, 99 Wave, 1089 Wave, etc.)
In Metamorphosis (the symmetrical distribution of primes)
In a cuboctahedron (each of the 4 "Bucky bowties")

As a confirming clue to a confirming clue, do you recall seeing "kata" elsewhere in the Monas?
I'll give you a hint. Dee uses the letters "c-a-t" from "cat-optrics."
(And it's not the kind of cat that meows)

On the Title page, Dee admonishes would-be critics of his book:
(He who does not understand should either be silent (*taceat*) or learn (*discat*).”
The first 3 letters of *taceat* are *tac* and the last 3 letters of *discat* are *cat*.
Tac and *cat* are transpalindromic syllables.
Not only do the syllables reflect each other, they also mean mirror!

(Give Dee a Genius Point for the cleverness of this clue.)

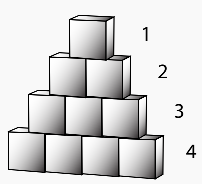


Curiously, when Queen Elizabeth asked Dee to explain the *Monas Hieroglyphica*
to her in private, she promised to “*discat* and *taceat*” (learn and be silent).

(Dee, *Compendious Rehearsal*, p. 12)

Having seen seen that Dee’s enthusiasm for the
reflection of the Greek ratios and the modern ratios,
let’s see how he hid various ratios in the *Monas* illustrations.

Another lesson using toy blocks.



Pythagorean tetraktys
(Dee’s Pythagorean Quaternary)
seen as ten toy blocks

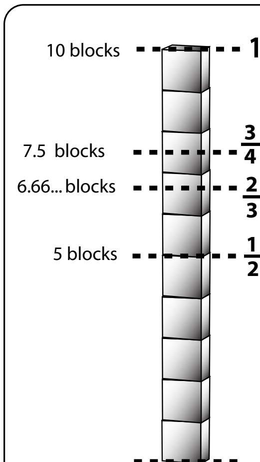
It’s obvious how these 4 “piles of blocks”
might be rearranged or “stacked up,”
to form the Pythagorean tetraktys

(or Dee’s Pythagorean Quaternary).

Another way to arrange these ten blocks is in one tall column.

Halfway up the column is 5 blocks, a whole number.
But two-thirds of the way up the column is
6.66… blocks, which is **not** a whole number.
And three-quarters of the way up the column is 7.5 blocks.
Again, **not** a Whole number.

Zeus would not be pleased with this arrangement.

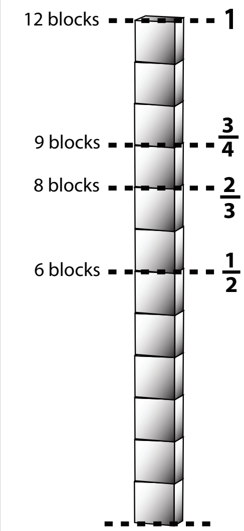


Ten blocks
don’t divide evenly
by 2/3 or 3/4
as “whole” blocks

The simple solution is, of course, to use 12 blocks instead of 10.
Here's where that highly composite number
12, (the docena) really shines.

Halfway up the column is 6 blocks.
Two-thirds of the way up is 8 blocks.
And three-quarters of the way up is 9 blocks.

All whole numbers – Zeus is pleased.



Twelve blocks
(The Docena)
divide nicely
by 1/2, 2/3 and 3/4

Do you recognize these numbers, 6, 8, 9, and 12?
They are the numbers used by the
Neoplatonics like Iamblichus and Nicomachus,
and by Boetheus,
(and by Rafael in his
"School of Athens" painting)
to express the 3 main harmonies,
diapaison (2 to 1), diapente (3 to 2) and diatesseron (4 to 3).

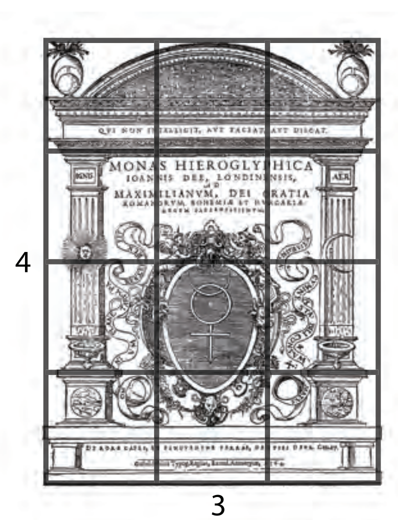
We have come full circle and in through the back door
using the same thought process that those ancient mathematicians used.
Only they expressed it in different ways (most noticeably, they didn't use toy blocks).

Let's see how Dee integrated the 3 Main Harmonies,
1/2, 2/3, and 3/4 in the illustrations of the *Monas*?

The "3 to 4" Harmony

The 3:4 harmony is most evident on the Title Page.
A rectangle that touches the outer edges of the architecture
(including the tips of the leaves which burst forth from the urns)
is exactly in the proportion of 4 to 3 (height to width).

The confirming clue that shows that this is no "accident"
can be seen by applying a 4 x 3 grid.
(It's actually easier to see if we momentarily delete
the emblem and all the words from the Title Page).



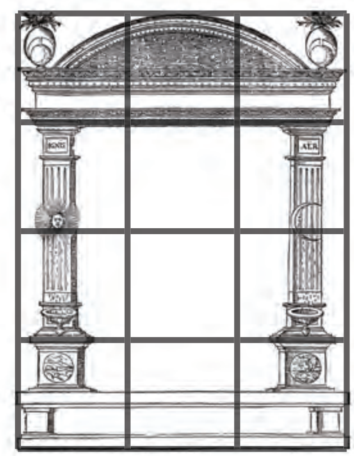
The horizontal dividing line that is
1/4 of the way up from the bottom
marks the base of the 2 columns
(or the top of the 2 pedestals).

The horizontal line that is 1/2 way up
cuts across the exact vertical middle
of the height of the columns.

And the horizontal line 3/4 of the way up
marks the top of the 2 columns
(or the bottom of the entablature
that surmounts the columns).

This is no accident.
The Title Page itself is very
"Quaternary rests in the Ternary"-ish.

4



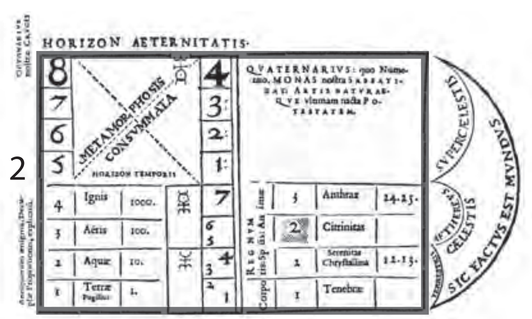
3

The 2 to 3 harmony.

Next, the 2/3 harmony is also easy to spot.

The "rectangular part" of the
"Thus the World was Created" chart is exactly
in the "2 by 3" (height to width) proportion.

2



3

The fact that this is no accident
can be seen by applying a "2 by 3" grid.

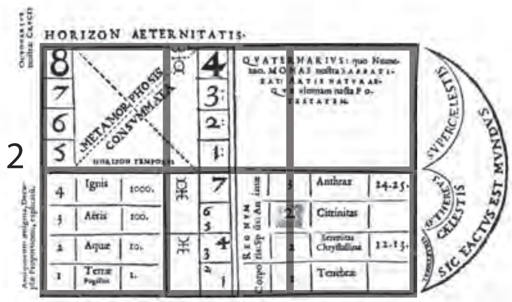
The midline of the height dimension
is the line separating Dee's "Above" from "Below."

The "thirdings" dividing the width fall at important places.

The 1/3 line coincides with a vertical line
next to the Solar and Lunar Mercury Planets symbols.

(This line was printed in the "engraving" pass through the press).

2



3

The 2/3 demarcation of the width is even more telling!
It runs vertically right through the digits in Dee's Artifical Quaternary!

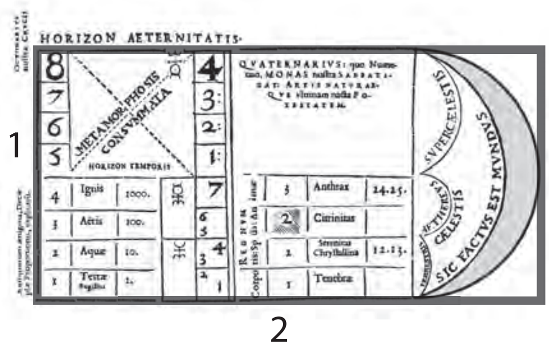
(including that important "Engraved 2"!).

The “ 1 to 2” harmony

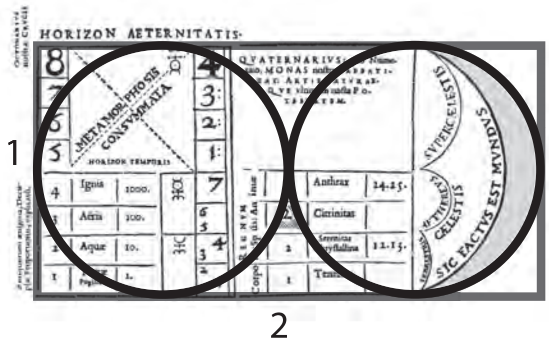
Dee has made the harmony of diapiason,
(or the “2 to 1,” or the “1 to 2” or the 1/2 proportion)
a little harder to find.

As described earlier,
one must first find the Metamorphosis numbers
in the circle segments on the right side of the chart.

As the largest encompassing circle segment
represents 360, the segment must be “ballooned up”
to become a more appropriate “half-circle.”

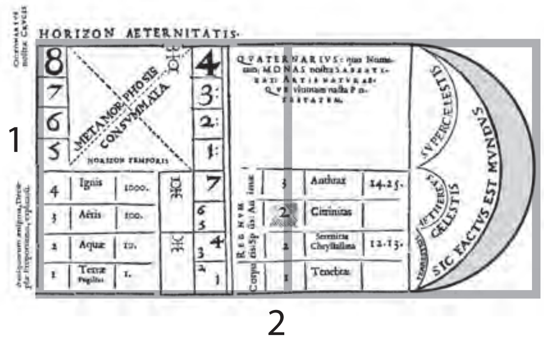


The confirming clue here is that the whole chart can
express two circles, a huge theme in the *Monas*.



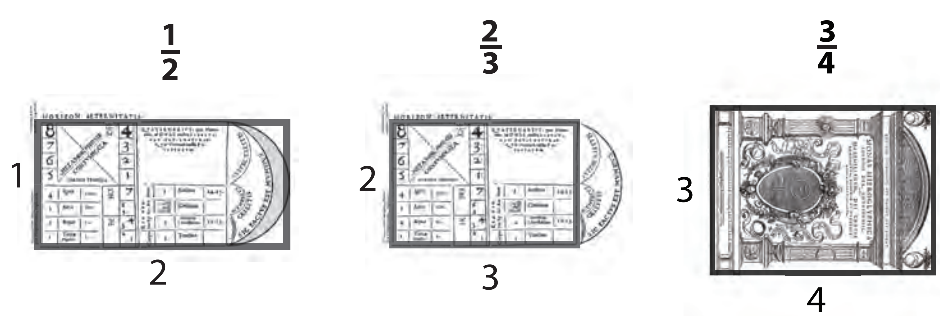
When a “1 by 2” grid is overlaid on this
now-expanded illustration, the vertical line
marking the middle of the width runs
right through the Artificial Quaternary again!

(Mathematically the reason for this is quite simple.
If the width of the rectangular part of the chart is x,
the whole chart, including the ballooned part, is (x + 1/3x),
which is then divided by 2.
This all is equivalent to to 1/2 x + 1/6 x,
which is 3/6 x + 1/6 x, which is 2/3 x,
which is the description of the line marking
2/3 of the width of the“rectangular” part of the chart.)



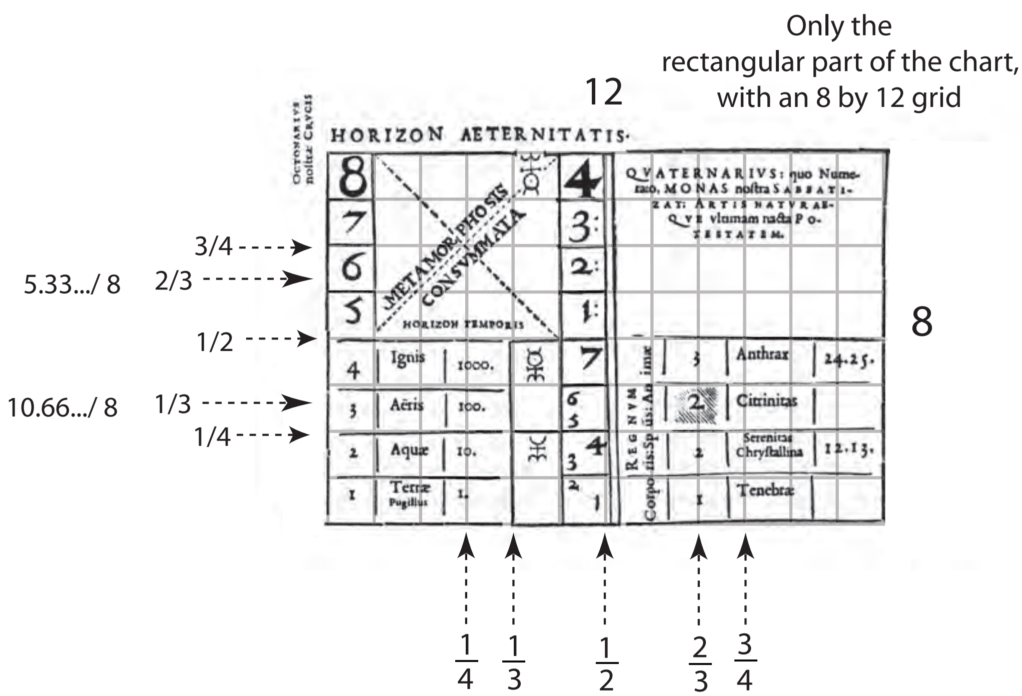
To summarize, Dee hid
the 3 Main Harmonies (1/2, 2/3, and 3/4) in the outer proportions
of the “Thus the World was Created” chart and the Title Page.

Expressions of the 3 main harmonies
in the “outer proportions” of Dee’s illustrations



The 3 main harmonies (1/2, 2/3 and 3/4) in the inner proportions of the "Thus The World Was Created" chart.

We've seen how the lines of the "2 by 3 grid" corresponds with important features in the chart.
Let's investigate even finer grids of the same proportion.
The "6 x 8" grid is nice, but the "8 by 12" grid is even nicer.
Note that the grid squares define the edges of the square boxes of the chart
which contain the engraved digits "1 through 7"(in the Below half),
and also "1 through 8"(in the Above half).



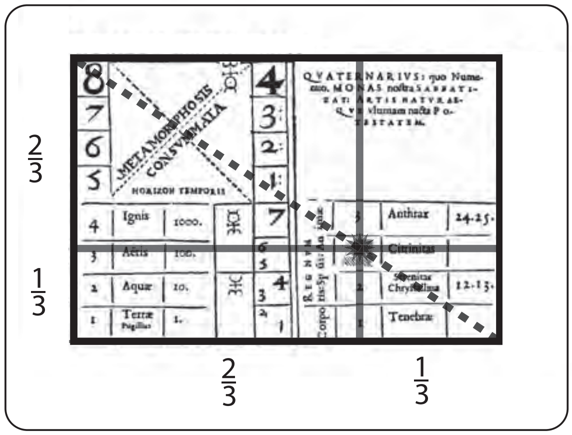
The "quartering, thirding, and halving" marks of the width all align with vertical grid lines
(as the width is that highly composite number 12).

But the "thirding" marks (1/3 and 2/3) of the height do **not** align with any of the horizontal grid lines
(because 8 is not evenly divisible by 3).

There is something suspiciously propitious in the "thirding" of the "rectangular part" of the chart.

Even though the "1/3 of the height line" does not correspond with any of the horizontal lines of the "8 by 12" grid, let's draw it in anyway.

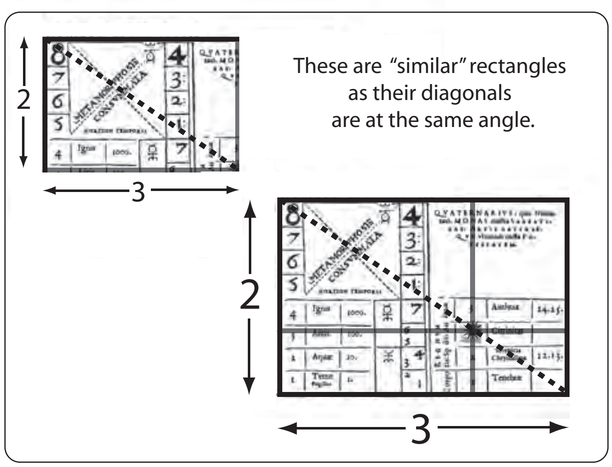
Notice that it intersects the vertical line marking "2/3 of the width" ***exactly*** on the "Engraved 2" of Dee's Artificial Quaternary. The (dotted line) diagonal (of this "Rectangular part "of the chart) also passes through the "Engraved 2!"



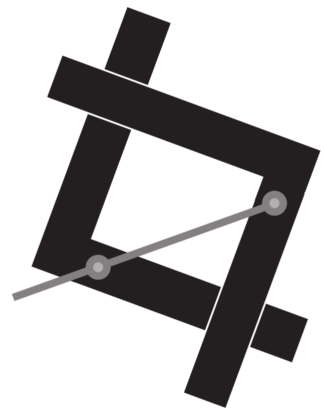
The reason this happens can be seen by examining the rectangle formed to the **to the upper left** of the "Engraved 2."

This smaller "rectangle" and the larger "rectangular part" of the chart are what geometers call "**similar**" rectangles.

As they are in the same proportion, their diagonals are at the same angle.



As a professional photographer, my art director clients would often give me specific proportions into which I had to fit all the elements of a photo so they would properly fit client's pre-designed layout.



Before capturing the image on film I would shoot Polaroid test prints to show the client. We would put a pair of cropping L's set to the clients specified proportion, on the print to assure the image would "fit" properly. These cropping L's were connected by a diagonal rod, so the "crop" could be enlarged or reduced, but remain in the same proportions.

With the advent of digital photography, constraining proportions for various croppings became as simple as "typing the proportions in the "Crop Tool dialog box." A simpler example of "constrained proportions" is using a zoom lens on a 35 mm camera. The "scene" can be "cropped" by "zooming" to any provided focal length, but the finished picture will always be in the "1 to 1-1/2" proportion that a 35 mm camera captures.

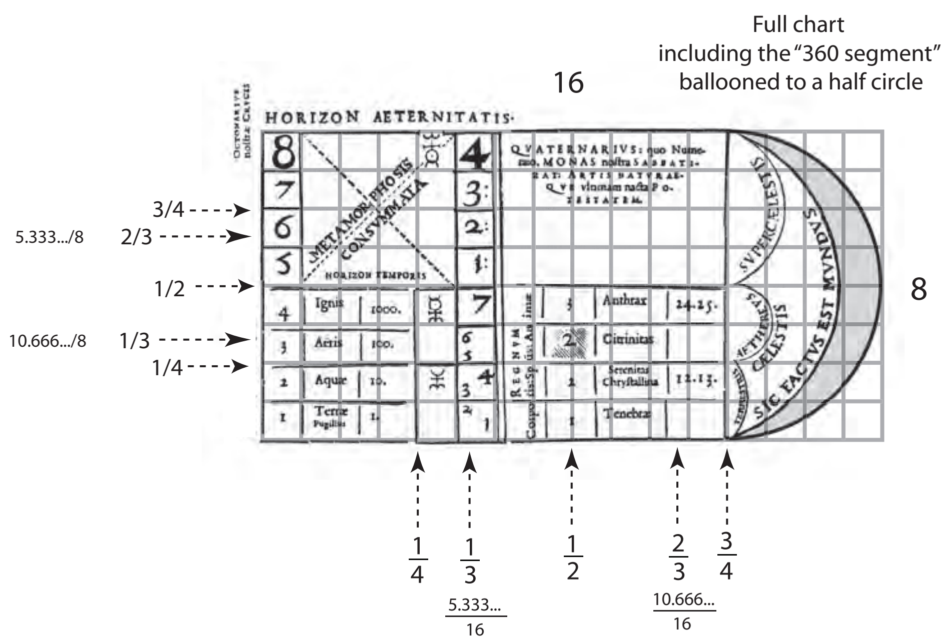
The only problem with this "sweet spot" was that it involved "thirding" nicely, but didn't involve halving and quartering at all. As it turns out this is not a problem, but a clue to the solution of a bigger puzzle.

Instead of only looking at the 2 by 3 "Rectangular part" of the chart, let's look at the full "1 by 2" chart.

Let's extend the "8 by 12" grid to an "8 by 16" grid in order to include the "ballooned 360 half circle" on the right side of the chart.

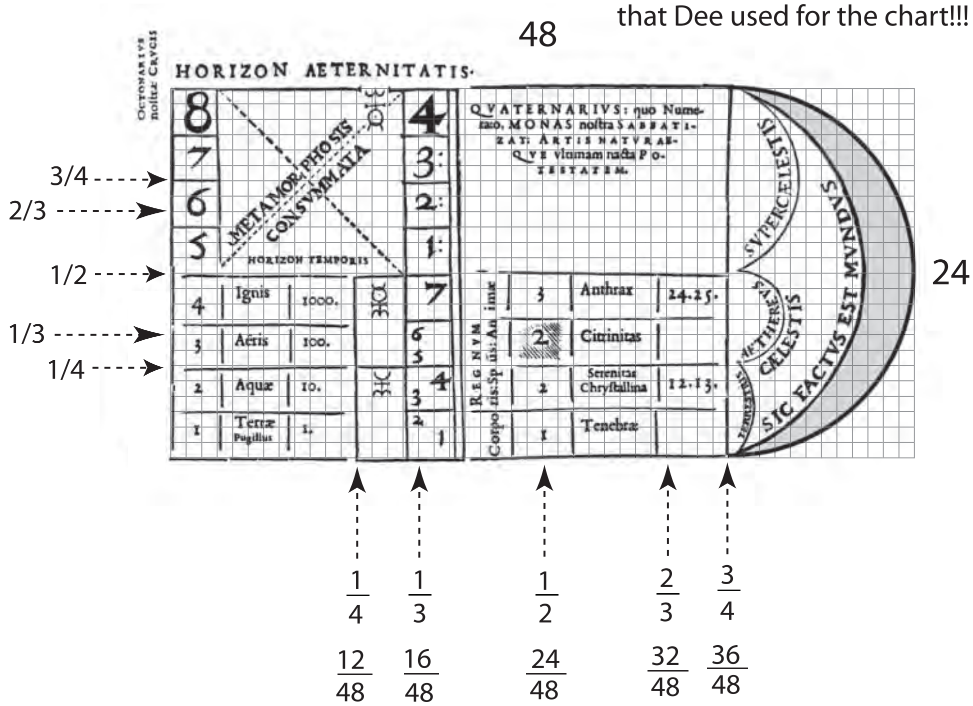
As we've seen, this puts the halfway mark of the width in line with Dee's Artifical Quaternary.

However, because 16 is not evenly divisible by 3, the 1/3 and 2/3 marks of the width do not align with any of the vertical grid lines.



This problem can be resolved if we use a grid
which is 3 times finer,
in other words, a “24 by 48” grid.

This appears to be the grid
that Dee used for the chart!!!



Suddenly all the vertical and horizontal
quartering, thirding, and halving marks all correspond with grid lines.
This appears to be Dee’s grid!

Let’s investigate each of these “division” lines individually.

Let’s start with the divisions of the WIDTH and see what they align with vertically.

The **1/4 mark** aligns with the engraved line to the left of the Solar and Lunar Mercury Planets symbol.

The fact that the **1/3 mark** goes through the small numbers 2, 3, 5, and 6 might seem insignificant, but it’s actually VERY significant!

The reason is that ***2 + 3 + 5 + 6 add up to 16***,
and 16 is a third of 48.
In other words, this 1/3 line is also the 16/48 line.
(More on this thrilling clue later).

Next, the **1/2 mark** aligns with Dee’s Artificial Quaternary (and thus a special member of that Quaternary, the “Engraved 2.”)

The **2/3 mark** seems to almost align with the 1 in the number 12 and the 2 in the number 24, two very important numbers for Dee. (The alignment is not perfect, but judging from the 4 short vertical lines to the left, there appears to have been some rightward drifting of the type in this section of the chart during the printing process.)

The **3/4 mark** aligns with the right edge of the “rectangular part” of the chart (because the “ballooned 360 half circle” section that was added expanded the width of the chart by 1/3 (and 3/4 of 4/3 is 1).

Next, we'll look at the quartering, thirding, and halving of the HEIGHT of the chart.

The **1/4 mark** corresponds with the important horizontal line which separates "Lunary Things" from "Solary Things" (both of which are in the "Below" half of the chart).

The **1/3 mark** of the height is very exciting. Following its progress from left to right, it just nicks the A in the word Aëris, crosses through the number 6, bisects the capital letter A in Animae (and more importantly the word "Anus,"or "Annulus," the Gold Ring of Gyges), and finally cuts right through the "Engraved 2."

Along with the vertical line that marks 1/2 of the width, they make like the crosshairs (like on a rifle scope) directly pinpointed on that "Engraved 2"! (This provides further evidence that the "Engraved 2" was made to look like "a mistake" **on purpose**, in order to highlight it.)

Next, the **1/2 mark** is just as exciting. It aligns with that grand division line (the Horizon of Time) which separates the "Below" half of the chart from the "Above" half of the chart. It separates "Earthly" things from "Divine" Things.

What's more remarkable is that, in this 24 by 48 grid, it marks 12/24 of the height. These two numbers 12 and 24 are of key importance throughout the Monas. For example, in Theorem 11 Dee explains that the "mystical sign of Aries" signifies the spring equinox when there are exactly 12 hours of daylight and 12 of darkness in a 24 hour day. He adds "Twenty-four Hours of Time divided in Equinoctial mode denote our most Secret Proportions."

Well, the idea that the 12/24 mark divides Earth from Heaven in this chart makes it very important indeed. (It might also be noted that the chunk of grid added to accommodate the "bal-looned 360 half circle is a "24 x 12" grid square section on this chart).

The numbers 12 and 24 are also important as the first two numbers of Metamorphosis. They are also both "results" of the Artificial Quaternary. And will see they play a key role in the design of the John Dee Tower!

Next, the **2/3 line** is thrilling for a different, more subtle reason. It underscores the number 6 and the number 2, thus mimicing that "1/3 line" which passes through a 6 and the "Engraved 2"). At first I thought these might be more representations of "twelveness" (6 x 2=12) until I noticed that the 2/3 line also cuts through the "letter A" in Metamorphosis, (just as the 1/3 line cuts through the A in Animae). Given Dee's fondness for the Latin alphabet letter/number code, these A's might be read as "1", as "A" is the first Letter of the Latin Alphabet (likewise Alpha in Greek and Aleph in Hebrew are "firsts").

Combining the 612 from the 1/3 mark and the 612 from the 2/3 mark makes 612612. This is very close to being 6126120, the eighth member of the Metamorphosis sequence, which when doubled by that Engraved 2, makes 12252240 the Exemplar number! This is all too fitting to be happenstance. But still Dee is off by a factor of 10. The zero in 6126120 is missing, and its unlike Dee not to include it. (It's there, but this is not the place to explain it. Can you find it?)

Finally, the "**3/4 of the height" line** divides the "Above" half of the chart into two horizontal sections. This line passes through the centerpoint of the large, "dotted-line X" in the upper right quadrant of the chart. It also seems to be a demarcation line above which all of the words in the "round" sentence are found.

The "leftover" area below the line (in that quadrant) is the only part of the chart where there is no information. This vacant area measures 6 grid squares high by 18 grid squares tall, which makes 108 grid squares – another key number in Dee's mathematics. (As 252 + 108 = 360, among other reasons).

To summarize, so many important features correspond with the 1/2, 1/3, and 1/4 marks, it appears that Dee used a **24 x 48** grid for his chart.

A confirming clue.

One final clue that Dee used a **24 by 48 grid** is that the two boldest numbers in the chart are **8 and 4**. Granted, they are "backwards," but Dee knew that anyone familiar with transpalindromes would see **84** as an expression of its opposite**, 48.**

(I have previously explained that the **Bold 8** and the **Bold 4** also express the "+4, -4 octave" rhythm inherent in the Base Ten numbering system. It's not unlike Dee to get two uses out of the same clue. It emphasizes the importance of these numbers.)

Picture this.
Dee is at his desk in his study at Mortlake,
surrounded by his library of wisdom from the past.

He wanted to summarize the mathematical
cosmology of Nature that he had uncovered.
On a blank sheet of paper, oriented horizontally,
he drew a grid of 24 by 48 small squares.

(I surmise that his grid squares
were each 1/2" tall by 1/2" wide,
making a chart 12 inches tall by 24 inches wide.
(If the grid squares were each 1 inch by 1 inch,
the chart would have been 2 feet tall by 4 feet wide,
unnecessarily large and cumbersome for a desktop.)

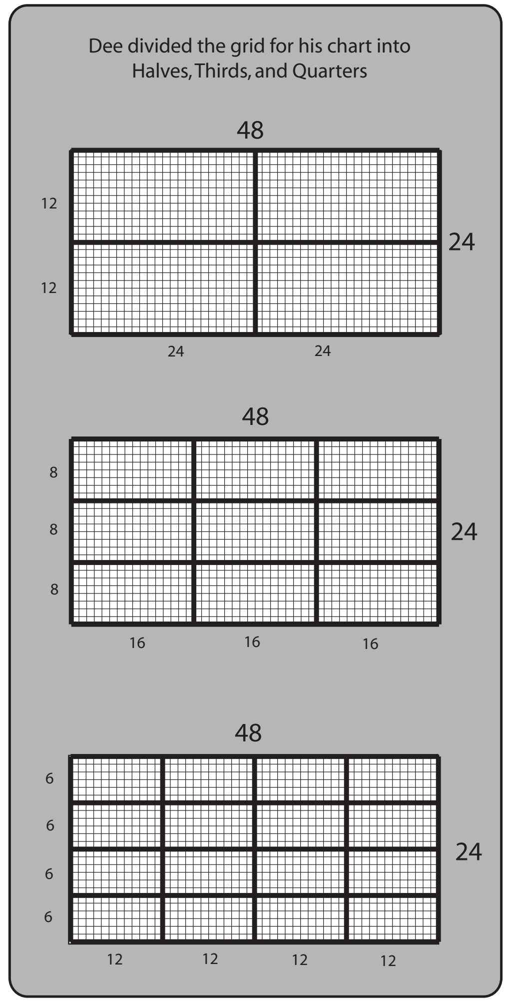
Thus, Dee's template for his important chart
summarizing the "Creation of the World,"
would include those powerful numbers 12 and 24.
A most propitious place to start.

A "sweet spot" in Dee's web of geometric harmony.

We've seen how Dee integrated the 3 main harmonies
in the **outer** proportions of his summary chart

Plus, we've seen how he integrated them
in the **inner** proportions.

But he integrated them *even further*
into the inner proportions
by using them to highlight
one particular "spot" in the chart.

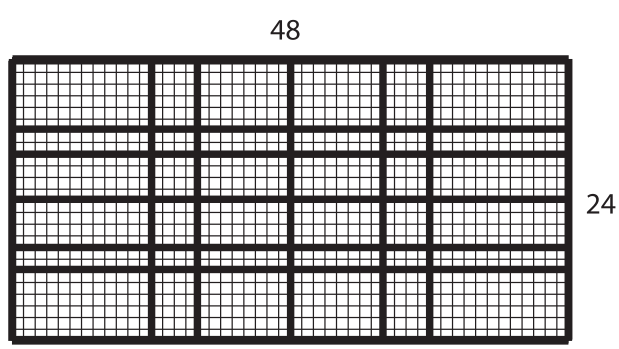


Dividing a "24 by 48" grid
in halves, thirds and quarters
makes smaller sections of
12 by 24,
8 by 16,
and
6 by 12,
respectively.

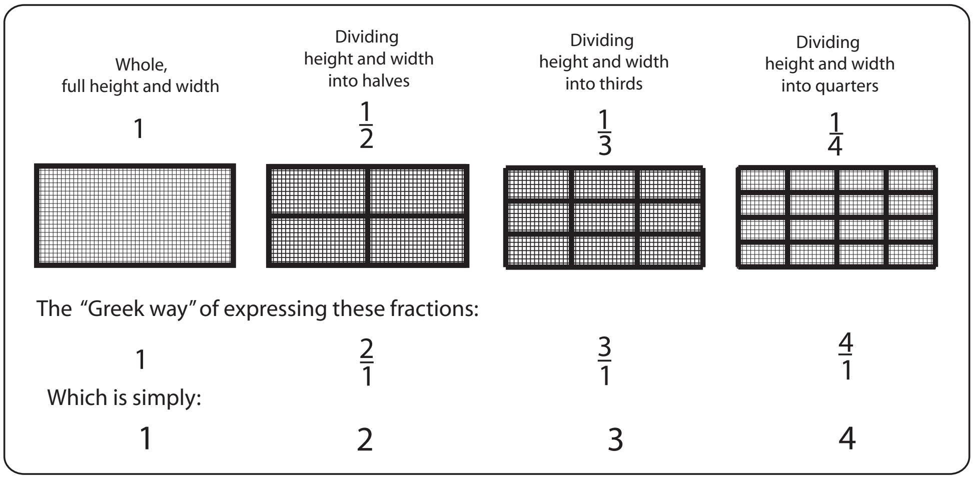
Superimposing all these subdivisions
makes a plaid pattern with 25 intersection points.

This is way too many possible "sweet spots."

So let's look at this problem in a new way
– following the path of how Dee's math works.



Here is a summary of the 24 by 48 grid seen as a whole, halves, thirds and quarters, which I've expressed as as 1, 1/2, 1/3, and 1/4. The fraction-flopping Greeks would have seen this as 1, 2/1, 3/1, and 4/1 (or simply 1, 2, 3, and 4).



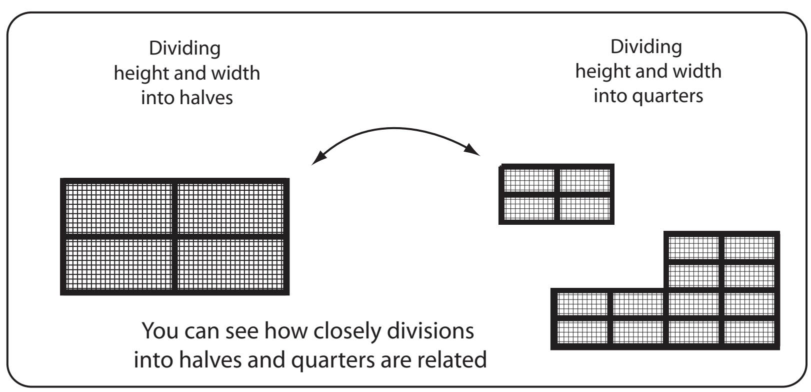
It's like the Pythagorean Quaternary seen as "divisions" instead of "wholes."

But remember, Dee "modified" the Pythagorean Quaternary (1, 2, 3, 4)
into his Artificial Quaternary (1, 2, 3, 2).
Dee saw that the 4 only "needs 2"
(as "another 2" already has been encountered previously).
The "essence" of 4 is 2.
(As we've seen, this leads to an understanding of the Metamorphosis sequence.)

In a similar, way,
Dee would have seen that the essence
of geometric quartering
is essentially halving.

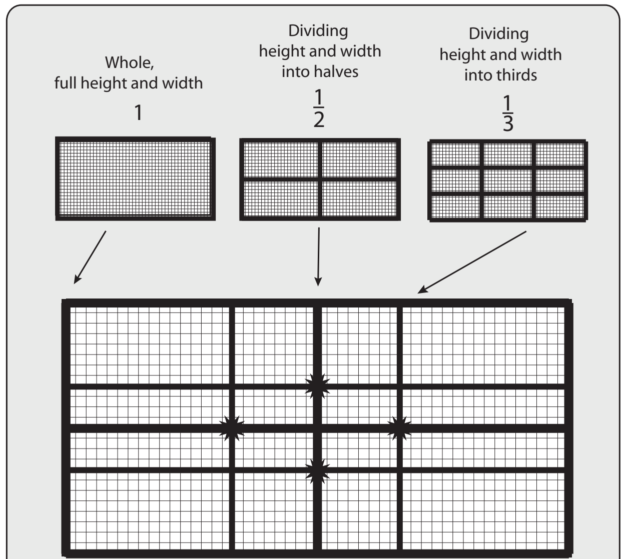
For example, here I've removed a "quarter chunk" from the full chart.

It looks exactly like the "halving" depiction, only on a smaller scale.



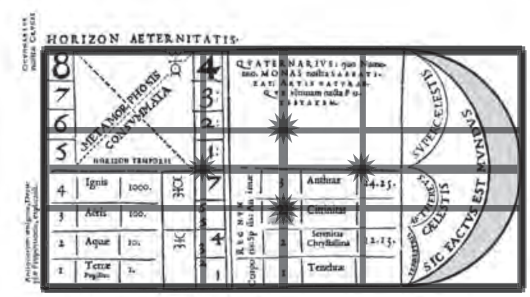
So let's remove the "quartering" grid for a moment.
What we're left with is the "halves" and "thirds"
(and of course the "whole", or full height and width).

Of the 9 intersections, there are only 4 that celebrate
the superimposition of halves and thirds,
(shown here with large asterisks).



There are four "hot spots"
where "halvings" and "thirdings"
intersect

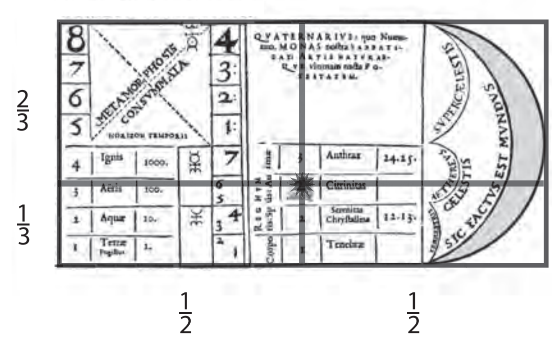
When this combo-grid is
superimposed over the actual chart,
we find that one of these "hot spots"
falls exactly on the "Engraved 2!"



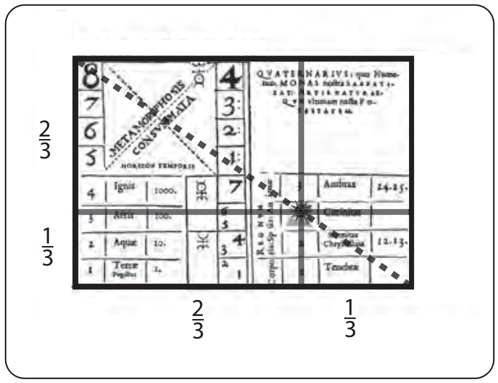
One of the "hot spots"
falls precisely
on the "Engraved 2"

This simplified version shows how this sweet spot
is related to "halving" and "thirding."
But, alas, it appears to be unrelated to "quartering."

One of the "hot spots"
falls precisely
on the "Engraved 2"



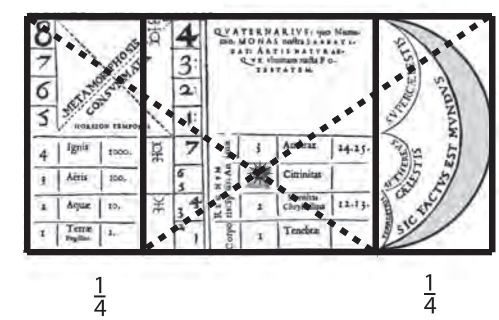
But not so fast.
there is a hidden geometric interconnection.
Remember that *diagonal line* from the analysis
of only the "rectangular part of the chart?"



Here we see that its lower, right-hand tip intersects
with the 3/4 mark of the width of the now-wider chart
which includes the "360" half circle.

In the same fashion,
the *other dotted-line diagonal* shown here
intersects the "1/4 mark" of the width of the chart.

"Quarters"are also involved,
as the diagonals of "3/4 of the chart"
pass through that nexus.



This really highlights
the "Engraved 2"
as a nexus.

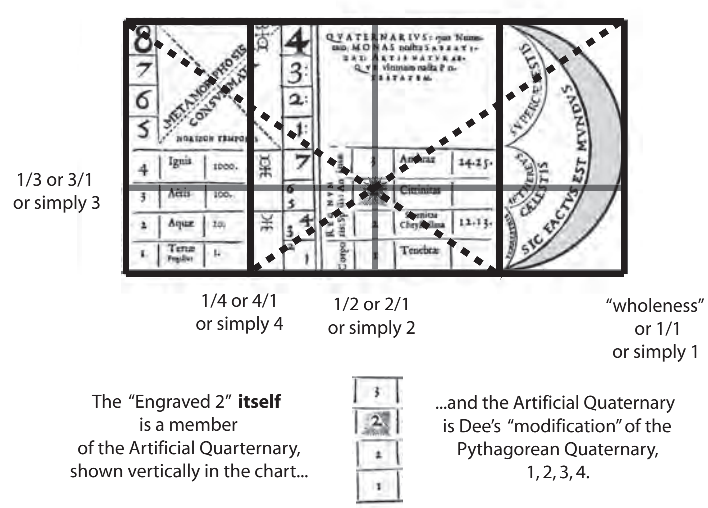
It's like a geometric
train station
with railroads
headed in
eight different directions.

Wholeness,
1/2, 1/3, and 1/4
are all involved.

Or as the Greeks
would have put it,
whole, 2/1, 3/1, and 4/1.

Or simply 1, 2, 3, and 4.

Dee's "Engraved 2"
is at the intersection of these lines
which involve the whole (1), and also the fractions 1/2, 1/3, and 1/4
or as the Greeks would have expressed it , 1, 2/1, 3/1, 4/1,
or in simplified terms, 1, 2, 3, 4



In short, this intersection point
is self referential.
Dee is graphically emphasizing
the idea that Number and Geometry
are two sides of the same coin!

Dee positioned the "Engraved 2" at the precise point where it would be "self-referential." It is a member of the Artificial Quaternary (1, 2, 3, 2), which is Dee's "modification" of the Pythagorean Quaternary (1, 2, 3, 4), which suggests wholeness, halving, thirding and quartering. Here at this "crossroads," Geometry and Number are singing the same song.

All this helps explain why Dee rearranged the sequencing of this Artificial Quaternary (1, 2, 3, 2) to (1, 2, 2, 3) in this chart. He switched the positions of the number "3" and the final number "2" so that 2 would fall on the "hot spot." This number "2," is the only thing that differentiates the Pythagorean Quaternary from the Artificial Quaternary. (Not to mention how Dee uses it as the final clue for reaching the Exemplar Number, 12252240) (And also as an expression of the "2 circles", the Sun and Moon, that help to provide the framework for the whole "24 by 48 grid" that is 2 times wider than it is tall.)

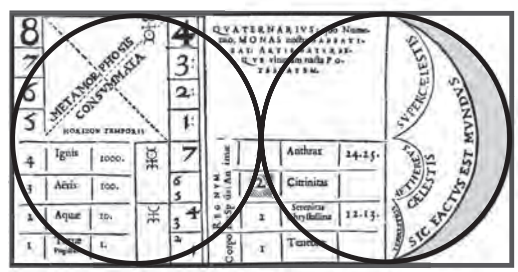
This also proves that Dee's crosshatching of the "Engraved 2" was not an accident. Dee made it appear to be a careless mistake that was corrected late in the printing process, but it's actually a disguise to hide his geometric goldmine.

In summary, Dee has woven a wonderful web
of the prime harmonies.

It is divine, visual music that he obscures
by hiding it in a cacaphony of other details
and by cryptically concealing
the true "1 to 2" dimensions of the chart.

The arithmetical and geometrical concepts
that Dee is dealing with here are so primal,
it becomes easier to undestand
why he called the study of these things:
"ARTIS SANCTAE"
(The Sacred Art).

"Wholeness"
is expressed
by the unity of the two circles
which are an integral aspect of the the chart.



The wondrous interrelationships among 1/2, 2/3, and 3/4 seen as glasses of milk and geometric rectangles.

In Aphorism 18 of the *Propadeumata Aphoristica*, Dee encourages the scholars (cryptically) to learn about the interrelationships among the three main harmonies 1/2, 2/3, and 3/4.

Noodling around with these three fractions (and their reciprocal Greek expressions), I came up with three interesting interrelationships.

I don't know about you, but to me its dizzying to look at these 3 equations and compare them to each other. It's clear that they are all describing the same basic intertwining, but there are too many flip-flops and switcheroos going on here to easily get a handle on what's going on.

Three interrelationships
between the 3 main harmonies

$\frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$	$\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$	$\frac{3}{4} \times \frac{2}{1} = \frac{3}{2}$

Note that the first equation begins with 1/2, the second begins with 2/3, and the final one with 3/4.

To give these fractions some traction, let's turn them into ***action verbs.***

Let's look at 1/2 as "cutting in half" or "**halving**" for short.

The fearless hero "halved" the poisonous snake.

Similarly, let's look at 2/3 as "**two thirding**."

The farmer found that only two thirds of his apples were fit to sell,

(the rest were barely fit for applesauce).

He "two thirded" his crop.

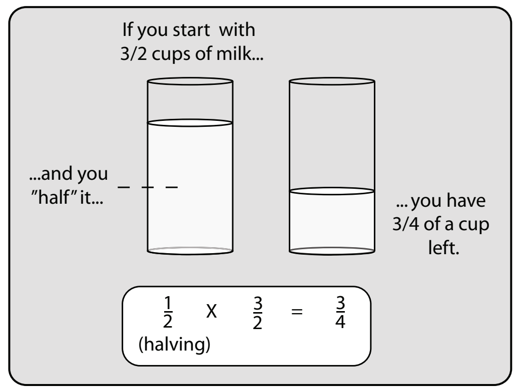
And finally, "three quartering," means taking three quarters of a whole.

At the movies, my friend "**three quartered**" the popcorn, then "shared" it with me.

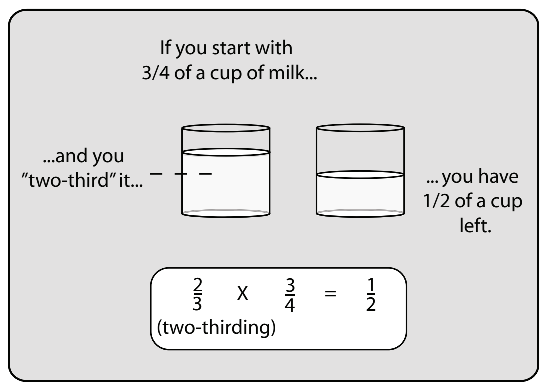
To visualize the first example, (1/2 x 3/2 = 3/4), let's use a slightly less dramatic "glasses of milk" analogy.

Let's say you had 3/2 of a cup of milk (meaning one and a half cups in a tall glass).

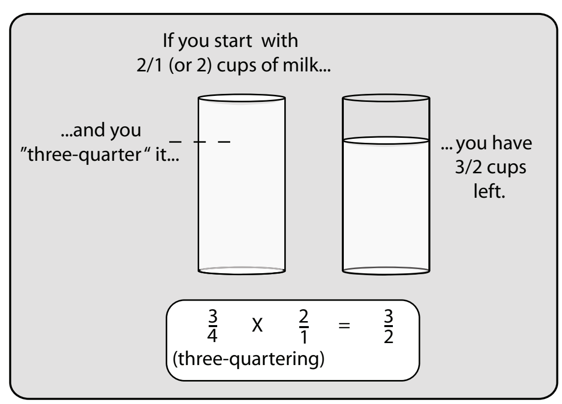
If you "halved" that quantity, you would have 3/4 of a cup.



Similarly, if you "two thirded" a glass of milk that was only 3/4 full to start with, the result would be a 1/2 glass of milk.

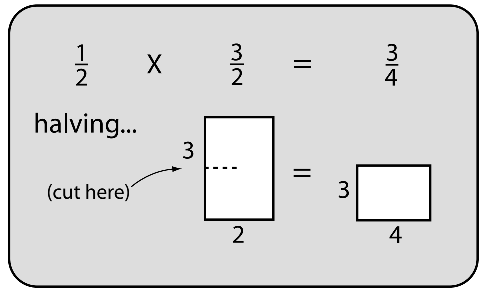


Finally, "three quartering" two cups of milk results in one-and-a-half cups.

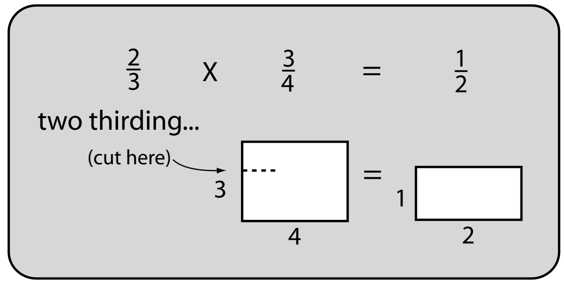


Now let's switch from white cow juice in glasses to geometric rectangles.

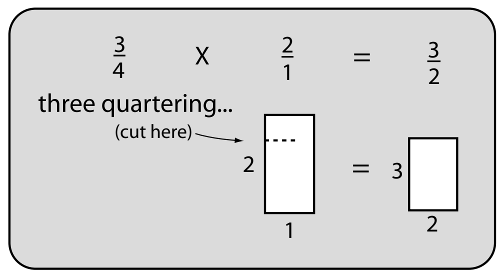
"Halving" a rectangle that has a
"height to width" proportion of 3 to 2
results in one that has the proportion 3 to 4.



Similarly, "two-thirding" a rectangle
with a "height to width" proportion of 3 to 4
results in one that has the proportion of 1 to 2.



Finally, "three-quartering" a rectangle
with a height to width proportion of 2 to 1
results in one that has the proportion 3 to 2.



Hopefully these simple demonstrations have made the three equations I found more tangible.
Because, hold on, there are more!

If the 3 equations are considered as the
"modern" expression of these fractions,
let's alternatvely look at them
in the "ancient Greek way."

In other words, with the ***reciprocals*** of all the fractions.

These same interrelationships
expressed the "ancient Greek way"
(in other words the same equations
expressed with the reciprocals of all the fractions)

$\frac{2}{1} \times \frac{2}{3} = \frac{4}{3}$	$\frac{3}{2} \times \frac{4}{3} = \frac{2}{1}$	$\frac{4}{3} \times \frac{1}{2} = \frac{2}{3}$

Summary of these 6 equations

$\frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$
$\frac{2}{1} \times \frac{2}{3} = \frac{4}{3}$
$\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$
$\frac{3}{2} \times \frac{4}{3} = \frac{2}{1}$
$\frac{3}{4} \times \frac{2}{1} = \frac{3}{2}$
$\frac{4}{3} \times \frac{1}{2} = \frac{2}{3}$

Seeing these as ***action verbs***,
there are doublings, one-and-a halvings,
and one-and-a-thirdings going on here.

I'll spare you the milk and rectangle demonstrations
and just give a summary chart of the, now, 6 equations.

But wait, that's not all!

Let's take these six equations
and switch the "order" of their multiplicands
(the two things that are multiplied).

Why bother?
Mathematical common sense tells us
the resulting product will be the same.

But remember, we're using the first multiplicand
as an ***action verb*** performing an action
on the second multiplicand.

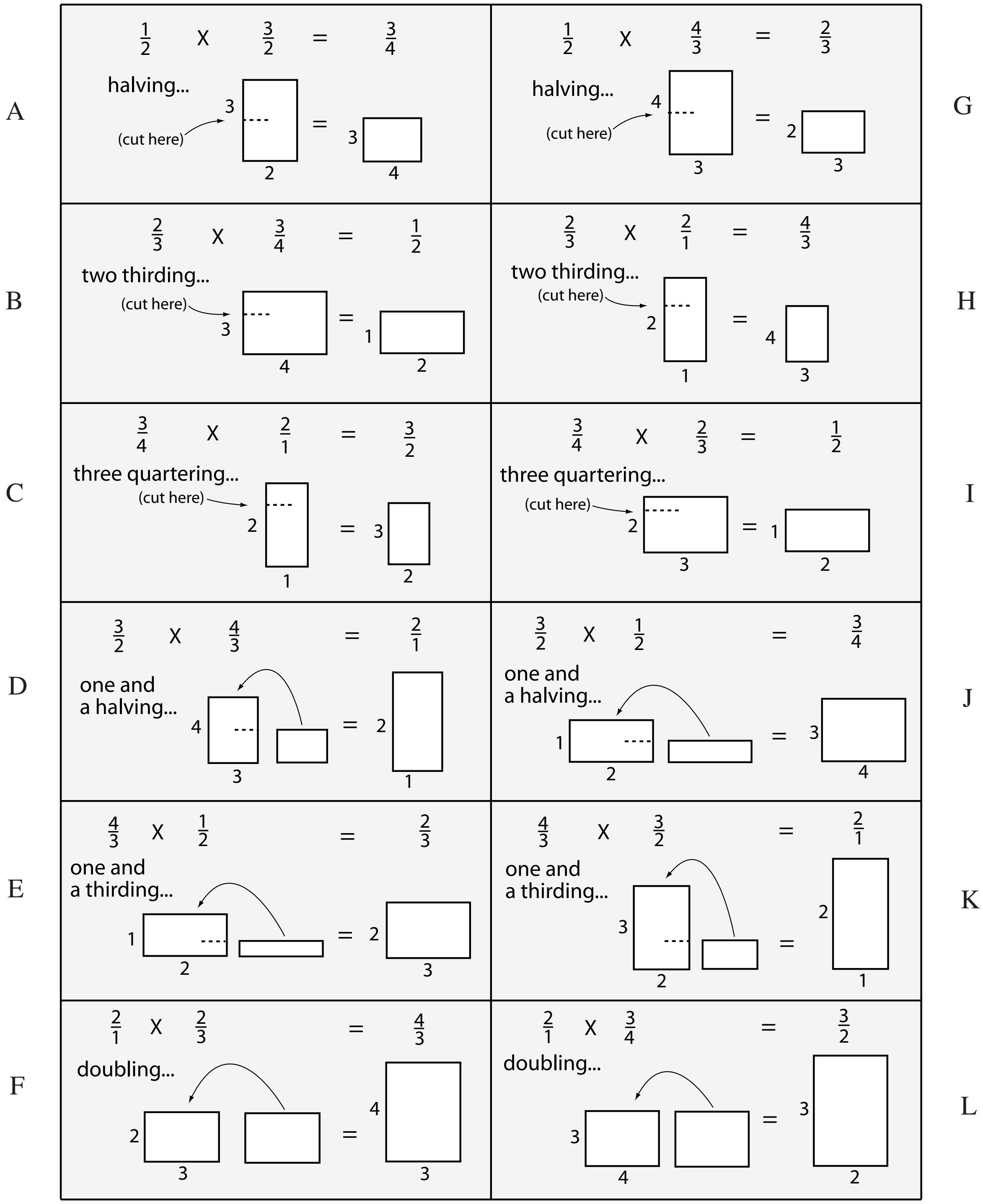
Thus, in a physical (milk) or
geometrical (rectangles) demonstration,
the, now, 12 equations all
describe different events.

The results are the same.
Only the order
of the multiplicands
has been switched.

$\frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$	$\frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$
$\frac{2}{1} \times \frac{2}{3} = \frac{4}{3}$	$\frac{2}{3} \times \frac{2}{1} = \frac{4}{3}$
$\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$	$\frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$
$\frac{3}{2} \times \frac{4}{3} = \frac{2}{1}$	$\frac{4}{3} \times \frac{3}{2} = \frac{2}{1}$
$\frac{3}{4} \times \frac{2}{1} = \frac{3}{2}$	$\frac{2}{1} \times \frac{3}{4} = \frac{3}{2}$
$\frac{4}{3} \times \frac{1}{2} = \frac{2}{3}$	$\frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$

Yet each of these expresses
a different "geometric event."

A dozen interesting interrelationships involving the 3 main harmonies



Even a cursory glance at this chart will tell you
that there are 12 different "events" going on here.

There are three reasons why I've delved into this matter so deeply.

The first: Dee enthusiastically recommended it in Aphorism 18.

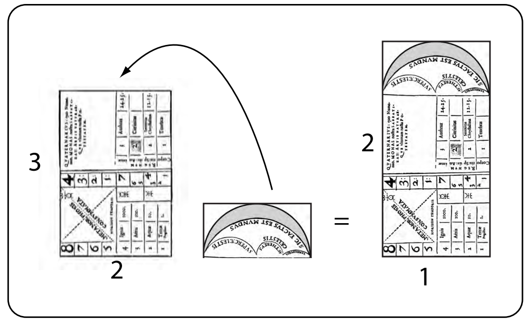
The second: "Look at all the fabulous interconnections between 1/2, 2/3, and 3/4"!!!

The third: Dee uses these "events" in various ways in the *Monas Hieroglyphica* illustrations.

For example, look at the event labeled "K."

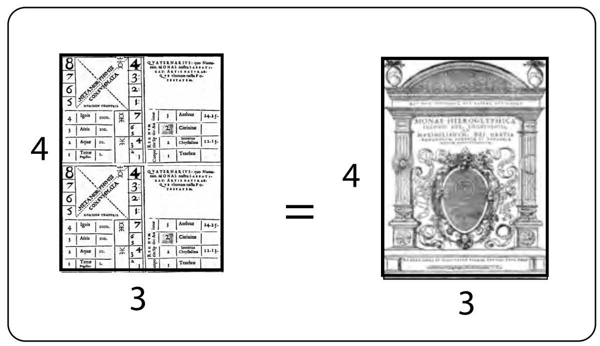
This is essentially what Dee is expressing by the two versions of his "Thus the World Was Created" chart.

Adding another "third" to the "rectangular part" of the chart"(2 by 3) will result in the "ballooned 360" version of the chart (1 by 2).

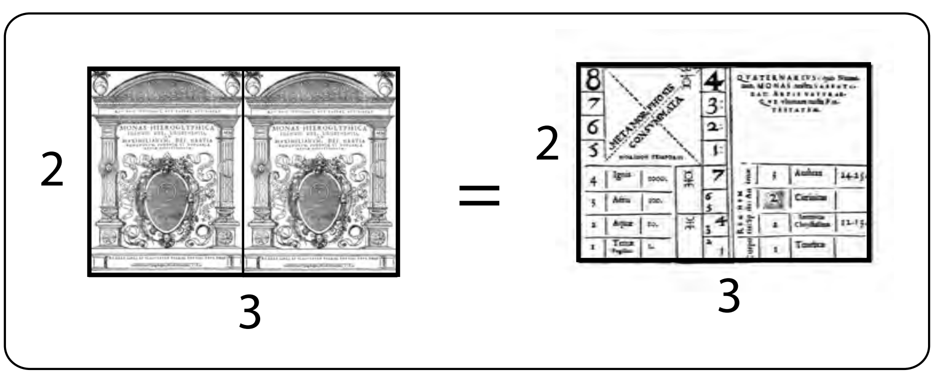


As another example, the events labeled "A, G, F, and L" can all be seen as expressions of either "halvings or doublings."

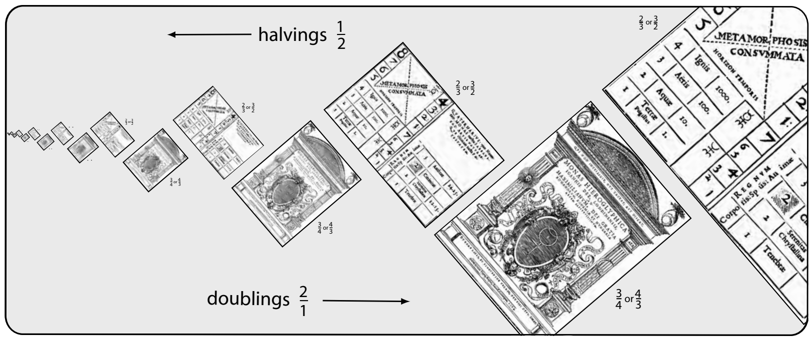
Events F or G might be seen as two "rectangular Creation" charts (each 2 by 3) fitting onto the Title Page (which is 4 by 3).

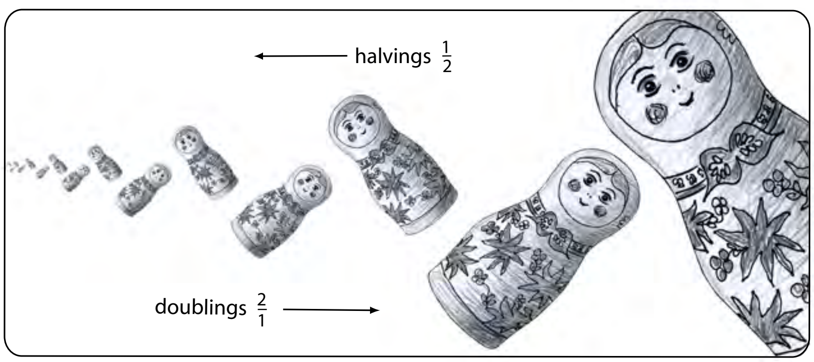


Look at the events labeled "A or L"
They mignt be seen as
two Title Pages (4 by 3)
fitting into the
"rectangular Creation Chart"(2 by 3)



I call this curious relationship the “Russian doll effect”
after the Russiian”Matryoshka” nesting dolls.
One fits in the next,
which fits in the next,
which fits in the next...





I realize that following all these fractions and shapes can get confusing.
The main point here is that 1/2, 2/3, and 3/4 are wondrously interwoven.

This visual inventory will provide clues to ***other*** relationships that Dee is expressing in his illustrations and their invisible, yet implied, grids.

There is another clue which Dee planted on the Title page
that exhibits this Russian doll “halving and doubling” phenomenon.

I’ll give you a hint:
“Tangentially, it’s related to a triangle (but this time, not an equilateral triangle).”

But first, let's look at that other category of ratios, the "part to part" ratio.

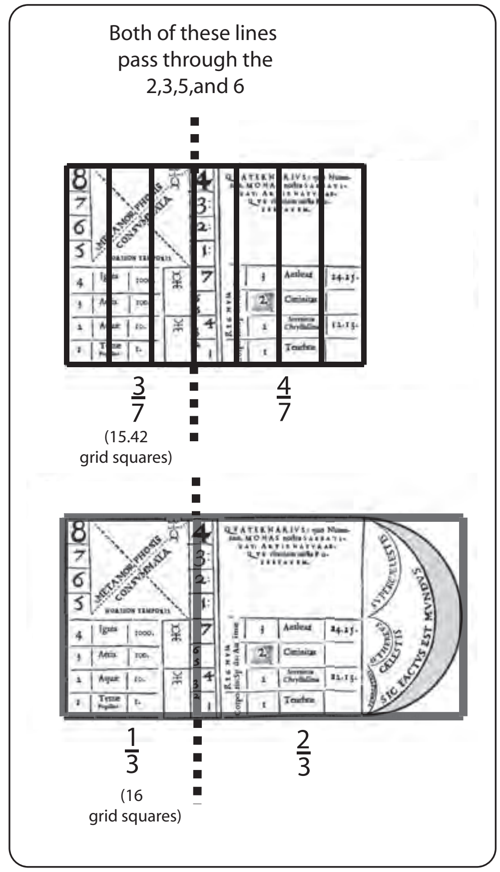
Let's take a quick look at some wondrous "coincidences" that occur with the "part to part" ratio 3:4 (actually 3/7 to 4/7) on the "Thus the World Was Made" chart.

The 3:4 "part to part" ratio "dividing line" of the width of the "rectangular part" of the the chart passes through those numbers "2, 3, 5, and 6."

We've already seen a vertical line, which takes the same route. That is, the "1/3 mark" of the width of the full "1 to 2" chart. How can these lines be the same? Well, actually they're slightly different.

One is actually 15 3/7 grid squares from the left edge of the chart, while the other is 16 grid squares.

But they are close enough that they both pass vertically through the "2, 3, 4, and 6 (which add up to 16)."



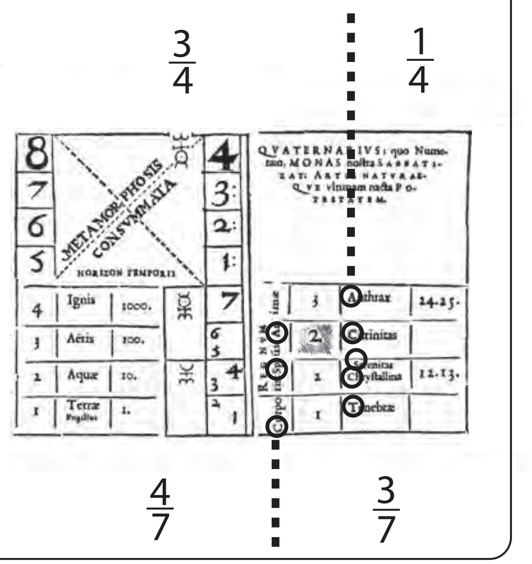
An even more revealing clue pops up when we investigate the 3:4 "part to part" ratio's alter ego, 4:3. A vertical line drawn from the 4/7 mark of the chart ascends through the tops of the vertically-written words Corpo-ris, Sp[irit]us, and Animae. It passes through the tops of the capitalized letters C, S, and A, but not the lowercase letters.

This result was suspiciously similar to another demarcation line I had seen – the line marking 3/4 of the width of the "Rectangular part" of the chart.

This line rose vertically straight through the capital letters starting the words Tenebrae, Chrystallina, Serenitas, Citrinitas, and Anthrax.

The word Serenitas is actual indented a bit, but including it, the capital letters along this line are: T, C, S, C and A.

The 3/4 "fraction" and the 4:3 "ratio" highlight Capitalized Letters that relate to A,S,and O



Though C, S, and A are common letters, it's very suspicious that they occur in both these sets of words, and that they are aligned with the 4:3 ratio mark and the 3/4 fraction mark.

This obvious clue puzzled me for quite a while. It wasn't until I was able to decipher Aphorism 18 of Dee's Propadeumata Aphoristica that I got what it meant.

As explained earlier, the A, S, and O of Axiom 18 are a shorthand code for "point line and circle."

To Dee, **point, line, and circle** represented **retrocity, one, and zero** or **oppositeness, the all, the nothing**, (the 3 essential parts of zero-one).

The "primary effect" of zero-retrocity one is 2, the "secondary effect is 3, and the tertiary "effect" is 4.

Dee's letters ASO, which create 2, 3, and 4, are a very important basis for his mathematical cosmology. It's only logical that he would include them in his chart describing the **Creation of the World**.

I think he wanted the reader to see the capital C's in Chrystallina and Citrinitas as two half circles, which could combine into the letter O.

Thus, his **ASO** is represented by **A**nthrax, **S**erenitas, and **C**hrystallina/**C**itrinitas.

In the three words Corporis, Sp[irit]us, and Animae, there is only one "half moon" C.

But the word Corporis itself can provide two O's, one of which he emphasized by the breaking word into syllables by a horizontal chart line (Corpo ris). Thus, Dee's **ASO** seems to be represented by **A**nimae, **S**p[irit]us and C**O**rp**O**ris.

I'll admit that this solution is a strange one, but "point, line, and circle" are critical to Dee's thinking.

They're not simply in Aphorim 18, but they comprise the very first 2 Theorems of the *Monas*.

And there doesn't seem to be any other reference to them anywhere else on this important summary chart.

(Incidentally, I think the T in Tenebrae is actually a decoy. If Dee had listed four alchemical stages that began with the letters A,C,C, and S, the correspondence with the CSA (of Corporis Spiritus Animae) would have made the clue **too** obvious).

Finding quarters, thirds and halves of the Title Page.

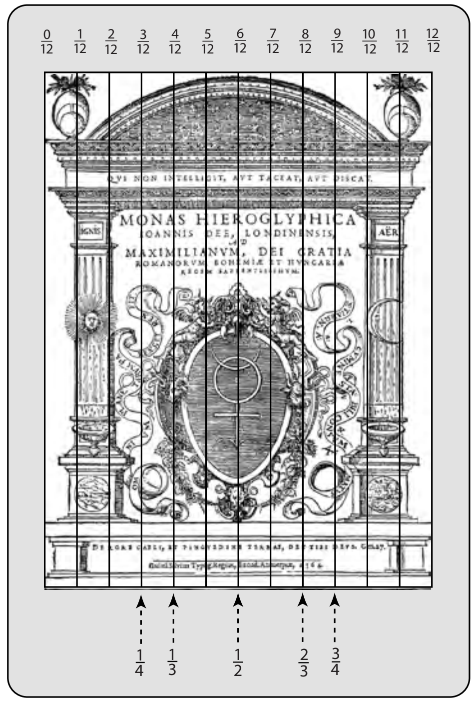
Let's now examine how the Title Page might be chopped up into quarters, thirds and halves.

To make the "vertical divisions" easier to see, I've divided the width into 12 parts.

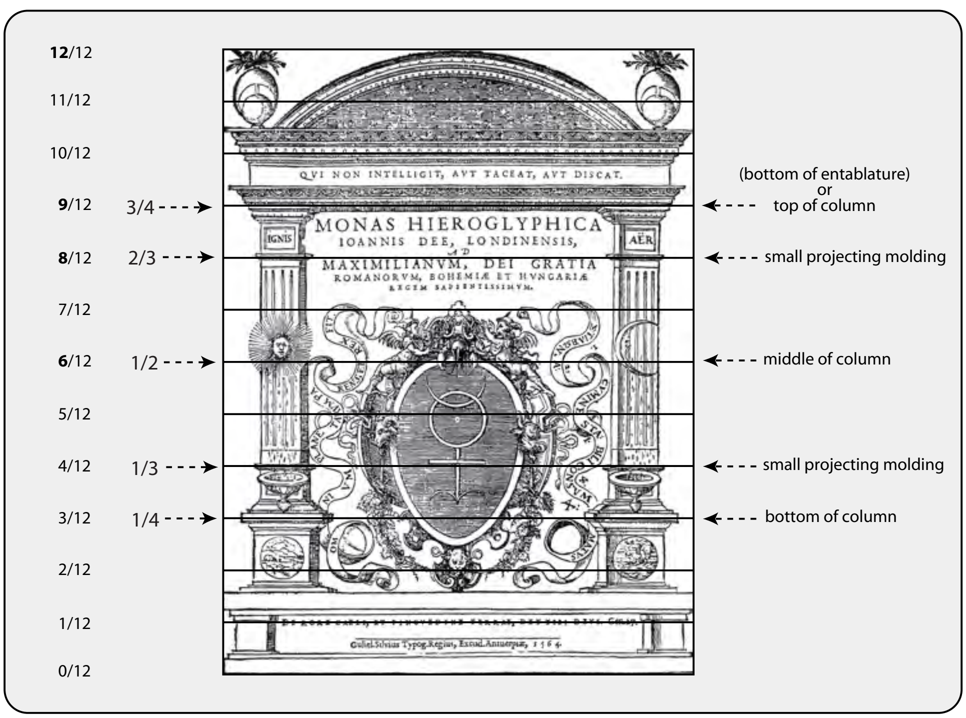
Notice that the 1/2 mark is the centerline of the page's side-to-side symmetry (except that the *Monas* symbol and emblem are ever so slightly "off" from the line – more on this later).

The 1/4, 1/3, 2/3, and 3/4 marks don't seem to align with anything in particular, however, the width of the **two architectural columns** seems to "fit the grid!" (between 1/12 and 2/12; and also between 10/12 and 11/12).

But even more exciting correspondences can be found when the architecture is sliced horizontally! Here the **height** is divided into 12 parts.



The **1/4 mark** aligns with the bottom of the column (or the top of the pedestal).
The **1/3 mark** aligns with a thin piece of projecting molding just above the two dew-collecting basins.
The **1/2 mark** aligns with the midpoint of the columns.
The **2/3 mark** aligns with another piece of projecting molding just below the words IGNIS and AER.
The **3/4 mark** aligns with the top of the columns (or the bottom of the entablature).



These alignments aren't readily apparent to the casual reader
because the columns are "busy" with decorative elements.
Also, because the anthropomorphized Sun and Moon symbols
are clearly not at the middle-height of the columns,
one might not suspect a grand underlying symmetry.

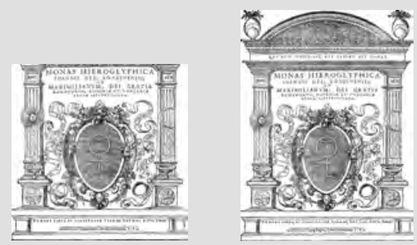
The "supported" architectural elements (entablature and "heavenly" dome)

Of particular significance is the 3/4 mark.

It separates the "supported" elements
(the spanning entablature and the "heavenly" dome).
from the "supporting" elements
(like the foundation which rests solidly on Earth,
and the strong columns which have
the names of the 4 Elements on them)

The "supporting" architectural elements (foundation and columns)

One way to see this as "3 to 4"
is by comparing the "supporting" elements
with the "whole" Title Page,
thus making an expression of
"Quaternary rests in the Ternary."

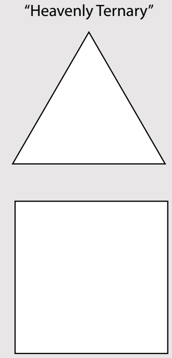


Comparing 3 "parts" to a "whole" of 4
is an expression of
"Quaternary rests in the Ternary"

If we express this 3:4 ratio "geometrically,"
it might be seen as "triangle:square."
It seeems that one reason Dee intentionally
placed the "3/4 mark" at this important place
was to make a cryptic expression
of this important theme in the *Monas*.

In a sense,
Dee is expressing the "Holy Trinity : Four Elements."
It's the the "Heavenly Ternary : Earthly Quaternary."
It's the triangle:square faces of the cuboctahedron,
(which are in a 4:3 ratio with "8 triangular faces and 6 square faces.")

"Heavenly Ternary"

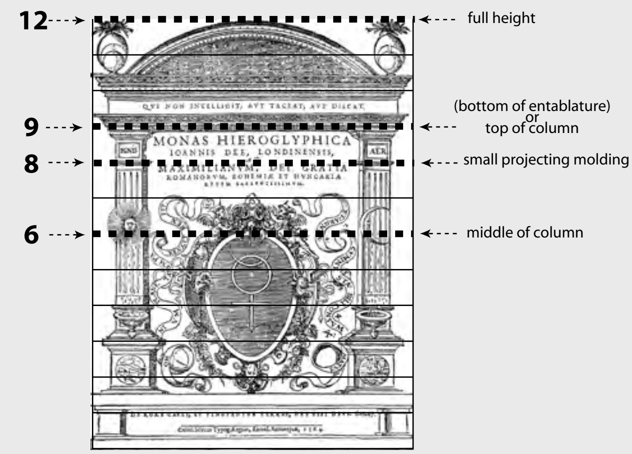


"Earthly Quaternary"

This is conceptually similar to
The "Earthly Quaternary"
rests in the "Heavenly Ternary"

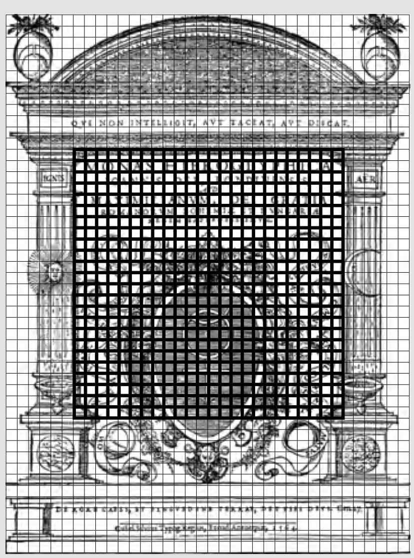
Another reason is that 3:4 is also 9:12.
As I have sliced the Title Page into 12 parts,
the 1/2 mark is at **6,**
the 2/3 mark is at **8,**
the 3/4 mark is at **9**,
and the full height is at **12**.

It seems as though Dee
is also expressing
Nicomachus' and Boethius'
"greatest and most perfect harmony,"
"6, 8, 9, and 12 ."



What do you think is playing at the theater?

Next, superimposing the "48 by 36 grid" on top of the Title page reveals a beautifully proportioned shape, which is there, *yet not there*.



The 24 by 24
"Theatre"
between the columns
(in the 48 by 36 overall grid)

The full height of the columns is 24 grid squares
(from pedestal to entablature).

The width of the space "between the columns"
is **also** 24 grid squares.

This means that what I call the "theater,"
the empty space in the middle of everything,
is a **24 by 24 square**.

I'm **not** including the empty space between
the pedestals in what I call the theater.

That empty space, along with the many other features of this
visually busy Title page make it challenging to perceive
that the theater is **square** and that it is **perfectly centered**
(both vertically *and* horizontally) on the Title Page.

Let's explore the interesting "play"
going on in Dee's theater.

THE TRIGONOMETRY OF THE MERCURIES' SPEARS

Oh no! I hate trigonometry.
I've long forgotten any trigonometry I ever knew!

Don't worry, dear reader, this chapter is easy as pie to follow,
even if you never studied trigonometry in the first place.
Even the mouthful of a word "trigonometry" is a turn-off.
But it's elegantly simple.

Tri means "three"
gon means "corner or angle"
metria means "measurement"

Trigonometry is the measurement of triangles.

On Dee's Title page, most of the architectural elements are at 90 degree angles to each other.
The most prominent angle is the one created by the two Mercuries' spears.
It looks like a 60 degree angle of an equilateral triangl, but it isn't.
It's about 67 or 68 degrees (it's hard to be more precise just using a protractor).

To understand what Dee is trying to tell us, we must
explore a certain aspect of trigonometry called a "tangent"

Conveniently, Dee illustrated a "tangent" in the emblem following Theorem 24.

It shows the **point** where a **line** is tangent to a **circle**
or as Dee pus it,
"Contact at a point."

A brief history of the word "tangent."

Dee's phrase "Contact at a point" might be boiled down to one single word: **tangent.**
This comes from the Latin verb *tangere*, which means "to touch."

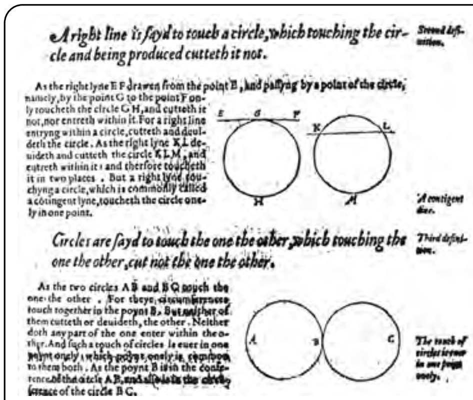
When Euclid wrote of a straight line that "is said to touch a circle" (Book 3, Definition 2)
he used the Greek word *ephaptesthai*, which means "to bind on to."

When Henry Billingsley translated this in his
1570 *Euclid's Elements*, he used the term "**contingent**"

Omitting the suffix "con,"
it's obvious that "tingent" and "tangent"
are simply different "pronunciations" of the same word!
The Latin word *contingere* means "to touch each other."
(*con*, "together with" + *tangere*, "to touch")

In its "participle adjective" form, it becomes *contact*.
Contingere also morphed into our English words con-
taminate
and contagious (to come in "contact" with diseases).

In other words, **"tangent = contingent = contact."**


A right line is sayd to touch a circle, which touching the circle and being produced cutteth it not. Second definition.

As the right lyne E F drawen from the point E, and passyng by a point of the circle, namely, by the point G to the point F only toucheth the circle G H, and cutteth it not, nor entreth within it. For a right line entryng within a circle, cutteth and deuideth the circle. As the right lyne K L deuideth and cutteth the circle K L M, and entreth within it: and therfore toucheth it in two places. But a right lyne touchyng a circle, which is commonly called a cōtingent lyne, toucheth the circle onely in one point. A contigent line.

Circles are sayd to touch the one the other, which touching the one the other, cut not the one the other. Third definition.

As the two circles A B and B C touch the one the other. For theyr circumferences touch together in the poynt B. But neither of them cutteth or deuideth, the other. Neither doth any part of the one enter within the other. And such a touch of circles is euer in one poynt onely: which poynt onely is common to them both. As the poynt B is in the conference of the circle A B, and also in the circumference of the circle B C. The touch of circles is euer in one point onely.

Henry Billingsley,
in his 1570 translation of Euclid's *Elements*,
calls the upper left drawing
"A contingent line"

The Danish mathematician Thomas Fincke (1561–1656) is credited
with being the first user of the word tangent (*tangentibus*)
in his 1583 *Geometraie Rotundi (Geometry of Circles)*.

Interestingly, Dee (who was 34 years older than Fincke)
owned an Ephemeris that Fincke wrote for the year 1582
(the year the John Dee Tower was being built).
(An ephemeris is an astronomical almanac of the angular positions of celestial objects).

Thomas Blundeville ,in his 1594 *Exercises,* writes
"Our modern Geometricians have of late invented two other right lines
belonging to a Circle, called lines Tangent and lines Secant."
A secant is simply a line which cuts a circle in 2 places.
(*secare* means "to cut"). (*OED,* tangent p. 72-3).

To summarize, **contact (at a point), contingent, and tangent**
all mean the same thing geometrically.
Dee is hinting that "tangent" is a ***clue*** to another of his puzzles.

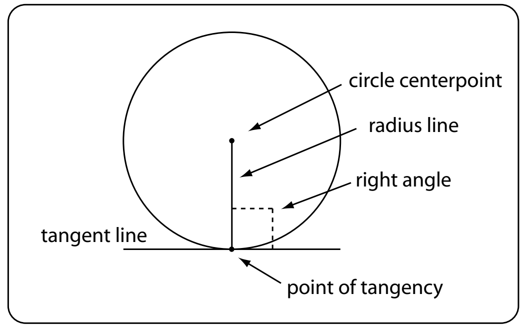
A Geometric "tangent" and a Trigonometric "tangent" are very closely related!

From high school trigonometry you will probably recall the names of functions called "sine, cosine, and tangent."

Using Dee's simple illustration, we will see how this Trigonometric "tangent" relates to Geometric "tangent."

Notice that Dee has drawn in two points: one at the point of contact and one the center of the circle. As per Theorem 2, it's obvious that these two points define a radius of the circle.

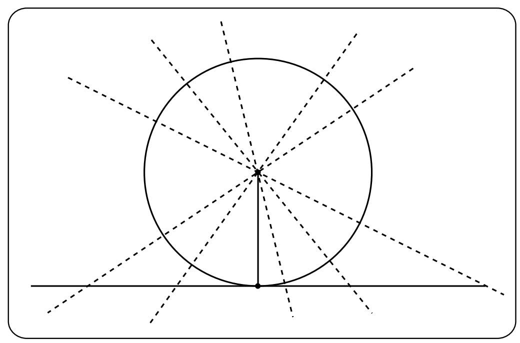
It's also clear that the "radius line" and the "tangent line" are perpendicular, forming 90° right angles where they cross.



This is fine, but we still don't have a triangle.

To make the "third side" of a triangle, any number of lines might be drawn through the circle's center which eventually cross the tangent line.

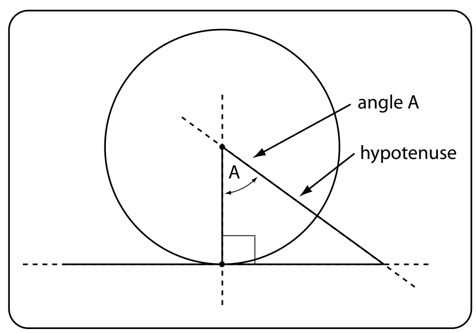
(Actually all lines, except the line parallel to the tangent line.)



Let's take one sample line, form a triangle, and rename the three sides in "trigonometric" terms.

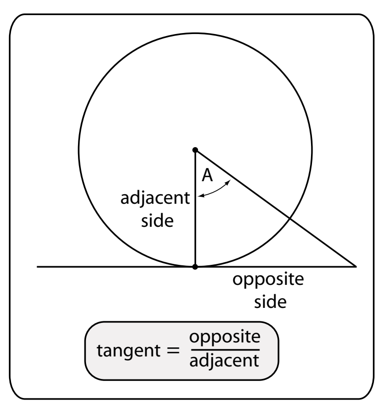
As we have a right triangle, the side opposite the right angle is the hypotenuse.

Let's call that angle whose apex is at the center of the circle "angle A."

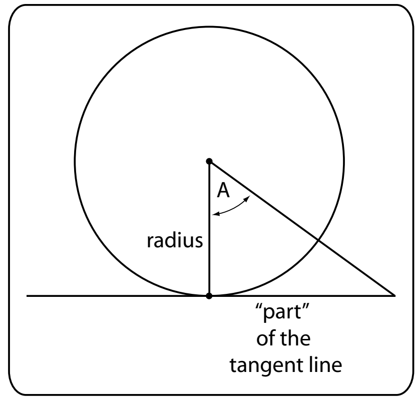


"Angle A" opens up to a side called "opposite." And right next to it is a side called "adjacent" (the radius of the circle).

The trigonometric function called a "tangent" is simply the relationship between the length of the "opposite" side and the length of the "adjacent" side.



So it's easy to see how a "geometric tangent" relates to a "trigonometric tangent."



We might also express this as:

$$\text{tangent} = \frac{\text{"part" of the tanget line}}{\text{radius}}$$

To understand how thrilling a tangent was for Dee,
let's take a brief excursion of the exciting "History of Trigonometry"
that takes us from Greece, India, Arabia, to near the "King's Mountain" in Germany,
and finally to Dee's library in Elizabethan London.

The action-packed history of Trigonometry.

Carl Boyer in *A History of Mathematics* explains that the ancient Egyptians and Babylonians studied the geometry of triangles. But as they "lacked the concept of angle measure," they primarily studied the interrelationships between the sides of the triangle.

Early ***Greek*** astronomers like Aristarchus (ca. 310 BC – ca. 230 BC) and Eratosthenes (ca. 276 BC – ca. 194 BC), (known for his sieve of prime numbers) worked with angles in trying to determine the size of the earth and the distances to the sun and the moon.

Neither Euclid (ca. 300 BC) nor Archimedes (ca. 287 BC – 212 BC) dealt with trigonometry "in the strict sense of the word," but they do have geometric theorems that are equivalent to specific laws of trigonometry.

It was ***Hipparchus*** (ca. 180 BC – ca. 125 BC) who appears to have developed the first trigonometric tables, earning him the title: **"Father of Trigonometry."** Theon reports that Hipparchus wrote a treatise in twelve books on "chords in a circle." (A chord is the straight line joining the two ends of an arc.) (Boyer, p. 162).

Its known that Menelaus (ca. 100 AD) wrote a treatise on Chords in a Circle, but only his text on spherical triangles (3 made by connecting 3 points on a sphere) has survived.

The most influential treatise on trigonometry is *Mathematical Synthesis* (*syn* "together" + *tassein* "to arrange") by by ***Ptolemy of Alexandria*** (ca. 90 AD – ca. 168 AD). This work was so significant that mathematicians called it the "megista" or the "greater" collection distinguish it from the earlier work of Aristarchus and Hipparchus, which was called the "lesser" work.

Later, the Arab mathematicians referred to Ptolmeys book as ***Almagest***, "***the greatest***", the name it has retained for centuries. (Note the similarity between the word Almagest and the Dee's word "magistral").

All of Ptolemy's trigonometric tables, and his descriptions of how he calculated them have survived. Ptolmey also wrote important books on geography, optics and astrology (referred to as the Tetrabiblos). (Boyer, p. 164-172).

It should be noted here that Dee owned copies of two works in Hipparchus and over 40 books by Ptolemy, including at least a half dozen copies of *Almagest*.

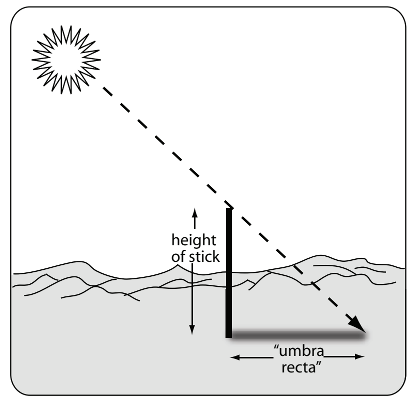
The Neoplatonist mathematicians didn't make many advances in trigonometry. The next developments took place in India. The ***Indian*** astronomer ***Aryabhata*** (476 AD – 550 AD) first defined "sine" as the relationship between half a chord and half an angle.

Arab mathematicians assimilated the Greek and Indian ideas. The caliph al-Mamun (809–833) is said to have had a dream in which he conversed with Aristotle. Subsequently he ordered his scholars to translate as many Greeks works as they could, including Euclid's *Elements* and Ptolemy's *Almagest*.

Al-Mamun built a study center called the "House of Wisdom" in Baghadad. One of the foremost teachers there was ***al-Khwarizmi*** (ca. 780 AD – ca. 850 AD) who wrote texts on arithmetic, algebra, the astrolabe, the sundial, as well as astronomical tables. Many refer to him as the "Father of Algebra." He inspired generations of Arab and European mathematicians. (Boyer, p. 227 and p. 230).

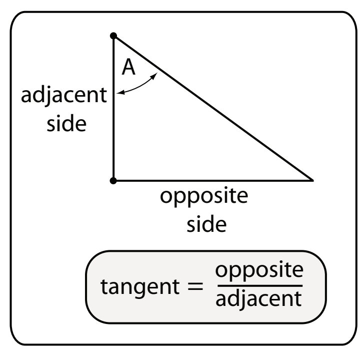
Greek, Indian, and Arab trigonometry up to this point was mostly concerned with sines and cosines, which involve the angles formed by chords of a circle.

Around 860 AD, the "tangent" function was explored in conjunction with sundials and horometry (time measurement). The length of the shadow made by a vertical stick (gnomon) in the ground was called an ***"Umbra recta"*** ("straight shadow" or "right shadow")



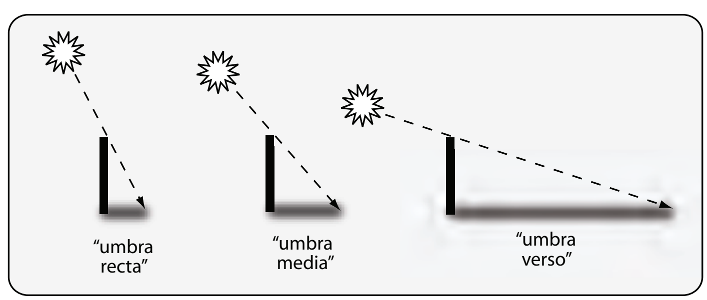
The proportion of the
"stick height" to the "umbra recta"
is same as the "tangent" function in trigonometry.

As shown earlier, in a right triangle, the tangent of angle A
is the opposite side compared to the adjacent side.



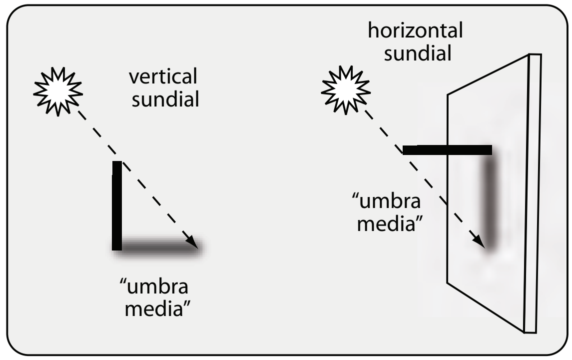
If the length of the shadow
was more than 45°, it was called "umbra verso,"
meaning "reverse shadow" or "turned shadow,"
(which corresponds to the trigonometric function of cotangent).

When the shadow's length was exactly 45°
it was called "umbra media"
(meaning "middle shadow").

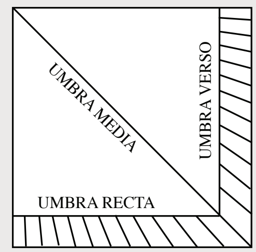


The shadow had various names because
there were two main types of sundials:
a vertical stick in the ground
or a horizontal stick coming out of a wall.

When the shadow lengths of these two sundials matched,
it was "umbra media." The sun was at a 45° angle.



Astrolabes and quadrants generally have a
"shadow square" engraved on them.
This is is a scale of these three kinds of umbras.



shadow square
engraved on astrolabes

Around 950 AD, **Abu'l-Wafa** devised a new mathematical method of calculating tables for sines, cosines, and tangents. While Ptolemy's sine tables were calculated to **3 places** (when converted to decimal). .Abu'l Wafa's sine tablets were calculated to **6 places**.

(O'Connor and Robertson, Abu'l Wafa).

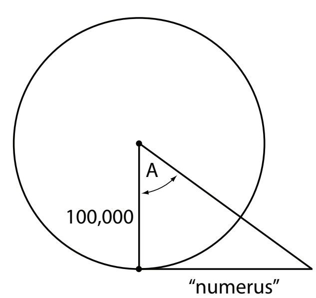
In the 900's and 1000's, Al-Battani, Al-Jayyani, and Omar Khayyam made further advances in trigonometric principles. In the 1300's Al-Kashi and Ulugh Beg developed tables calculated up to the equivalent of **8 decimal places**.

In the early Renaissance, Europe finally started to explore trigonometry. Johann Müller, of Königsberg was probably the most influential mathematician of the 1400's. He preferred to be called **Regiomontanus**, the Latinization of "King's Mountain," (which is what Königsberg means).

He studied mathematics and astronomy in Leipzig, Vienna, and Rome. He set up a printing press in Nuremberg with a goal of reprinting the works of all the Greek mathematicians. He completed a new Latin version of Ptolemy's *Almagest* which had been started by his teacher, Georg Peurebach.

But perhaps Regiomontanus' greatest accomplishment was his *De triangulis omnimodis*. This work contained over 50 propositions about triangles, but concentrated on sines and cosines. He wrote another book *Tabulae directionum,* specifically on tangents.

Regiomontanus doesn't actually use the word "tangent", instead uses only the word "numerus." To avoid fractions, he used the number 100,000 as the radius of the circle, and the "numerus" is the length of the tangent line for any given degree. (Boyer, p. 275).



Regiomontanus'
table of tangents

In the 1500's, a contemporary of Dee's, George Rheticus of Austria (1514–1574), was passionate about the study of triangles. In 1542, he published *On De lateribus et angulis triangulorum (On the Sides and Angles of Triangles)*, which were the sections of Copernicus' *De revolutionibus (On Revolutions)* dealing with trigonometry.

In 1551, he published *Canon of the Science of Triangles*, which was an introduction to his magnum opus *The Science of Triangles*. This ambitious project involved computing, by hand, about 100,000 ratios to at least 10 decimal places. Rheuticus died before its completion, but his student Valentin Otto saw that its 1500 pages were finished and printed. These tables were so accurate they were used up to the early 20th century for astronomical computations.

(Wikipedia, Georg Joachim Rheticus).

In the mid-1700's, Leonhard Euler further analyzed the wonders of triangles in his *Introductio in analysin infinitorum*.

Now-a-days, finding trigonometric functions is as easy as pressing a button on a good **hand calculator**.

Dee wrote about Trigonometry

But Dee was no casual book collector. He was an astute geometer who used these theorems and tables for his work in astronomy and navigation. Not only that, he wrote about his knowledge. Prior to writing the *Monas* he had written 2 books called *De nova Navigationum (A New System of Navigation)* and 3 books about entitled *De Trigono Circinoque Analogico (The Triangle and the Analogical Compass)*.

Dee mentions these works in the dedication of the *Propaedeumata Aphoristica*, though they were apparently never published, and the original manuscripts are longer extant.

Trigonometry books in Dee's library

This brief history of trigonometry serves as a background to understanding why Dee was so excited about tangents as demonstrated in his "Contact at a Point" emblem (and in other ways, as we shall soon see).

First, it should be noted that Dee personally owned many of these classic works on trigonometry. Roberts and Watson's catalog of Dee's library shows that Dee owned the works of the **Greeks**, **Aristarchus** (102, B298) and **Hipparchus** (270, M43f).

Dee owned over 30 books by **Claudius Ptolemy** including a half dozen copies of *Almagest*. He owned a manuscript copy of **Al Khwarizi's** *Tabulae astronomicae (Astronimical Tables)* which Roberts and Watson call "a handsome book, perhaps Dee's finest..."

(Roberts and Watson, p. 171).

But most significantly, Dee owned 7 books by **Regiomontanus** (1436–1476) including the table of sines, *De triangulus omnimodus*, and the table of "tangents" *Tabulae directionem*. Roberts and Watson note that Dee's first edition copy of Regiomontanus' 1551 *Tabulae directionem* was stolen when looters raided his library in 1583. Dee apparently bought a replacement copy, and signed it John Dee, 1602, which now resides in the Library of the University of Sidney in Australia. (Roberts and Watson, p. 157, D-15).

Dee also owned the Copernicus' treatise on trigonometry that **Georg Rheticus** had published in 1541 (catalog number 768). He also owned two copies of Rheticus' *Canon of the Science of Triangles* published in 1551 (catalog numbers 1274 and 1848).

Three more clues about shadows (and thus about tangents).

Before explaining what all this business of "tangent tables," "umbra recta," and trigonometry have to do with the *Monas* (beyond the "Contact at a Point" emblem), let me point out several important clues which suggest that Dee had "trig" in mind.

On two back-to-back pages of is *Letter to Maximillian* (p. 9 and p. 9 verso), Dee weaves the word "shadow" into the text 10 times: **Umbralites, Umbra, Umbras, Umbrae, Umbris, Umbras, umbralite, Umbrarum, Umgram, Umbratiles.** Using one word this inordinate amount of times sure suggests he wants us to be on the lookout for a clue involving shadows. (Dee also use the word "recto" throughout the *Letter to Maximillian*, but a stronger clue is his capitalizations of the somewhat synonymous Greek word ORTHOTOMEIN, "to cut in a straight line")

We've also seen that Dee makes three cryptic references to the **camera obscura** in his admonitions to the professions of Astronomers, Opticians, and Experts on Weights.

(Dee, *Monas*, *Letter to Maximillian*, p. 6).

He cryptically asserts that the camera obscura is useful for astronomers to study the movements of the "Caelestium Corporum," or "Heavenly Bodies." A key "heavenly body" is the sun, and its motions can be observed by following the solar disc projected inside the camera obscura.

A camera obscura used this way is essentially an "inside-out sundial," with the "hole" **acting like the "tip" of a gnomon**. If the solar disc is tracked as it moves across the horizontal floor, its "umbra recta" can be studied. If it is tracked as it moves across a vertical wall, its "umbra verso" can be investigated.

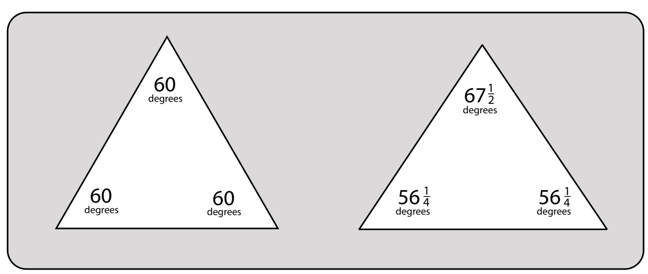
Finally, a very graphic clue that Dee wants us to look at shadows is the fact that the architectural illustration on the front cover has a dramatic sense of light. The light appears to be coming "from the left and above," as it forms shadows under the entablature and on the right side of the columns. Even the two urns reflect that directional light, which most assuredly is not emanating from the smiling sun engraved on the left column.

The Mercury spears sing "Quaternary rests in the Ternary."

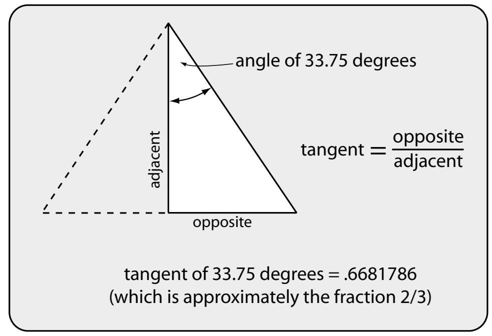
Now that we have a clearer picture of how Dee felt about the wonders of "triangle-measurement," we can better grasp why Dee carefully positioned the spears of the two Mercuries of the Title page to define an specific angle.

With a large plastic protractor, it's apparent that the angle of the spears is **more than 67 degrees, but less than 68 degrees.** As a starting point, let's call it call 67 1/2 degrees.

If 671/2 is the apex
of an isosceles triangle (with 2 sides equal),
the other 2 angles would be 56 1/4 degrees each.
Here's how it looks compared
with an equilateral triangle.

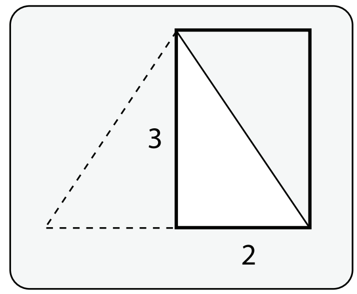


We can't use trigonometry on this triangle
as "trig" requires right triangles.
But, because it's an isosceles triangle,
we can split it in half vertically
and use "trig" on one half of it.

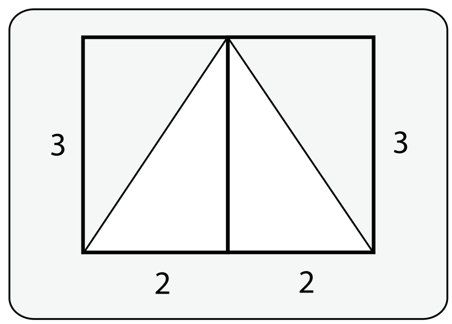


Half of 67.5 degrees is 33.75 degrees.
Pushing the "tan" button on my hand calculator,
I found that the **tangent of 33.75 degrees was**
.6681786
(rounded off to 7 digits)

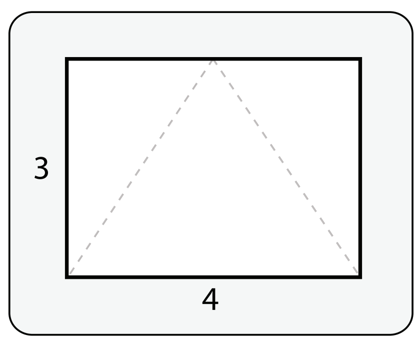
This was the clue that unlocked
the door to the Title Page illustration!
Here's how:



The decimal .6681786 is very, very close to .666666...,
which is one of Dee's favorite fractions, the harmony 2/3.
Using the hypotenuse as a diagonal, let's draw
a rectangle 2 units wide by 3 units tall.



As the other right-triangle is the same proportion,
let's add another same-sized rectangle.



Combining them makes a horizontal rectangle
in which the height (at the apex) to width (at base)
is in the proportion of 3 to 4.

Suddenly we have Dee's super-favorite
"Quaternary rests in the Ternary" proportion!

In other words, those two silent spears,
when seen as the apex of an isosceles triangle
are geometrically singing the song
"Quaternary rests in the Ternary!"

What the books in Dee's library would have told him about this angle and its tangent.

Remember, my measurement of the angle of the Mercuries' spears
of "**about 67 1/2**" degrees was made visually with a protractor.
So, half of that angle, "**about 33 3/4 degrees,**" is still an estimate.

My hand calculator informed me that to get a tangent which is .6666666,
"angle A" must be 33.690065 degrees.
We can safely round that off to 33.69
(which is almost 33 7/10, slightly less than my 33 3/4 estimate).

Fortuitously, the nearby John Hay Library at Brown University had copies of the same books
on tangent tables by Regiomontanus and Rheticus that Dee had in his library.
Regiomontanus' data (from the mid 1400's) says that to get
a tangent which is .6669170, an angle of 33 3/10 is required.
His result differs from my calculator's result only by about 4/10 of a degree.

(Regiomontanus doesn't actually use the word tangent.
He calls this table "*Canon Fecunda*" or "Fruitful Catalog").
(Regiomontanus, Joannes, *Primus liber tabularum directionum, Tubingae, Apud Haeredes Virici Mor*hardi, 1554)

But Rheticus' data (from the mid 1500's) is even more accurate.
He says that a tangent which is .6660768 results from an angle of 33 2/3 degrees.
His result differs from that of my calculator by only 2/100 of a degree!
(Georg Rheticus, *Canon doctrinae triangulorum*, Lipsiae,1551)

Doubling Rheticus' figure, makes 67 1/3 degrees.
This appears to be the angle that Dee actually intended for his Mercuries' spears.
This is extremely close to my hand-measured estimate of 67 1/2 degrees.
The difference is negligible, especially in a hand engraved illustration.

[Don't be confused. That 33 2/3 degrees result is ***no***t "one third" of 100.
That would be **33 1/3,** (like the old phonograph records).
It appears to be only coincidental that these numbers are so close.]

To summarize, by setting the two Mercuries' spears to 67 2/3 degrees,
Dee wants us to see the relationship between 2/3 and 3/4
which are two of the three main harmonies,

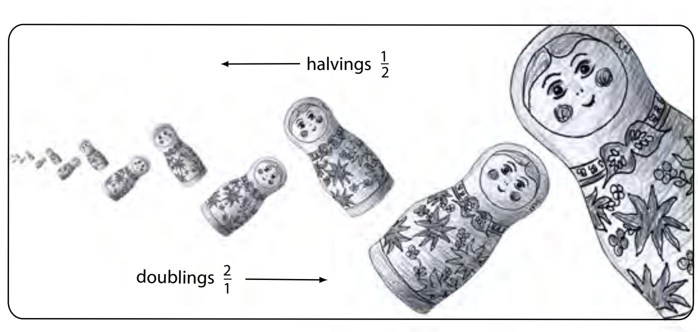
But what about that third harmony, 1/2?

Actually, This 1/2 (or 2/1) harmony is implied in the relationship between 2/3 and3/4.

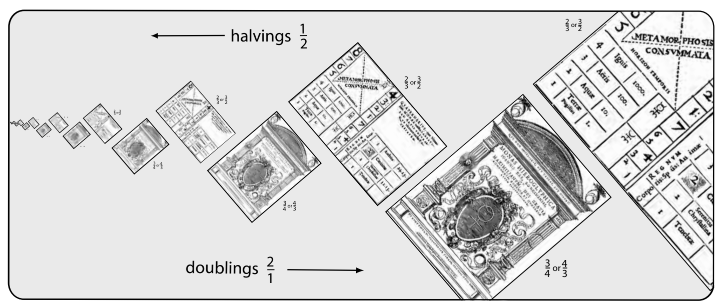
Halving 3/4 makes2/3.
Doubling 2/3 makes 3/4.

This interrelationship among the 3 harmonies is implicit in the Russian Doll's "halving and doubling dance..." which is the "dance involving two of Dee's illustrations"... which is the "dance of the two Mercuries":

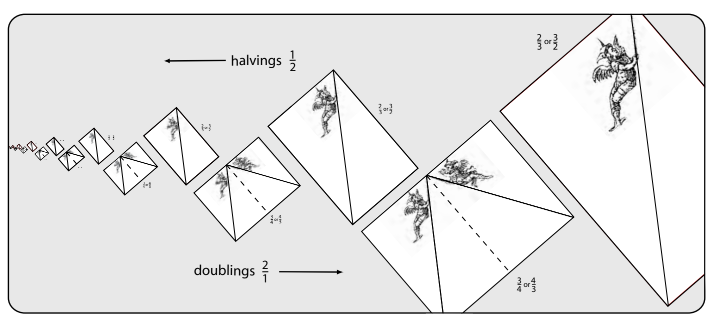
Just like the Russian Dolls do a "halving and doubling dance"...



...the Title Page and the "rectangular part" of the "Thus the World Was Created "chart do the same "halving and doubling dance"...



...and the Mercuries do the same "halving and doubling dance."



A tangible way to ge a feel for this “dance” is to take a piece of 8 1/2 by 11 paper (oriented vertically) and trim off approximately 1 inch from the right or left edge to make it 7 1/2 by 11, which is approximately the ratio 2:3. This is the proportion the “rectangular part” of the “Thus the World Was Created”chart.

Fold it in half, and it becomes the 3:4 shape, proportions of the Title Page. Fold it again, and it’s back to a 2:3 shape. Fold it again, and it’s back to 3:4. Contiue folding until you can’t fold any more. This demonstrates the dance of “halvings.

Now, undo all those folds, and you will experience the dance of “doublings.”

All this is a very nice way to see the harmony 1/2 integrate with 2/3 and 3/4, but I think Dee wanted us to see 1/2 integrate with these other harmonies in an even grander way!

To explain, let’s first examine where the Mercuries’spears would point if they were elongated downwards.

The extension of the right spear seems to nicely intersect the lower right corner of the Title Page. But alas, the extension of the lower left spear clearly “misses the mark” of that lower left corner.

(Note: The “hole in the sheild” where the Mercuries’ spears meet is about 28 grid squares from the bottom of the Title Page.)

The Title page appears to be a symphony of symmetry, so why is the emblem askew like this?

(This is no printing error, as the emblem and the architecture are both engravings.)

Take a look at the two “flowing ribbons.” The right ribbon has some “breathing room” between itself and the right column (next to the Moon).

The left ribbon actually flows behind the left column (next to the sun).

Similarly the bottom of the emblem (just below the central Lion’s face) is awkwardly nipped off by the top of the architectural “foundation.”

Furthermore, the whole emblem is slightly askew, clockwise, with respect to the prominently right-angled architectural “frame.”

This seems contradictory to Dee’s proclivity for Symmetrical perfection.

Why would Dee, who is so concerned with the perfect execution of every “jot and title,” allow this to happen. My conclusion is that it was done intentionally, as a clue.

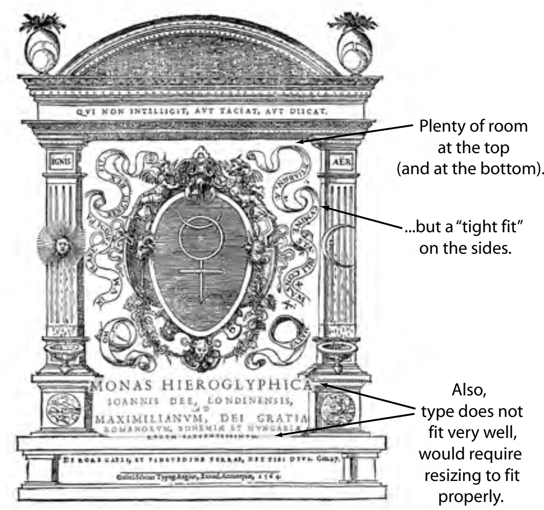
As explained earlier, when I first saw Dee's Title Page, I sensed that there was something "wrong" or "visually disturbing" about it. Here was this exquisite emblem with airy flowing ribbons that felt visually "heavy" in the space between the columns. The bases of the columns seemed to be "pinching" the emblem which yearns to be set free, to "float in the breeze."

It felt as though the emblem and type should be reversed with each other, so the square-shaped emblem would fit into the square-shaped "theater" between the columns.

But there were some problems with this.

The emblem would fit with plenty of room at the top and bottom, but the fit on the sides would still be tight.

(Also the type, as set, would not fit very nicely in the bottom space between the pedestals. The *Monas Hieroglyphica* title would have to be set in smaller type, and the spacing between the rest of the lines of type would have to be made tighter.)



My first conjectured "restoration" of the Title Page felt right, but still had some problems.

Now, another "compromise solution" became apparent. The type might be "split up." The title *Monas Hieroglyphica* would remain fitting nicely at the **top,** and Dee's name and the dedication to King Maximillian would fit nicely **below**, between the pedestals. The emblem still floats, but fits snugly in its allotted space.

Dee's Title page
(as printed in 1564.)

The emblem "feels" too low and "heavy.
All of the type is clustered above it.

The "restored" Title Page.

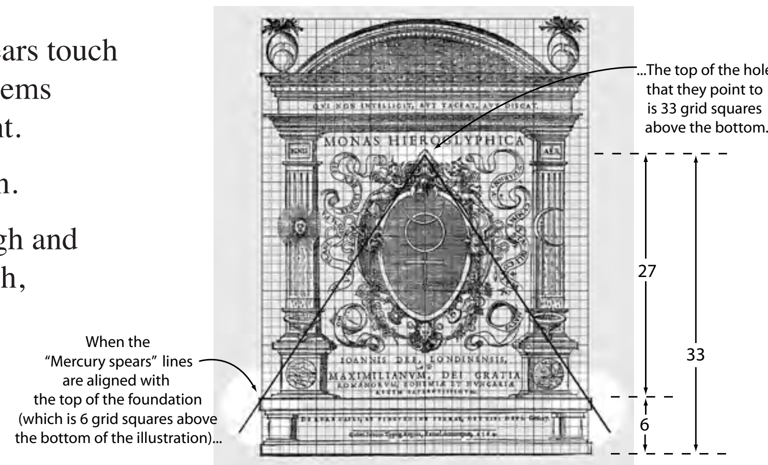
The emblem floats nicely between the columns.
The type is broken up, with the title on top
and Dee's and Maximillian's name below.
The whole design has an "airier",
more "balanced" feel to it.

The bottom of the flowing ribbons now swirl freely, as if blown by a gust of wind deflecting off the angled bases of the columns. Also, the Sun and Moon on the columns seem aligned with the gracefully "indented swirls" in the middle of the ribbons.

The fact that the extended Mercuries' spears touch the top corners of the foundation seems to confirm that this was Dee'intent.

A triangle with a solid foundation.

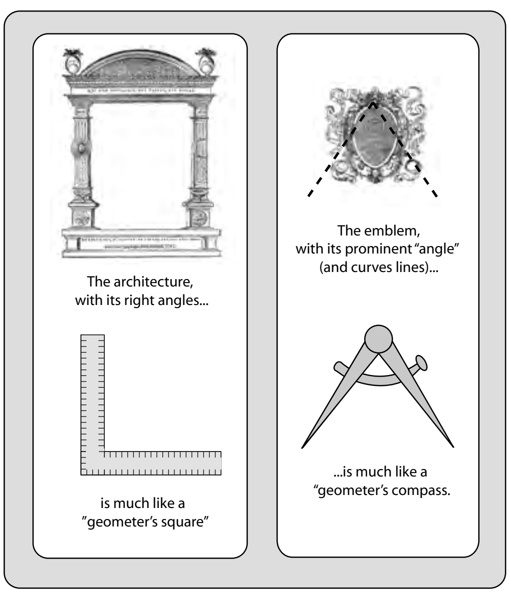
As the foundation is 6 grid squares high and the triangle is 27 grid squares high, this puts the hole at the tips of the Mercuries' spears at 33 grid squares high.



Looking at this comparison diagram, I think even a person who is not a graphic designer would agree that the 'restored" Title page just "feels" better.

But why would Dee do this? Just for the fun of it?
I don't think so. That's not Dee's style. This is clearly a clue. But to what?

Dee knew that most readers would assume such a finely-crafted Title Page to be "frozen," or "final" or "printed the way the author approved it." But by this restoration, it's clear that Dee wanted the reader to "loosen things up a bit" and see the architecture and the emblem as two separate things. Tearing them entirely apart helps see what he is describing.



Except for the gently curved dome, the architecture involves straight lines, which are all either at right angles or parallel to each other.

It is reminiscent of a "**geometer's square**," also known as a carpenter's square.

The emblem, on the other hand, involves mostly curvy things with no right angles (except for the Cross of the Elements of the Monas symbol).

The one angle it does feature is the 67 2/3 degree angle of the Mercuries' spears. This prominent angle makes it seem like a "**geometer's compass**" splayed open to 67-2/3 degrees.

Although this seems to be describing something angular, remember that a geometer's compass is used to draw curved lines.

It should be noted that the carpenter's square and the drafting compass have been used symbolically for centuries. In ancient China, the mythical scholar Fuxi carried a carpenter's square in his hand. Albrect Dürer incorporated one in his famous etching "Melancholia" symbolizing the apostle Thomas who was the patron saint of builders.

Medieval manuscripts and even the artist William Blake depicted the Creator as a geometrician using a drafting compass to construct the globe.
(Hans Biedermann, *Dictionary of Symbolism*, p. 75 and 321).

Frequently the square and compass are combined, representing a Union of Earth (square) with Heaven (circle). This union of square and circle can be seen in the design plan of the Temple of the Heaven in Beijing China.

Nowadays the square and compass are important in the symbology of Freemasonry, which was officially founded in 1717, long after Dee's time.

You can be assured that the geometer and navigational expert Dee had many differently-sized squares and compasses on his drafting table.

William Blake's painting "Ancient of Days" features a geometer's compass opened to a right angle

Utilizing this "built-in" geometer's compass.

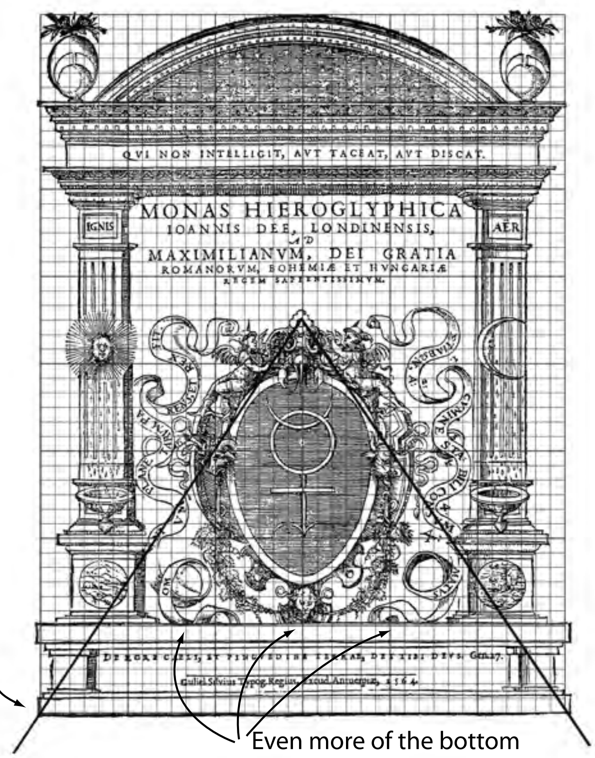
And what did Dee want us to measure with the "geometer's compass"?
He wants us to use it to **measure the architecture**.

So let's "keep it loose" and use the compass to **"measure" the "foundation"** of the architecture. Here, the extended lines of the Mercuries' spears are aligned with the bottom corners of the Title page.

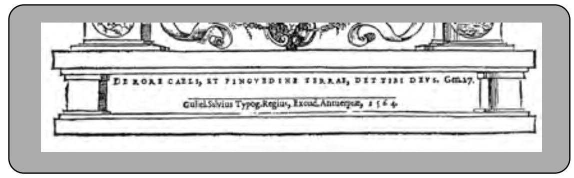
This seems to be an awkward fit because much of the emblem is cut off at the bottom.

(That's because the hole the Mercuries' spears point to is now 27 grid squares high, whereas in Dee's original printed Title page, it was 28 grid squares high.)

But we shouldn't be concerned with that cut-off, as here we're only using the emblem as a "tool," like a caliper or measuring device.



Another confirming clue that Dee intended the reader to see a triangle 27 grid squares tall can be seen in the Biblical quote Dee included in the foundation of the architecture.



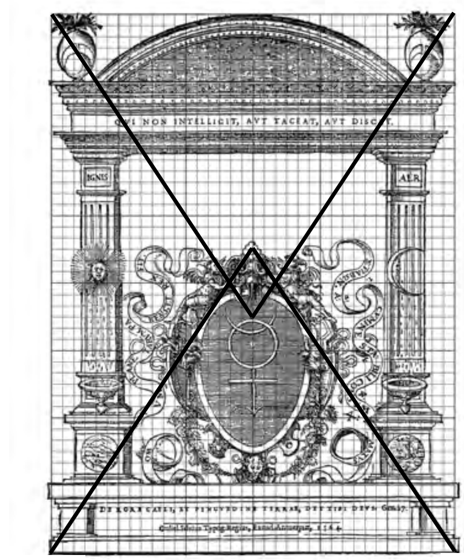
It "just happens" to be a quote from Genesis 27 (which he indicates).

The hole being 27 grid squares high doesn't seem to relate with the height of 48 grid squares.

The fraction 27/48 is equivalent to 9/16, which is not one of the 3 main harmonies.

One idea might be that Dee wanted us to see **two** 67 2/3 degree triangles overlapping, making diamond shape "window" by their intersection. (Elizabethan leaded windows were often diamond-shaped.)

And look who is peering through the window. It's Dee himself, in the guise of his astrological sign of Cancer the crab (or lobster).



2 triangles
nicely frame Dee's
"crustacean self-portrait"

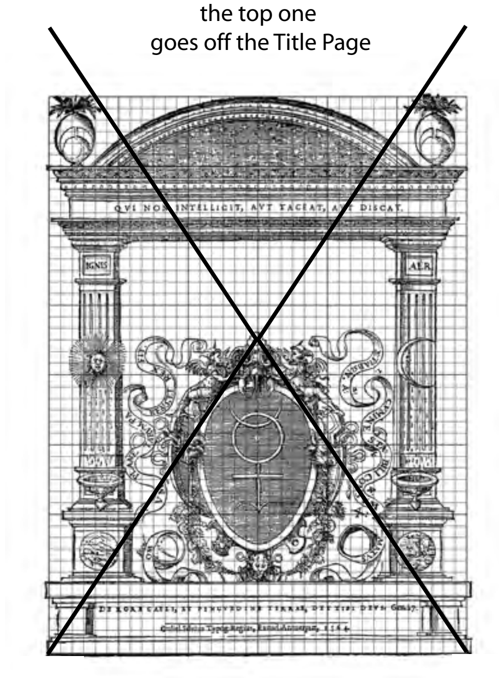
As interesting as this is, the diamond overlap still seems a little awkward, geometrically speaking. It would be more in keeping with Dee's thinking if the two triangles were "tip to tip."
(much like the tip to tip tetrahedra of the cuboctahedron, even though we're not dealing with equilateral triangles here)

This would make two equal sized triangles: a "perfect pair," just like the Sun and the Moon; or the upright and inverted Monas symbols; or indeed, the two Mercuries.

However, two tip to tip triangles would position the upper triangle's base **off the top** of the Title page, (up to 54 grid squares in height).

Going off the page like this seems strange, but what a perfect way for Dee to hide something. It's implied, yet not even there.

When the 2 triangles
are tip to tip,
the top one
goes off the Title Page



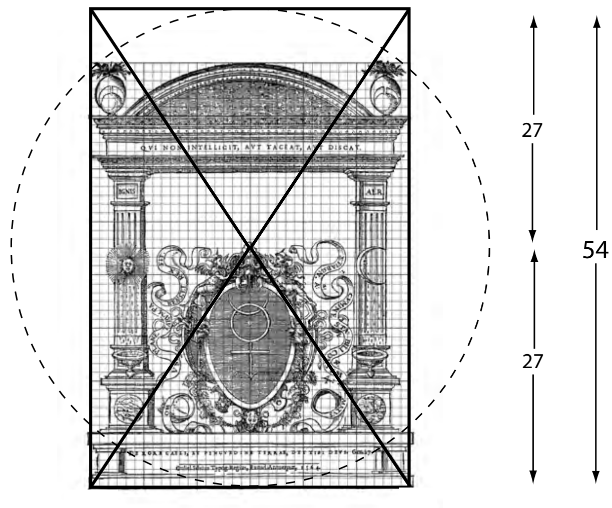
There are several confirming clues that reinforce
the idea that this is what Dee had in mind.

Using the common tip of the triangles as a center, if we draw in that circle that is **tangent** to the bottom edge of the Title Page, the top of it will extend off the top of the page.

Here I've drawn in another line **tangent** to the top of the circle, which obviously is 54 grid squares tall (27 + 27 = 54).

(A clue for you: An important aspect of the number 54 is that it is evenly divisible by 9, but the number 48 is **not**. This is a clue to solving another part of this geometric puzzle. This is best explained from a slightly different tack, which we'll get to in a moment. But for now, I'll give you a clue: The spine of the *Monas* symbol has 9 parts.)

The 2 tip to tip triangles require 54 grid squares of height



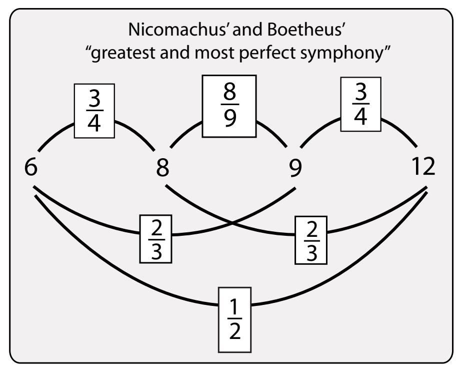
The other clue can be seen by taking a closer look at Nicomachus' and Boethius' "greatest and most perfect symphony," the interrelationships between the numbers 6, 8, 9, and 12.

We have previously investigated the 2:3 (diapente), 3:4 (diatesseron) and the 1:2 (diapason or octave) proportions of these numbers.

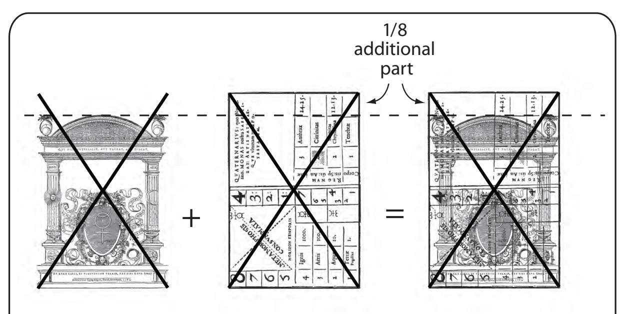
But, there's another proportion, hidden right in the middle of all this action, which Boethius referred to by the Latin name "**epogdous.**"

This is the proportion of 9 to 8, or **containing a whole and an eighth**. Boethius adds that in musical notation, this is called *tonus* (or a "tone") as this "one-eighth-of-an-octave" is the "common measure of all musical sounds."

(Michael Masi, *Boethius on Numbers*. P. 187).



In terms of Dee's illustrations it means that the verticalized "rectangular part" of the Creation chart is "one eighth" larger than the Title Page.



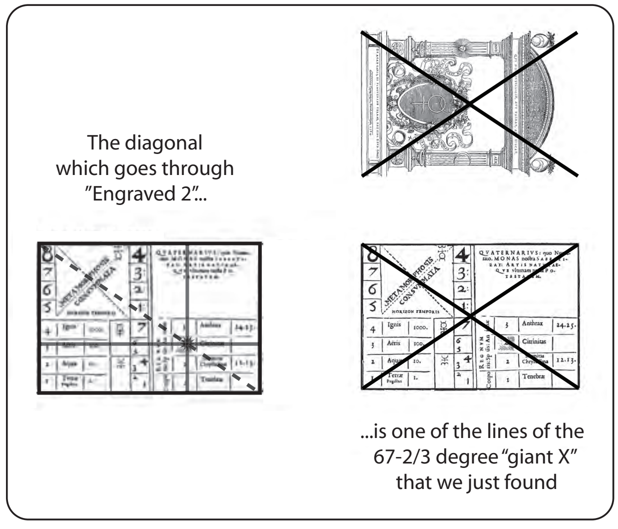
Dee's illustrations demonstrate the "epogdous" 9:8 ratio

A confirming clue in the bellybutton of the "Thus the World Was Created" Chart

There is another confirming clue that we're on the right trail here.

In an earlier analysis of the "rectangular part" of the Creation chart, we saw that a diagonal of the chart went right through that important Engraved 2 in the lower-right quadrant.

We can now see that the two diagonals of the chart form two angles of 67 2/3 degrees, the same angle as the Mercuries' spears from the Title Page!



The "epogdous" relates to the 3 main harmonies in other ways

Like the other ratios, "epogdous"can be written using the "ancient Greek" expression, 9/8, or in the "modern" way as 8/9.

These fractions relate to the 3 main harmonies quite nicely.

$$\frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$$

$$\frac{3}{2} \times \frac{3}{4} = \frac{9}{8}$$

(These equations do not include the harmony 1/2 (or 2/1), but we've seen how the harmony 1/2 is involved with the harmonies 2/3 and 3/4 in the Russian doll effect.)

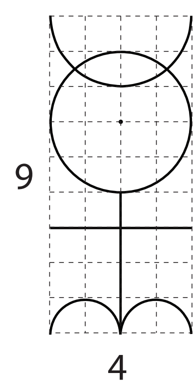
The obsevant reader will notice that 1/2 times the epogdous is 8/18, which is equivalent to 4/9 and this is the the proportions of the Monas symbol!

The Monas symbol is **self-referential,** in the sense that **its parts multiply to express the whole**

To understand what this means, let's first review how various parts of the Monas symbol express the 3 main harmonies.

$$\frac{1}{2} \times \frac{8}{9} = \frac{4}{9}$$

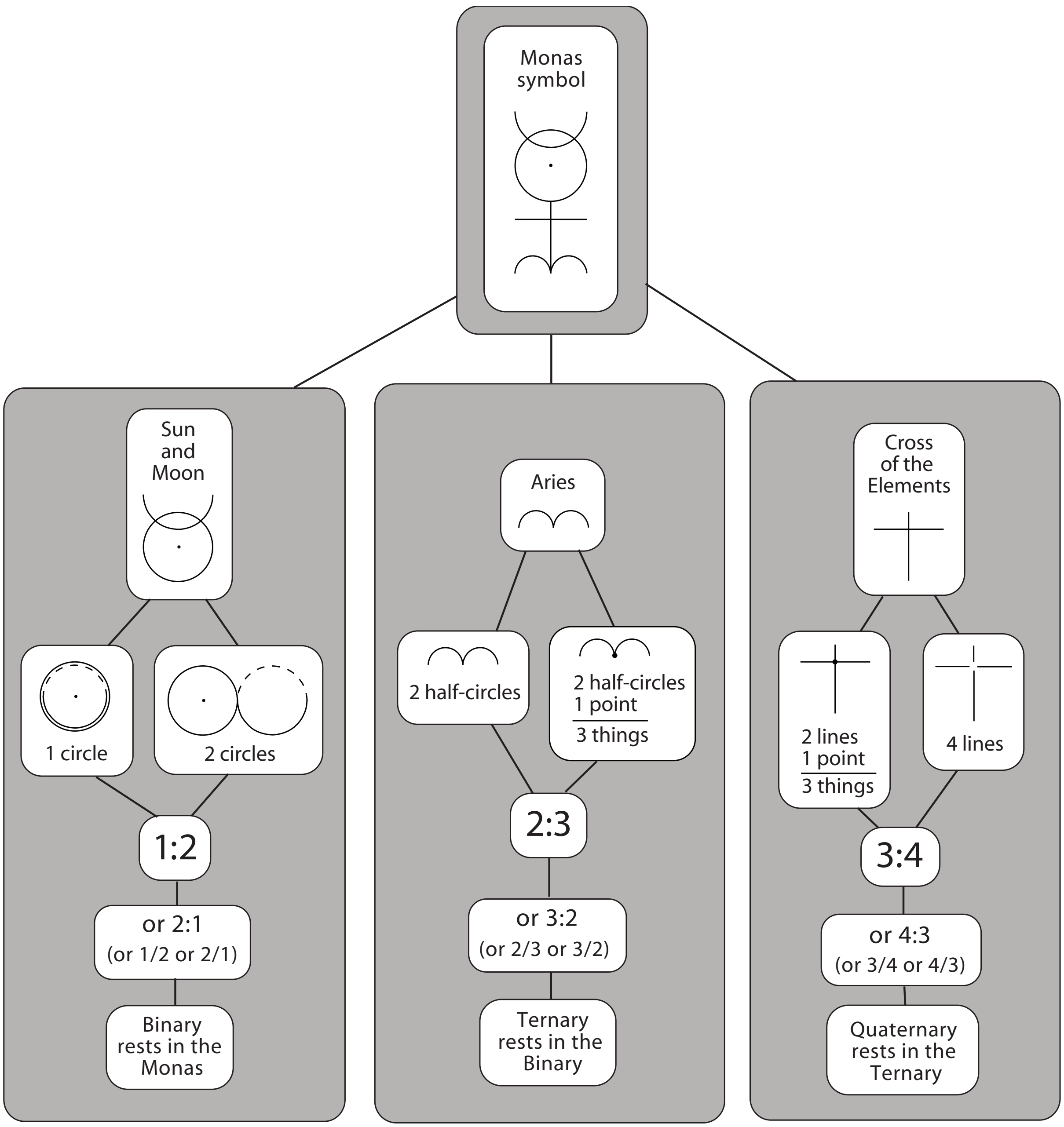
$$\frac{2}{1} \times \frac{9}{8} = \frac{9}{4}$$



Multiplication of the various parts of the Monas Symbol

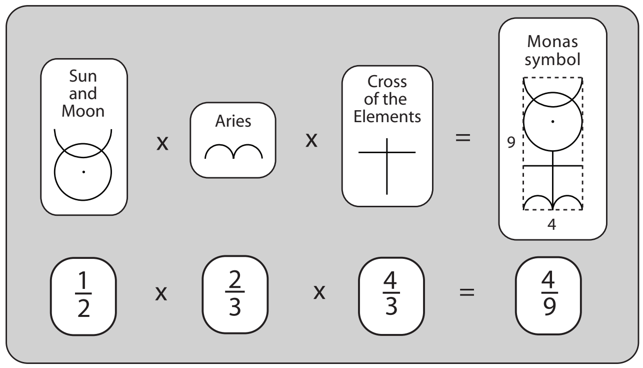
Remember, Dee also saw the various parts of the Monas symbol as expressing the 3 harmonies (expressed in "modern" fractions ***or*** in "ancient Greek" fractions).

The Sun and Moon express 1/2 (or 2/1).
The Aries symbol expresses 2/3 (or 3/2).
The Cross of the Elements expresses 3/4 (or 4/3).



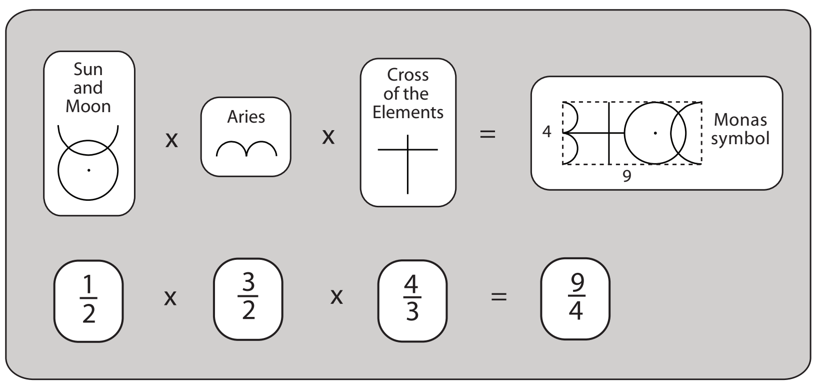
When various expressions of these harmonies are multiplied,
the result is 4/9, the proportions of the Monas symbol.

(Note that 1/2 and 2/3 are expressed the”modern”way,
but 4/3 is expressed the “Greek “way.)



Here’s another way this relationship might be seen.

(Note that this time, 1/2 and 3/4 are expressed the “modern”way,
but 3/2 is expressed the “Greek” way.)



Can you think of an even better way to express this amazing interrelationship?
(which Dee wants us to find, in order to open more splendid mathematical doors)

To explain what I’m getting at, let’s look at all these “fractions”
in terms of the shapes of Dee’s illustrations.

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O’Conner, John J. and Robertson, Edmund F. MacTutor History of Mathematics Archive.
Boyer, Carl B. A History of Mathematics (1991, John Wiley and Sons).

THE 252 ON THE TITLE PAGE

Restoring the Title page emblem to its "floating" position brings to light some other interesting "connections."

A straight line connects the word IGNIS (FIRE) with its "opposite" element, WATER (illustrated in the circle).

Another straight line connects AER (AIR) with its "opposite" element EARTH (also illustrated in a circle).

The two lines intersect at the center point of the Cross of the Elements of the central Monas symbol, forming a "giant X" (though not exactly at a 90 degree angles).

Another "giant X" connects what I call the "Gold and Silver Urns" with the elements of Earth and Water.

These lines intersect at the lowest point of the Moon half-circle. This point is also contained in the interior of the Sun circle.

The Sun circle and the Moon half-circle overlap so much, if I had a pick one point that said Sun meets said Moon, this would be the point.

I shall refer to this point as the **Sun/Moon point** of the Monas symbol.

Curiously, this point does **not** intersect with a line drawn between the "center of the radiant Sun face" and the "center of the crescent Moon face" on the columns.

It seems like Dee could have made this a confirming clue that the emblem has been properly restored.
But he didn't.

Dee had something else up his sleeve.
Can you figure out what?

(I'll give you a hint: the line connectin the "center of the smiling Sun" and the "center of the crescent Moon" is 25 grid squares from the bottom of the chart.)

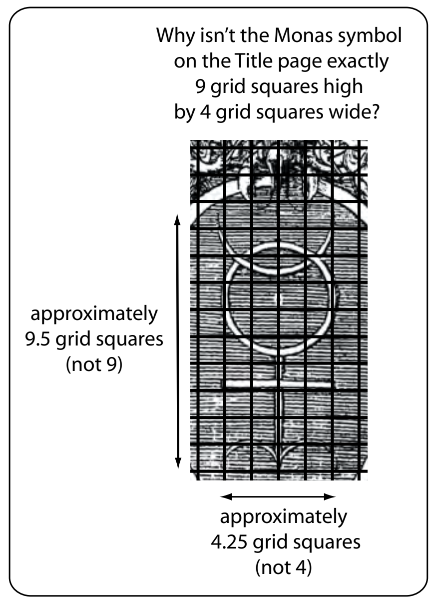
The solution involves another unusual inconsistency found on the Title Page.
The Monas symbol doesn't relate to the 48 by 36 grid.

One might suspect that the Monas symbol would be 9 grid squares tall by 4 grid squares wide.
(or 36 grid squares).

But it's not.

It's about 9.5 grid squares tall by 4.25 grid squares wide
(or a 40.4 grid square area).

Dee enlarged the Monas symbol by about 6 percent for some reason.

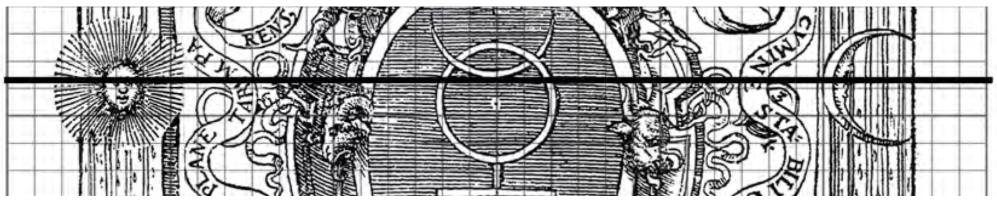


[I found out first-hand during my first attempts at "cracking the code" of the Title page. The Monas symbol is really the only thing on the Title Page for which Dee gives very specific geometrical proportions in the text of the *Monas*.

If one tries to determine the whole Title page grid using the Monas symbol grid as a template, it leads to the wrong grid. Dee fit the architecture to his 48 x 36 grid, but enlarged the Monas symbol as a "red herring" to throw someone looking for an easy solution "off the track."]

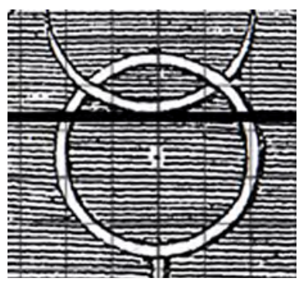
But he didn't enlarge the Monas symbol randomly.
One reason he enlarged it the amount he did was so the
"giant X" connecting the "4 Elements" would intersect the center point of the cross
and
the other "giant X" connecting the "2 urns and 2 elements" would intersect
at what I call the "**Sun/Moon point** on the Monas symbol."

Here's a horizontal line drawn at that "**Sun/Moon point.**"
There are more good reasons why Dee put the line here.
Can you **see** them?



This line intersects the EYES of the Sun and the Moon.

eyes of the sun...



...the Sun/Moon point...

...eye of the Moon

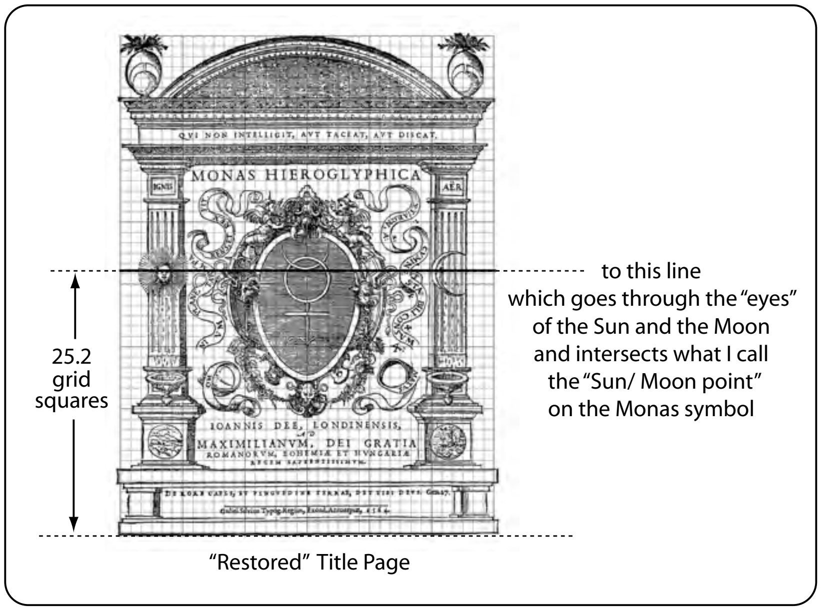
The camera obscura is a powerful example of the oppositeness
in the realm of light and physics is an important (yet cryptic) theme of the *Monas*.

We've seen that in Dee's "advice to Opticians,"
he describes his understanding of how vision works.

Dee was fascinated with optics, mirrors, burning mirrors,
the camera obscura, solar disc calendars and of course the human eye.
Dee even makes a pun on the word " eye" in his geometric construction
of the Monas symbol of Theorem 23, where he labels the "cyclops eye" with the "letter I."

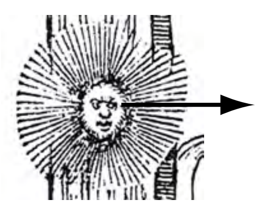
But there's another huge theme in the *Monas*
that Dee is representing by this line – the number 252.
This line through the eyes is 25.2 grid squares from the bottom of the Title page!

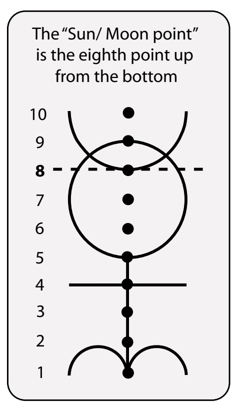
(I'm not suggesting Dee wrote 25.2 in decimal notation,
but he definitely knew 25 2/10 was only a "factor of 10" from being 252.)

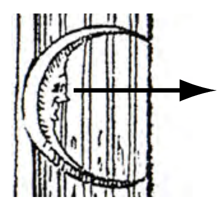


"Restored" Title Page

[As an interesting side note, exactly 5.25 charts would be required
to accomodate 252 grid squares of height, as 48 x 5.25=252]







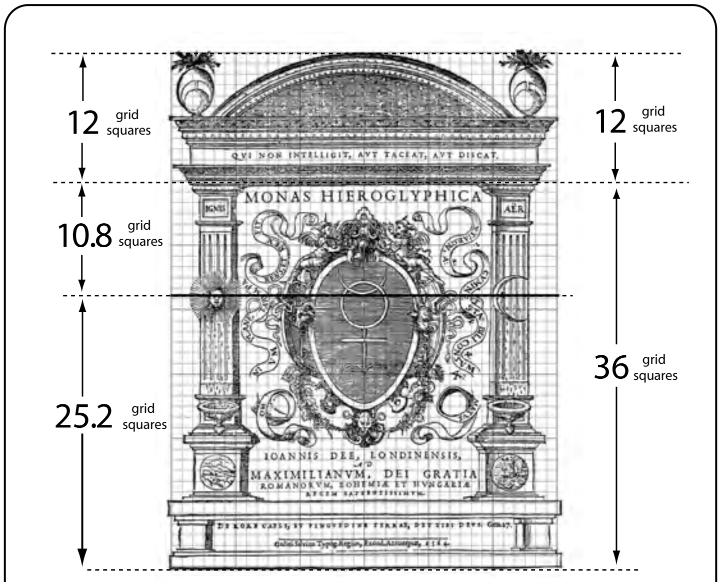
Here are several confirming clues:
Notice how the Sun's two eyes are looking to the right, as if suggesting this "line of vision."

The same goes for the Moon, whose face is shown in profile.

Furthermore, counting upwards from the bottom of the spine of the Monas symbol,
this Sun/Moon point is point 8, and eight is the "octave" of Consummata.

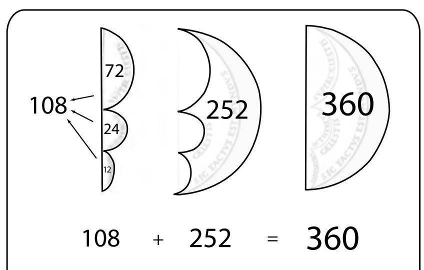
Another confirming clue is that the top of the columns (or the bottom of the entablature)
is 36 grid squares in height.

Subtracting 25.2 from 36 makes 10.8.
This is a subtle way of showing the relationship between these powerful numbers:
(108 + 252 = 360).



"Restored" Title Page

This is exact same relationship
Dee hid in the circle segments
on the right side of his
"Thus the World Was Created" chart.



The proportions of the "ballooned" part
of Dee's "Thus the World Was Created" chart

Yet another confirming clue can be seen in the number
of remaining grid squares of height, which is 12, the docena. (25.2 + 10.8 + 12 = 48).

We've seen all the importance of 12 in Metamorphosis and
as the first transpalindromable number.

Bumping up these numbers by a factor of 10 makes (252 + 108 + 120 = 480).
And you might recall why 120 is infamous.
(Besides Dee's 120 Aphorisms in his *Propadeumata Aphoristica*).

The reflective mate of 120 is 021.
And 120 x 21 = 2520, that very special Metamorphosis number.
The same numbers keep popping up, over and over again.

That Dee was one clever buckaroo!
He might have polished up the text of the *Monas* in 12 days
in Antwerp, but it's clear that he thought a lot longer than that
about all these intricate interrelationships in his illustrations.

His clues never go too far (which has resulted in their claimed obscurity),
but there's always just enough to go on, (and the confirming clues are just as subtle).

One final note of interest:
This "Sun's eyes to Moon's eyes" analysis involves the emblem in its "**restored**" position."
When this same line is drawn across the placement of that emblem
as it was **originally** printed, **it passes through Dee's brain!**

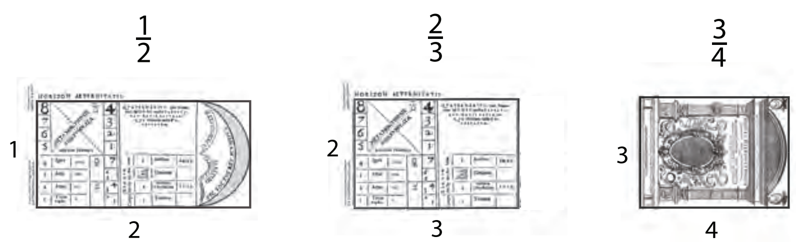
(I am referring to the crustacean who is
overseeing all the action of the Title Page).

Dee literally had 252 on his mind.

If you contemplate the small triangle
formed by that line and the Mercuries' spears,
you can see the Russian Dolls dancing.

THE INTERRELATIONSHIPS OF THE 3 MAIN HARMONIES COME ALIVE... VISUALLY!

Dee made the three harmonies come to life
by representing them as rectangles.



The proportions of Dee's three illustrations seen as fractions

The harmony 1/2 is the "one to two" proportion of the "ballooned 360 Thus the World was Created" chart, which I will call simply the "extended" Creation chart.

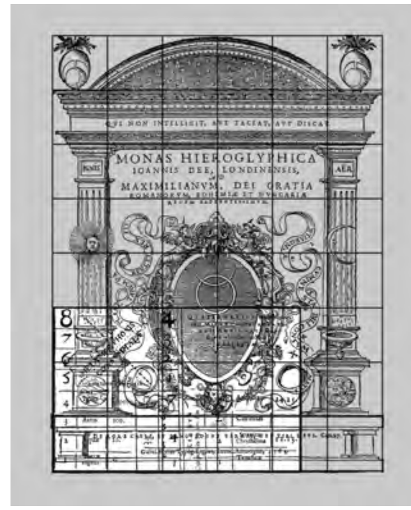
The harmony 2/3 is the "rectangular part" of the "Thus the World was Created" chart, here called "the rectangular Creation chart."

The harmony of 3/4 is the Title Page (even though its height to width is 4/3, when oriented horizontally its height to width is 3/4).

These harmonies are quite beautiful in their own right, but their interactions are even more fascinating. To see this visually, it seems as though that Dee wants us to **superimpose them.**

The “rectangular Creation chart,” (in the scale that Dee printed it),
is quite small compared to the Title Page.
(it only takes up 18 x 27 grid squares on the 48 x 36 title page)

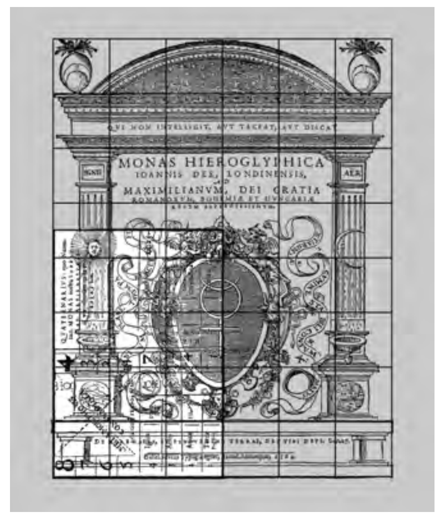
You can see by the large 8 x 6 grid drawn here
that its height is 3/8 of the height of the Title Page.



When the “rectangular Creation chart” is verticalized,
its height is 27 grid squares.

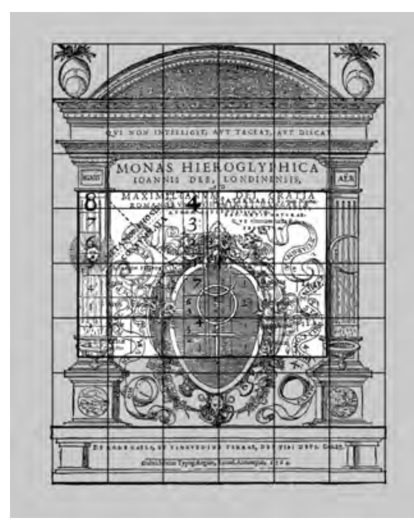
This is basically depiction of our earliear exploration of the “tangent.”

Note that the upper right corner of the “rectangular Creation chart”
is now positioned where that hole which the two Mercuries’ spears point
to when “repositioned” and used as a “drafting compass”.



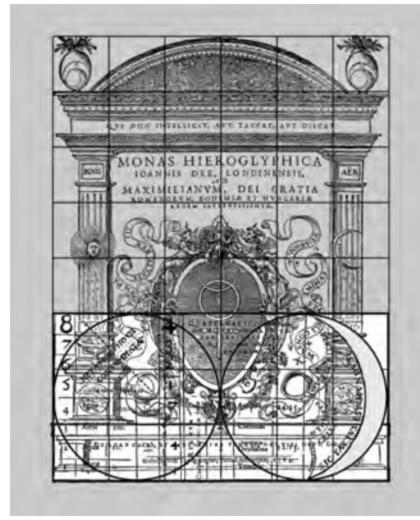
It’s interesting that the horizontal version of the
“rectangular Creation chart” measures from the
outside edge of one column to the inside edge of the other column.

But all-in-all, **none of these three superimpositions
is really very exciting.**

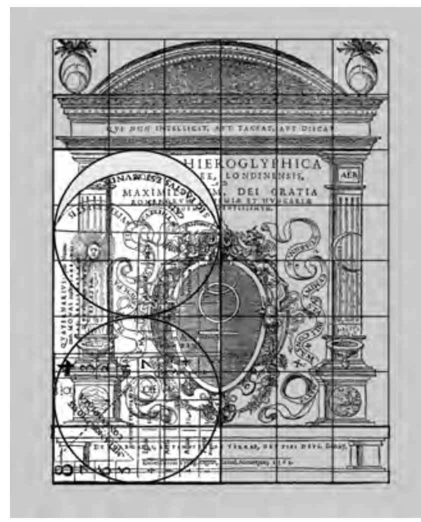


When the “extended Creation chart” is superimposed on the Title page, things get a little more interesting.

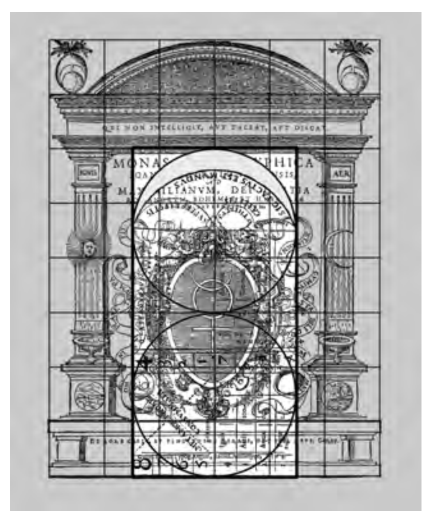
Oriented horizontally, it is the exact width of the Title Page. (36 grid squares wide)



When oriented vertically, it rises to the height of the top of the columns.

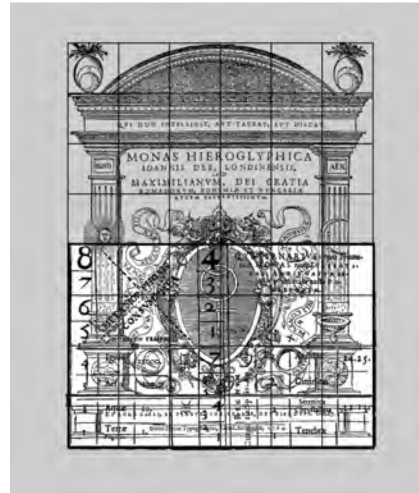


When it is centered, it might even be seen as supporting the entablature and dome assembly.



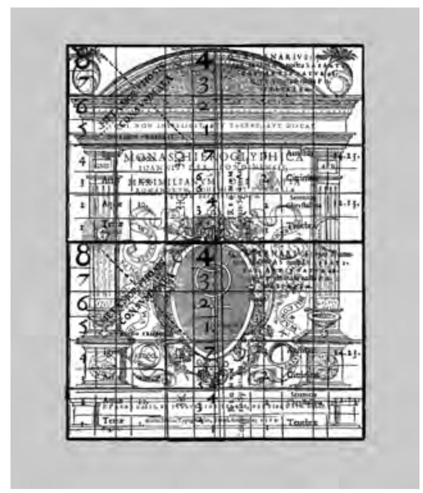
Next, suppose we enlarge the “rectangular Creation chart”
by ***one third*** in both dimensions.

(In other words, the 18 x 27 original version,
times 4/3, makes it 24 x 36.)



This means that **two** of these “***one third enlarged***” charts
would completely fill the Title page.

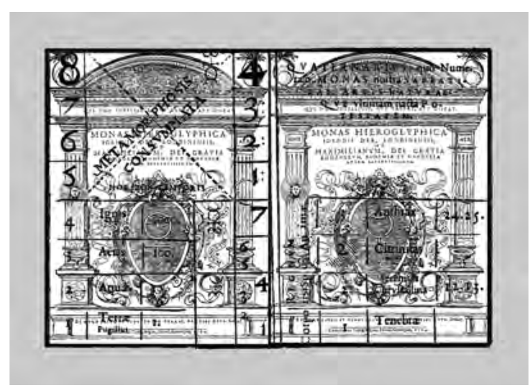
This is an example of the “Russian doll effect” we saw earlier,
where two rectangles in the 2/3 proportion made a 4/3 proportion.
(2/1 x 2/3 equals 4/3)



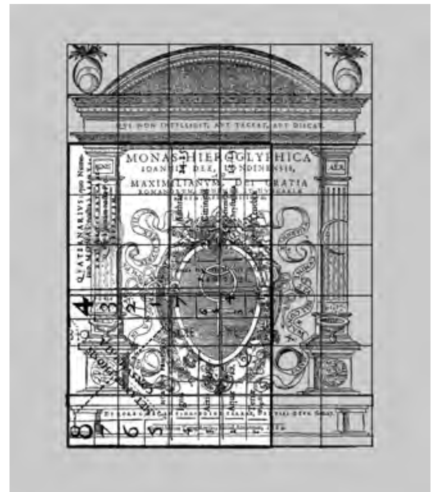
Now, its width is equal to the full width
of the Title page.

And in height, it now rises to one-half
of the height of the Title. Page.

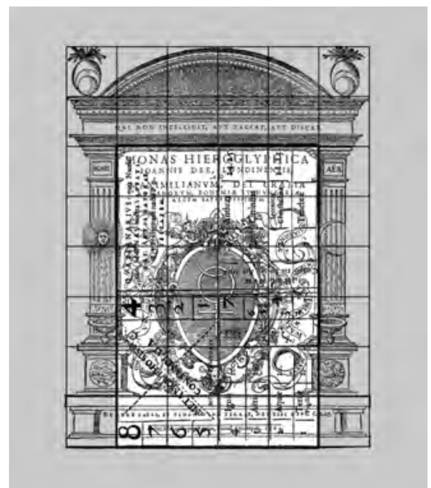
The “Russian doll” that is one “step” smaller
would be two of the 4/3 proportioned Title pages
sitting in the 2/3 proportioned “rectangular Creation chart.”



Orienting this "***one third enlarged***" chart vertically,
it obviously rises to the height of the top of the columns.



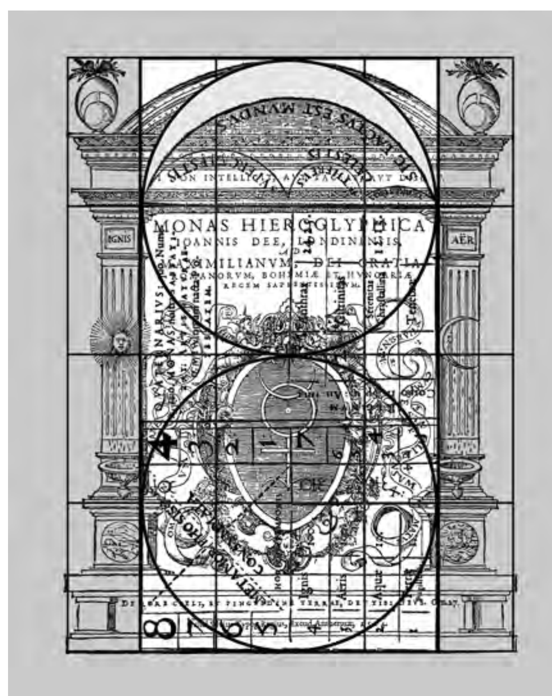
But, as it's width is 24 grid squares,
it also fits perfectly between the columns.
Now ***this*** is an interesting visual correlation!



When the "extended Creation chart"
is treated the same way
(enlarged by ***a third***, oriented vertically, and centered)
another wondrous correlation appears.

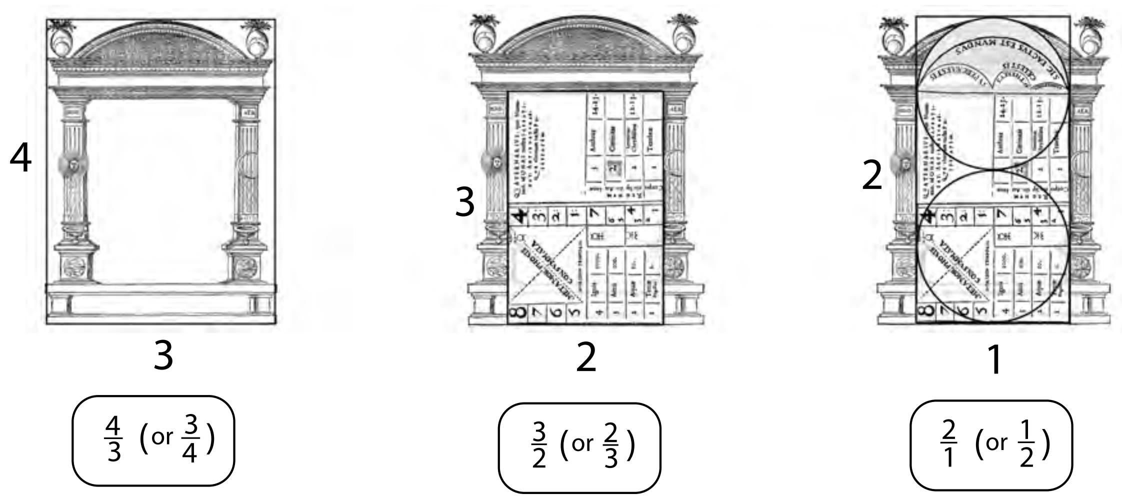
The tip-top of the "ballooned 360"
semicircle is now tangent with the
architectural dome of the Title Page!

They may not be the same arc,
but they are both graceful curves
which "contact at a point".



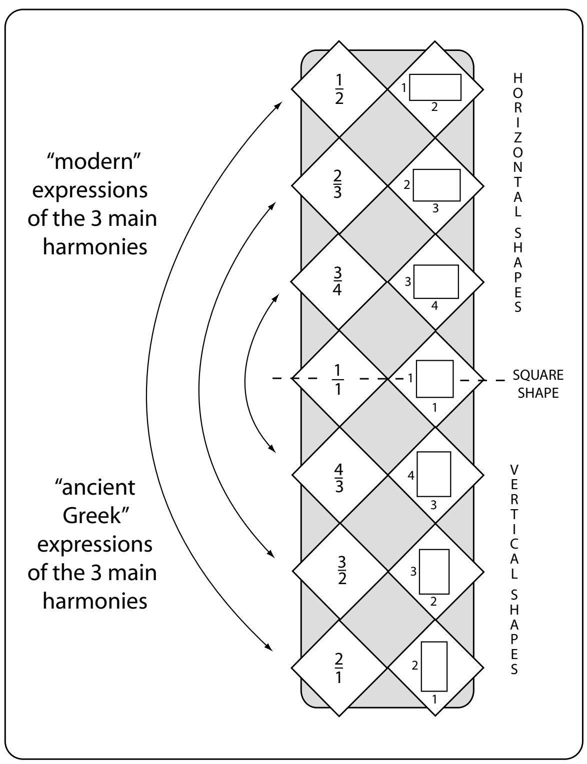
To visually emphasize the way these three harmonies interrelate on this summary chart, I have deleted the emblem and the type of the Title page, leaving only the architecture.

Interrelationships between the
Title Page and the "Thus the World was Created" chart
showing the 3 main harmonies.



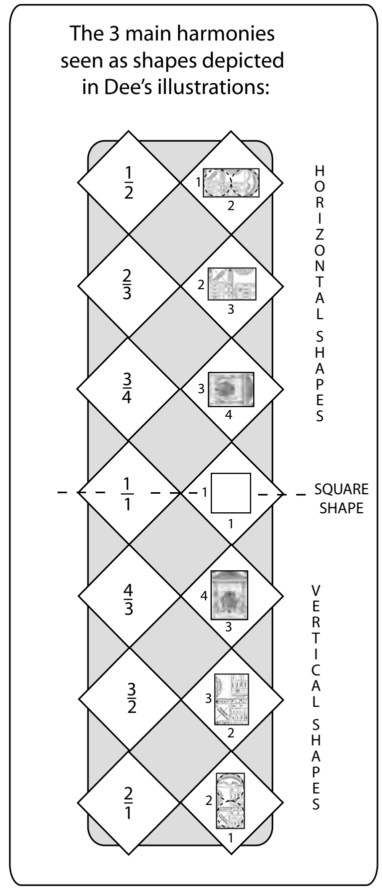
Interrelationships of Dee's 4 illustrations: 1/2, 2/3, 3/4, and 4/9

We've seen how the Greeks expressed these three main harmonies "upside down" from how we generally describe them nowadays.



In this diagram, the Greek expressions are shown as vertical shapes and the "modern" expressions are seen as horizontal shapes.
In the middle is the "1 to 1" proportion -- a "perfect square".
(We might even see or the Inferior Astronomy chart from Theorem 13 as that "perfect square.")

Next, I've replaced all those shapes
with some of Dee's key illustrations.

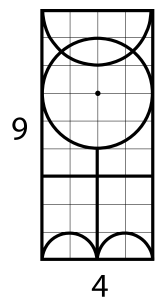


But there's a very important proportion
missing from this inventory.

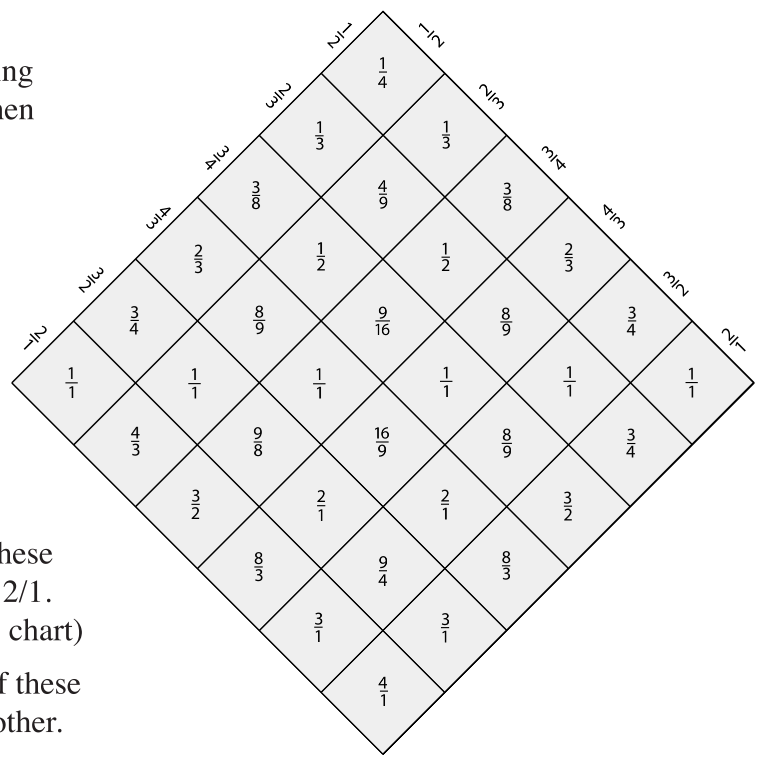
The proportion of Monas symbol.

It measures precisely 9 units tall by four units wide.

How does the 9/4 (or even 4/9) fit into the picture?



The answer can be seen by studying
how these fractions interrelate when
"paired up with themselves."
(Here, a few good graphics
will save thousands
of words.)



There are 36 possible "pairings" of these
six items: 1/2, 2/3, 3/4, 4/3, 3/2, and 2/1.
(see upper left and right edge is of this chart)

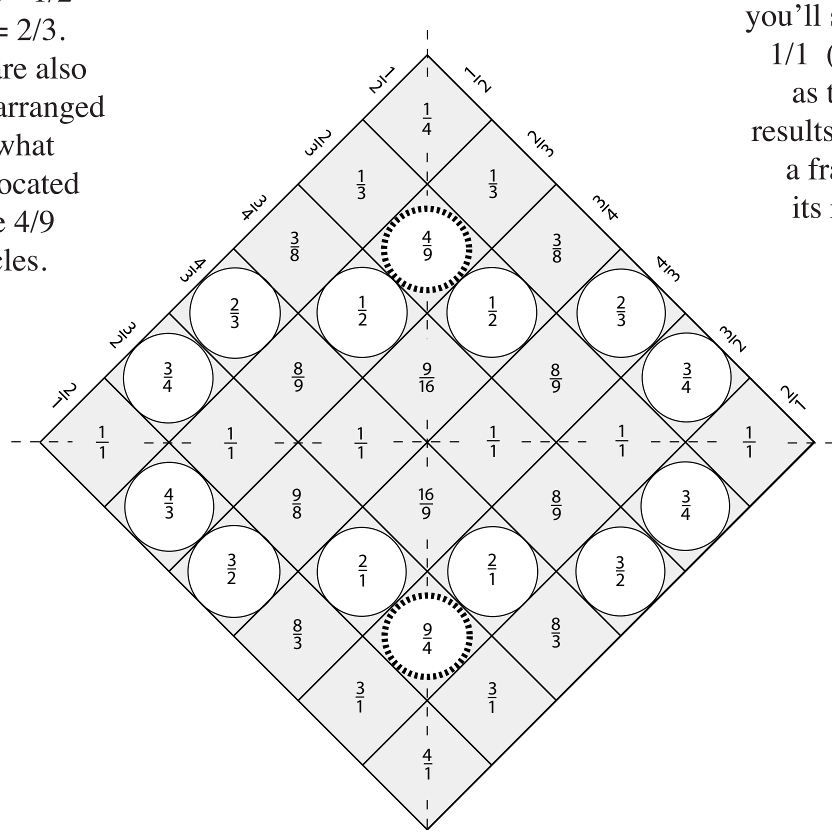
The boxes in the chart are the results of these
pairs of numbers multiplied by each other.

This might just appear as a jumble of numbers,
so I've highlighted a whats important to notice.
The fractions 4/9 (which is 2/3 x 2/3)
and 9/4 (which is 3/2 x 3/2) hold
prominent positions along the "spine"
(the vertical axis of the chart).

Notice that 12 of the results (circled here)
are the very same numbers we started with!

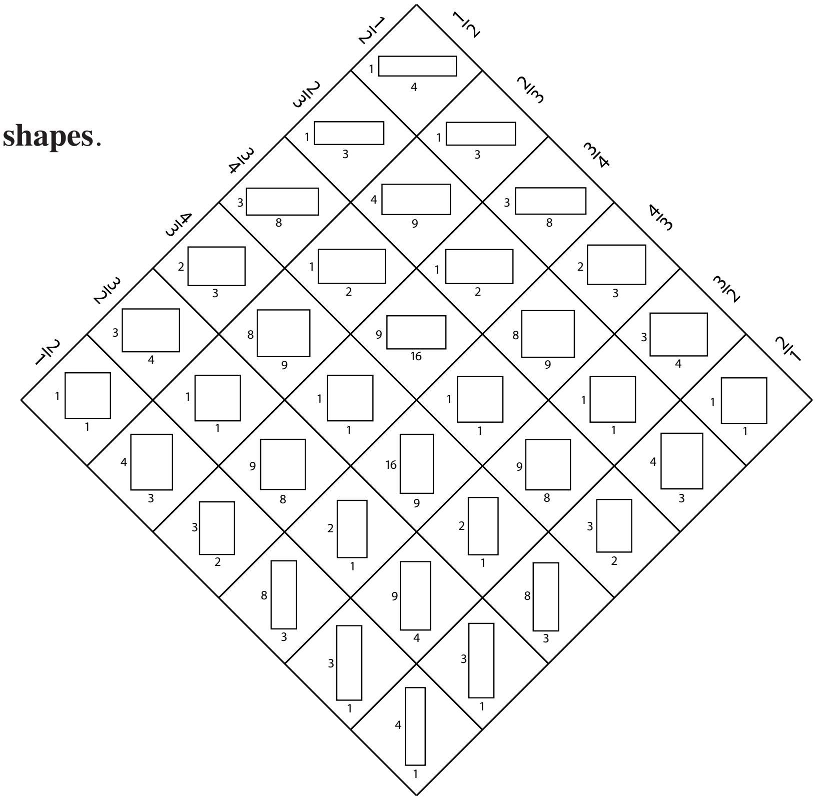
For example,
find 3/4 x 2/3 =1/2
or 4/3 x 1/2 = 2/3.
These results are also
symmetrically arranged
in the somewhat
oval pattern located
between the 4/9
and 9/4 circles.

All along the
central horizontal axis
you'll see the fraction
1/1 (or simply 1),
as these are the
results of multiplying
a fraction times
its reciprocals.



Also, note that arrayed above the 4/9 circle
are the fractions 1/3, 1/4 and 1/3.
Similarly, below the 9/4 circle
are the fractions 3/1, 4/1 and 3/1,
or simply 3, 4 and 3.
These results, that seem to be singing that
"Quaternary rests in the Ternary" tune,
seem to further enclose the 4/9 and 9/4 circles.

Here are those same 36 results seen as **shapes**.

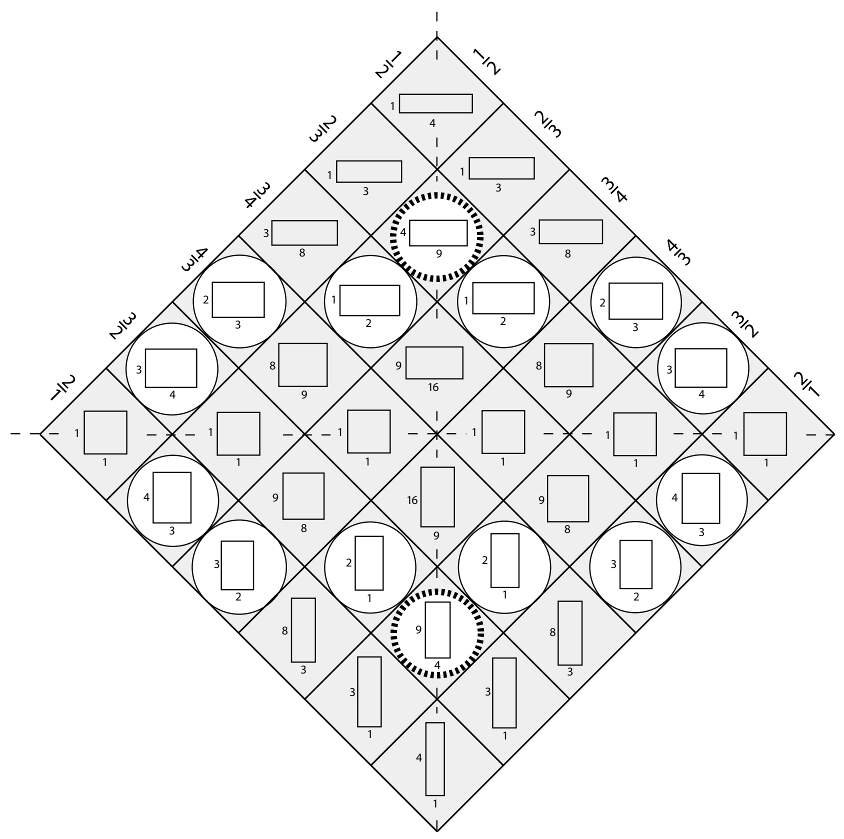


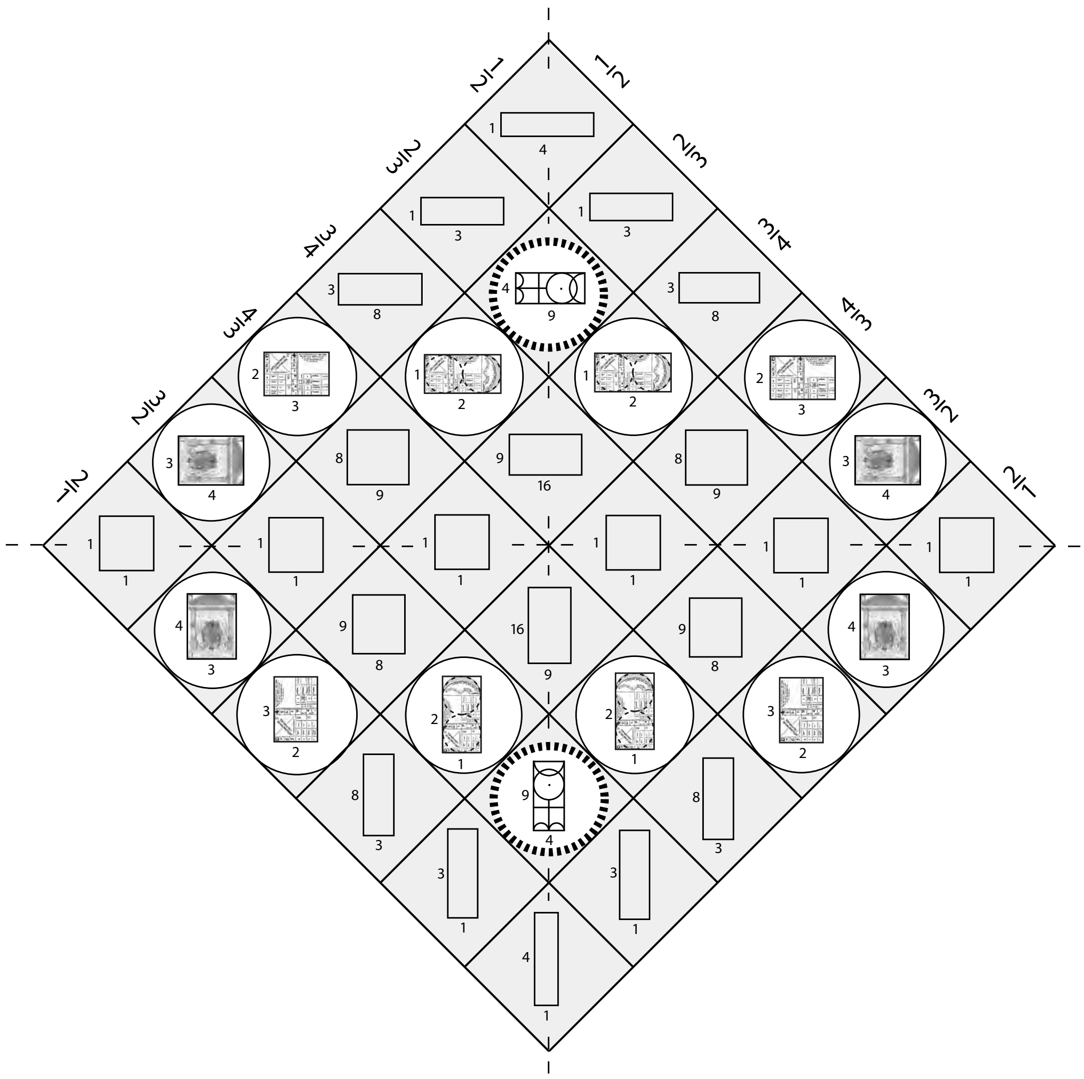
Notice that the horizontal centerline is filled with "1 to 1" **squares**.

Above that line, all of the shapes are **horizontal** (like Modern ratios).

Below the line, all the shapes are **vertical** (like Greek ratios).

Again, I've highlighted the "significant" results by circling them.





Now, let's replace those important results
with shapes from Dee's illustrations:

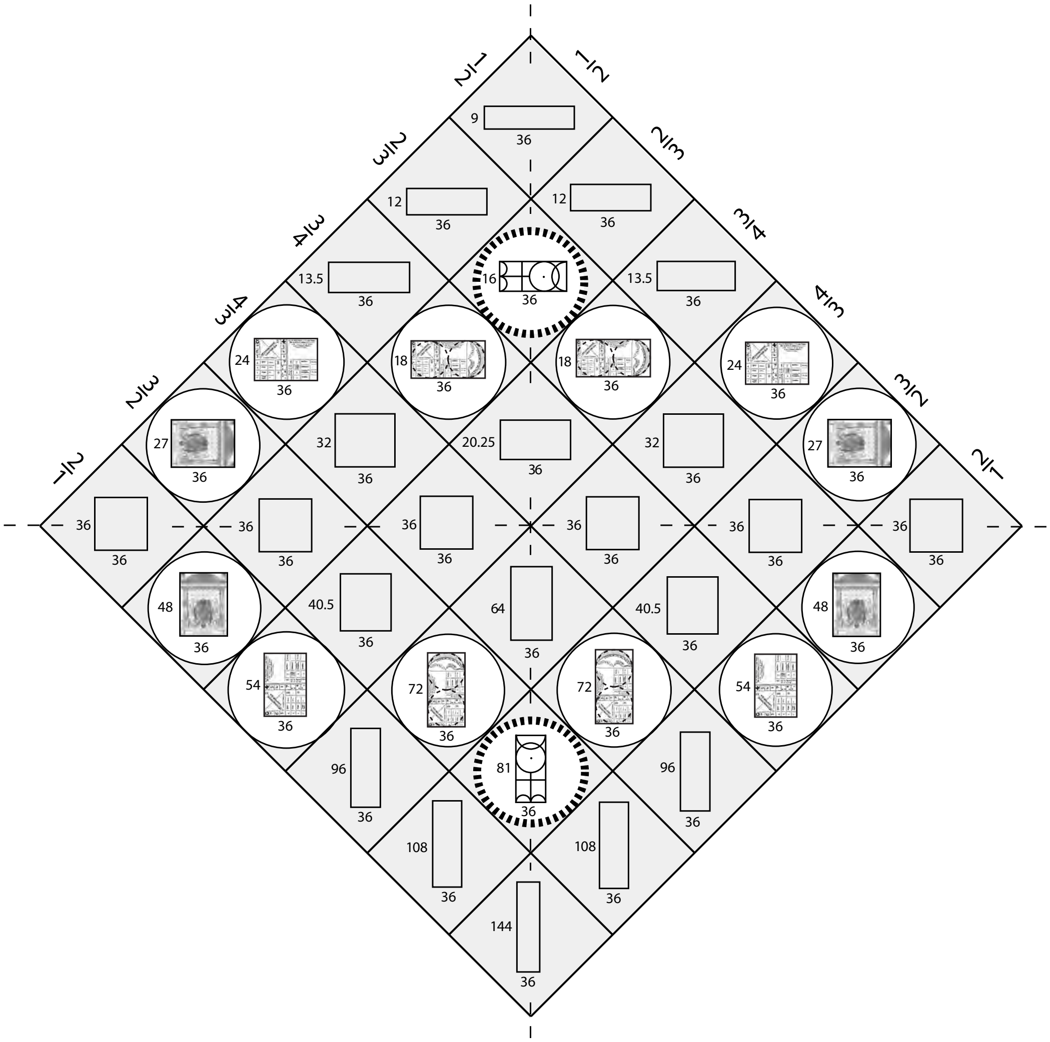
the Title page,

the "rectangular part of the Creation chart,"

the "ballooned Creation chart,"

and the Monas symbol itself.

The modern expression of the Monas symbol (4/9) is **laying on it's side,**
but the ancient Greek expression (9/4) (prologous/upologous) is proudly **upright.**

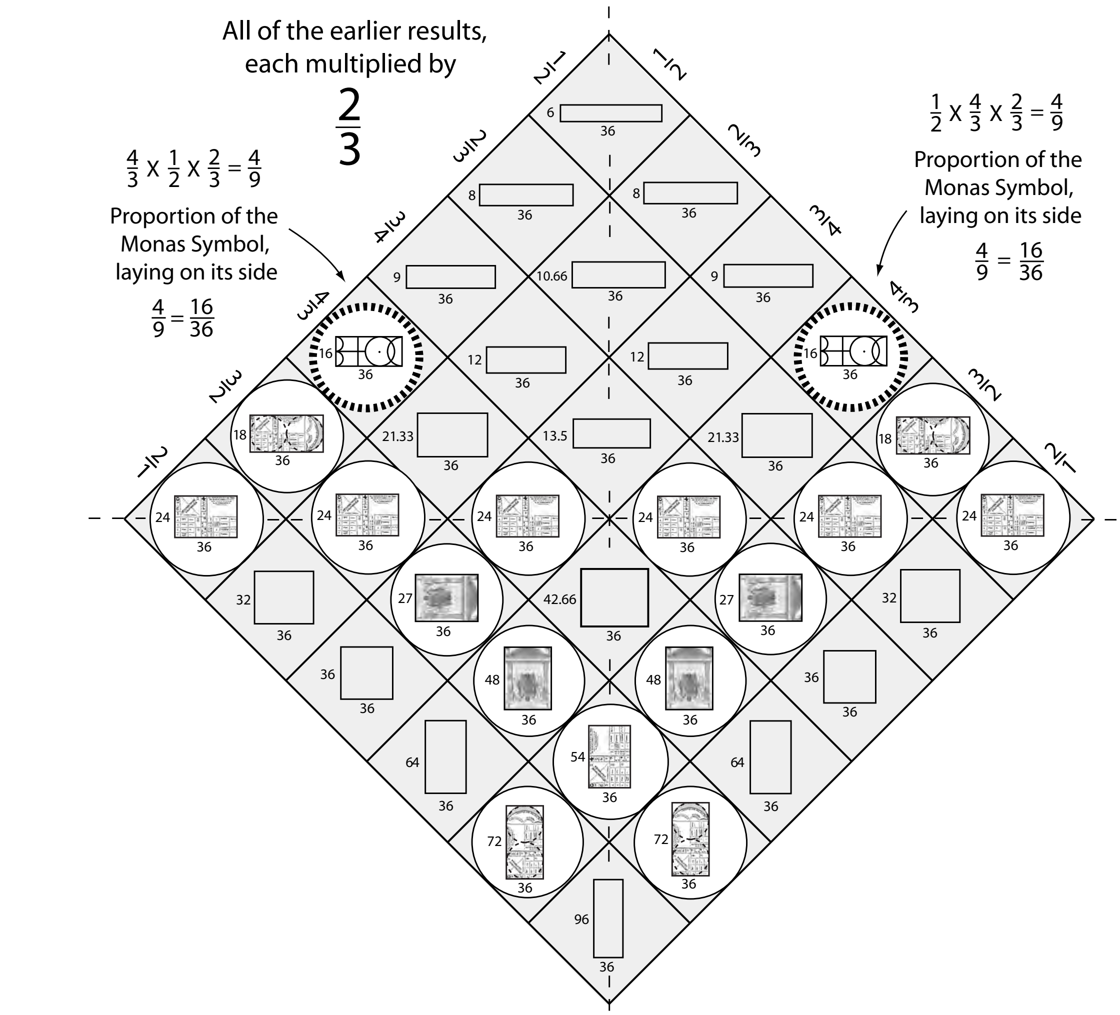


Next let’s have all these proportions “speak the same language”
by giving them a “common denominator” of 36
(that is, making the width of **all** the rectangles 36).

(You’ll note that these representations are not all the same “ scale.”
What’s important are their relative proportions.)

These charts paint and nice picture of how the modern and Greek
expressions of the three harmonies relate to the Monas symbol.
But somehow these expressions of the Monas symbol are not satisfying.
Something’s missing.

They make it seem as though the Monas symbol is only related to 2/3 and 3/2,
but not to those other wonderfully harmonious expressions 1/2, 2/1, 3/4 and 4/3..



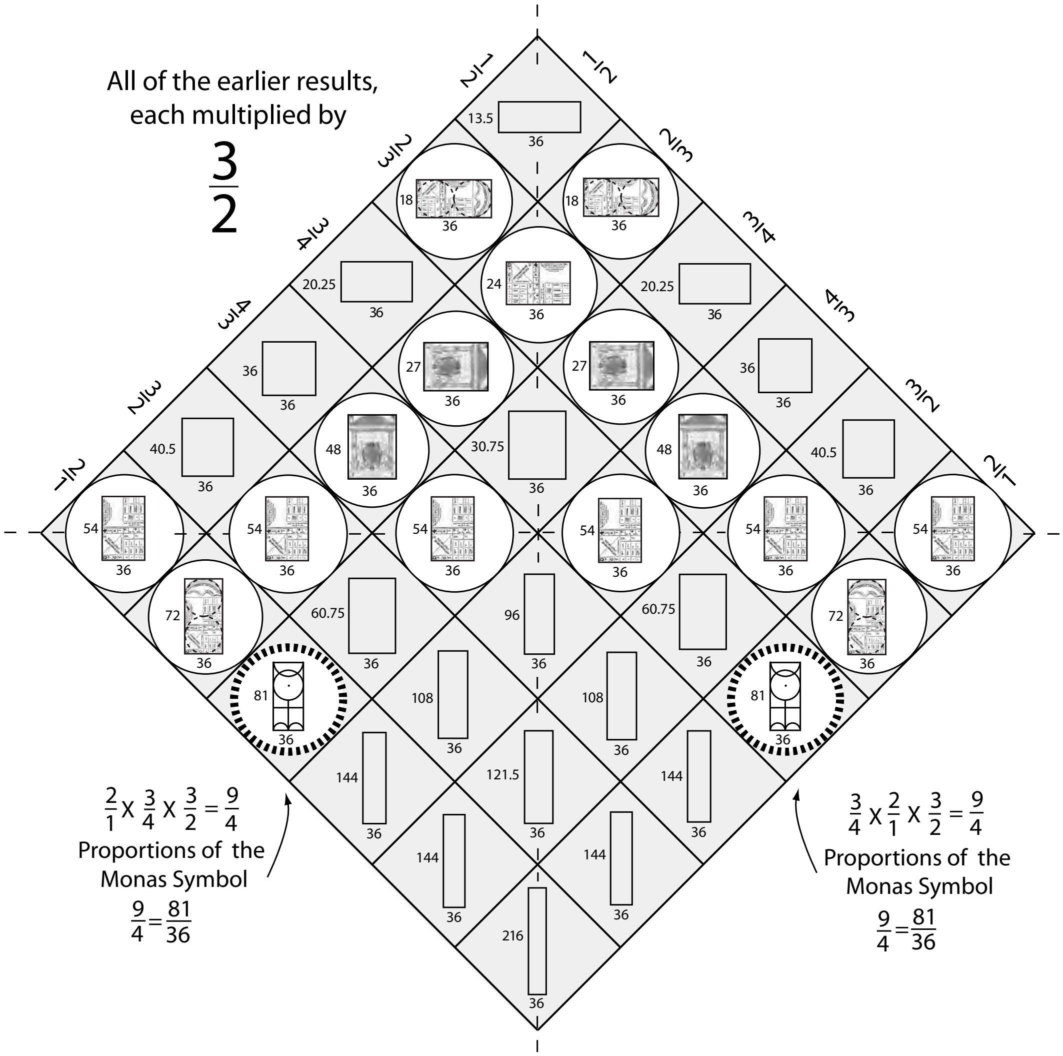
To show all the ways in which these
expressions can be combined in "groups of threes"
would require 6 more charts, each with 36 results.

I'll spare you 4 of those charts which don't yield results
that are as important as the 2 which follow.

This first chart is simply all of our original results multiplied by 2/3.

Notice first that the "horizontal centerline row" is filled with 24/36 boxes.
For example, the furthest box to the left is a result of the reciprocals 2/1 x 1/2,
then that result times 2/3, or the 24/36 proportion.

Besides all the duplications on this "horizontal centerline row,"
all of the proportions we started with (1/2, 2/3, 3/4, 4/3, 3/2 and 2/3) are still all present.
Nestled in with them are the two Monas symbol proportions, but they are laying on their sides.



The second chart of "triplets" shows all of the earlier results multiplied by 3/2.

The "horizontal centerline row" is all in the proportion 3/2.
And again all of the proportions we started with (1/2, 2/3, 3/4, 4/3, 2/3 and 2/1) are represented.

Only this time, the two Monas symbols are standing proudly upright!
They're both actually a product of the same fractions.

The only difference is the sequence in which they are multiplied
(2/1 x 3/4 x 3/2 is pretty much the same as 3/4 x 2/1 x 3/2).

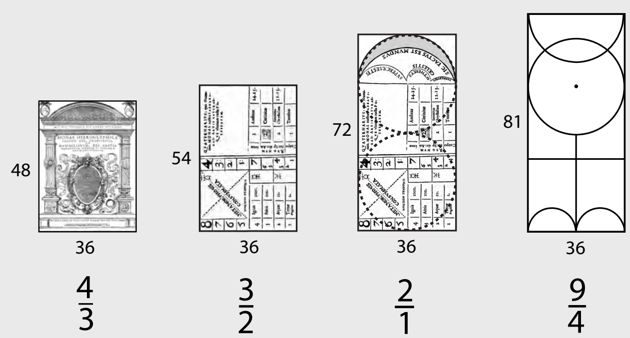
To summarize,
these busy charts are challenging to follow.

The main point is that the Monas symbol 9/4 (or 4/9)
is ***very*** involved with the 3 main harmonies 1/2, 2/3, and 3/4.

Seeing Dee's 4 illustrations (1/2, 2/3, 3/4, and 4/9) as an equation

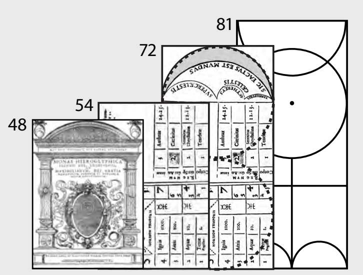
Here, expressed the ancient Greek way,
are the four key proportions that Dee features
(cryptically) in his illustrations.

If their widths are all 36,
their heights are 48, 54, 72 and 81, respectively.



Four key proportions featured
in the *Monas* illustrations

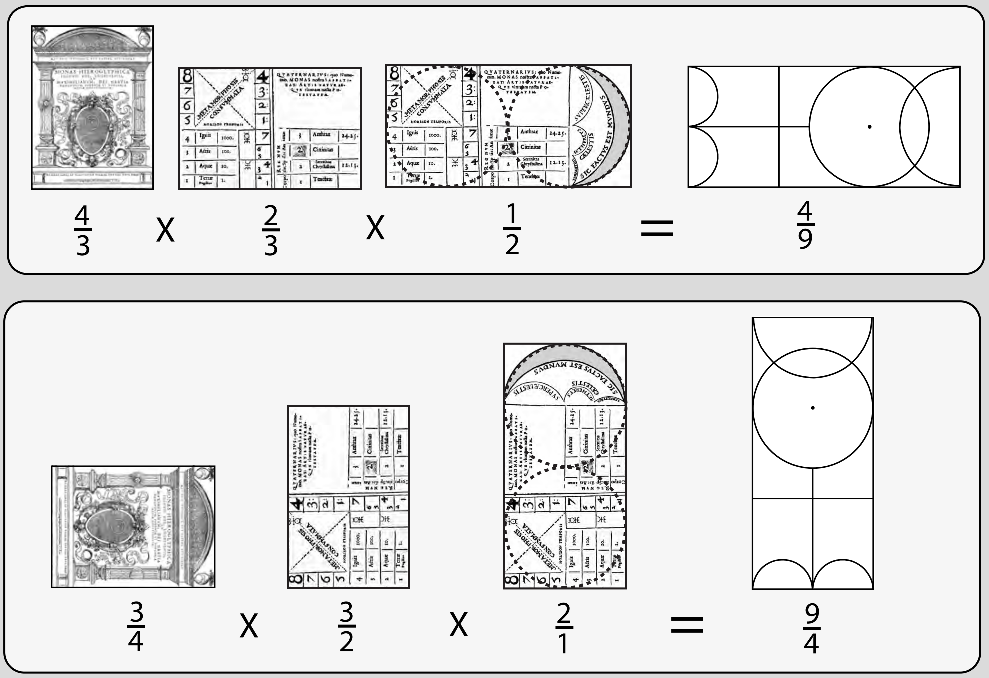
The question is, how does Dee want us to see them all as interrelated?



How do these shapes interrelate?

Here is a summary of two basic ways that all four interrelate.

In the top one, the Title page and both representations of the "Creation" chart are oriented correctly, but the Monas symbol is lying on its side.



Two ways in which all 3 harmonies
(seen as Dee's shapes)
multiply together
to make the proportion of the Monas symbol

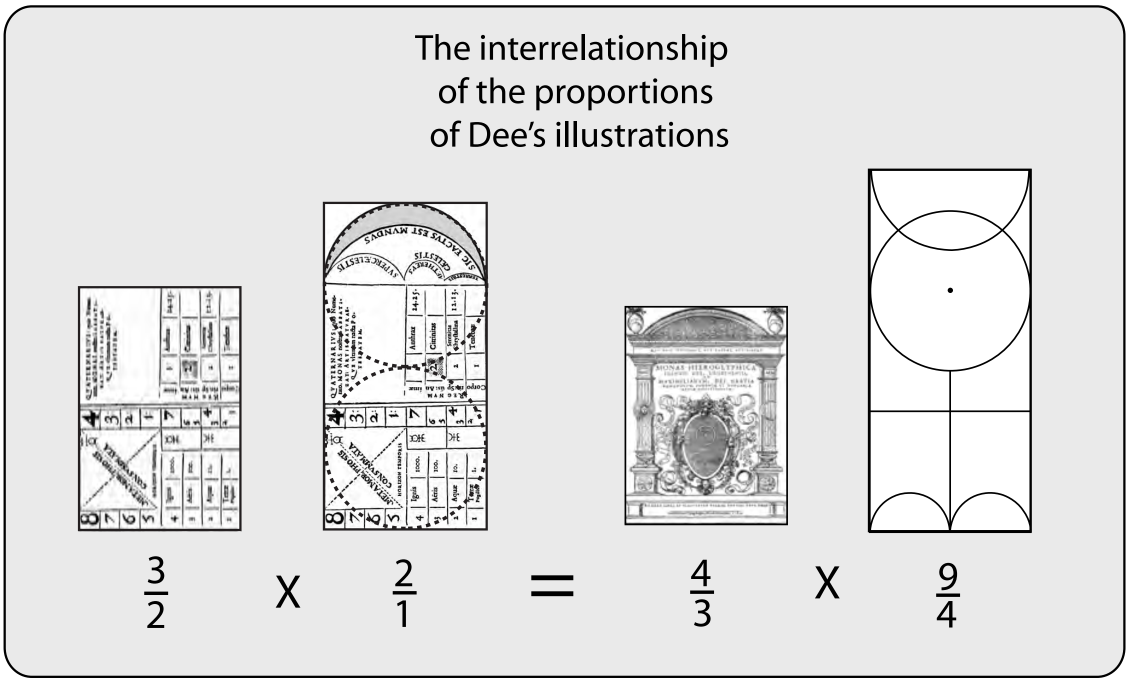
In the bottom one, the first three representations are all oriented differently than they are in the *Monas* text, but the Monas symbol is proudly upright.

But here Title page lying on its side looks odd.

But, there's yet another way to see this "equation" of shapes.

We can drag the Title page to the right side of the equation
(while "verticalizing" the shape, which is just like "inverting" the fraction from 3/4 to 4/3).

This results in a much more "balanced mathematical picture."

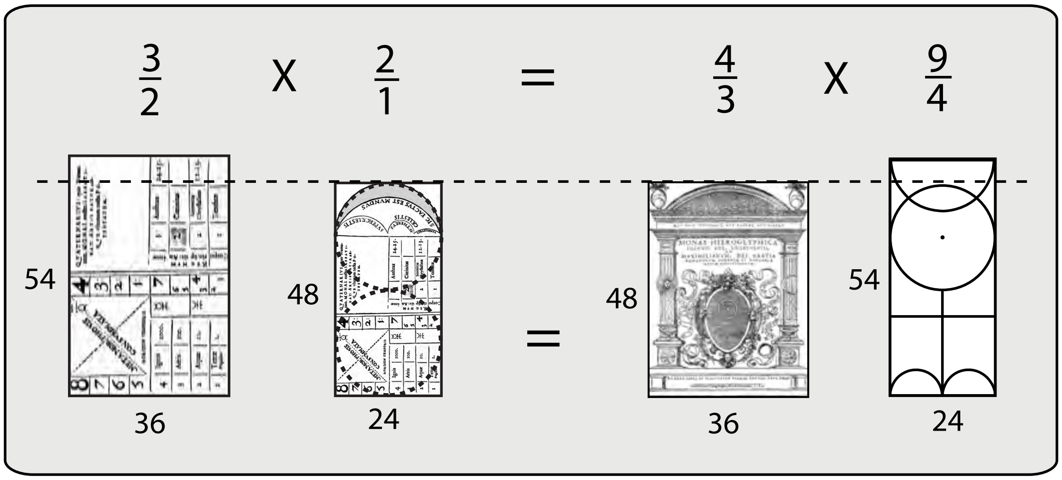


However, one visually disturbing thing about this arrangement is that the
Title page looks "miniature" compared to the "**jumbo-sized**" Monas symbol.

There is such a wide variety of heights here that it's hard
to picture how all these four shapes might interrelate further.

Dee wouldn't have brought us this far without having
all these puzzle pieces fit together more neatly than this.

I'll cut to the chase.
One way to bring all the heights into the same "range" is to reduce the scale of both the "ballooned Creation" chart and the Monas symbol ***by one third***.



Scaling the "ballooned Creation" chart from 72/36 down to 48/24 makes its height the same as the height of the Title page (the two inner illustrations here).

Likewise, scaling the Monas symbol from 81/36 down to 54/24 makes it the same height as the upright "rectangular Creation" chart (the two outer illustrations here).

These four shapes might not have the same"common denominator" (that is, the same widths), as some are 36 and some are 24.

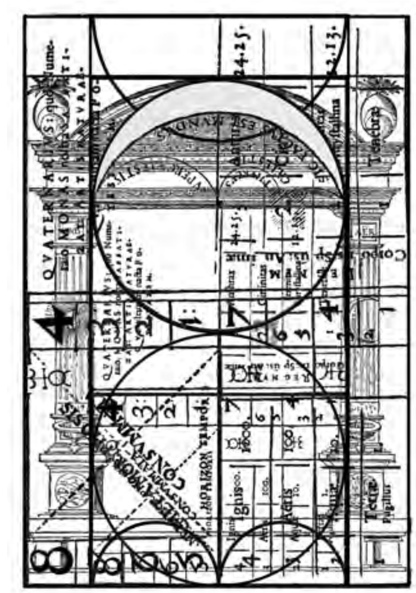
But at least they are all closer to being the same height.

If these four shapes were instruments in a musical quartet all playing at the same time here's what they would sound like (that is, visually superimposed).

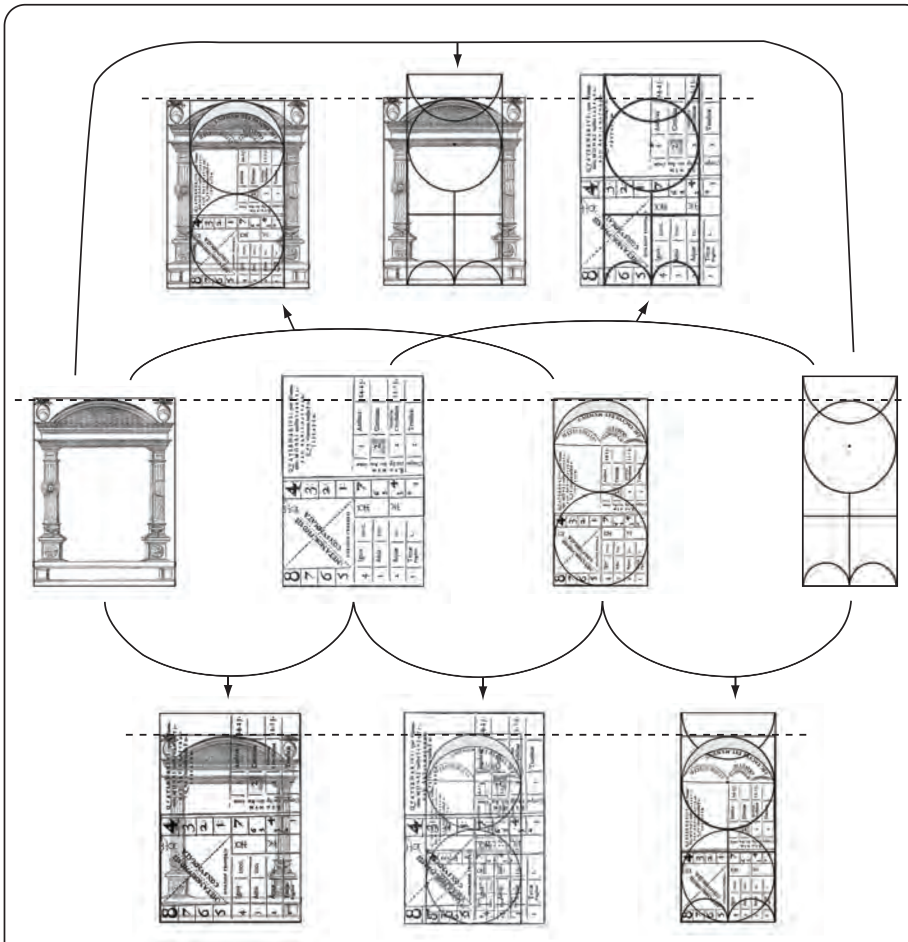
We've seen that the the two columns of the Title Page exactly 24 grid squares apart from each other.

Now these new ***"shrunk by a third"*** shapes (the "ballooned Creation chart and the Monas symbol) fit perfectly "between the columns."

This might appear to be a jumbled mess, but there are wonderful harmonies to be seen.



This visually shows all the various "pairings" of the four shapes
which are shown in the middle of the chart.



Various "pairings" of Dee's shapes

Along the top, we see that the "extended Creation" chart fits snugly between the columns. It's "ballooned half circle" echoes the architectural dome beneath it.

Next to that, the Monas symbol also fits perfectly between the columns, and the top of the Sun circle echoes the architectural dome. The only somewhat odd feature is that part of the "horns" of the Moon extend beyond of the edge of the dome. (This is actually an important clue)

The third pairing along the top shows that this Monas symbol and the "extended Creation" chart are the same height.

Along the bottom, the first pairing shows that the "rectangular Creation" chart and the Title Page have the same width, but not the same height.

The next pairing might not appear to be very synchronous, but remember, they are both "versions" of the very same chart.

The final pairing might be the most poignant of all.

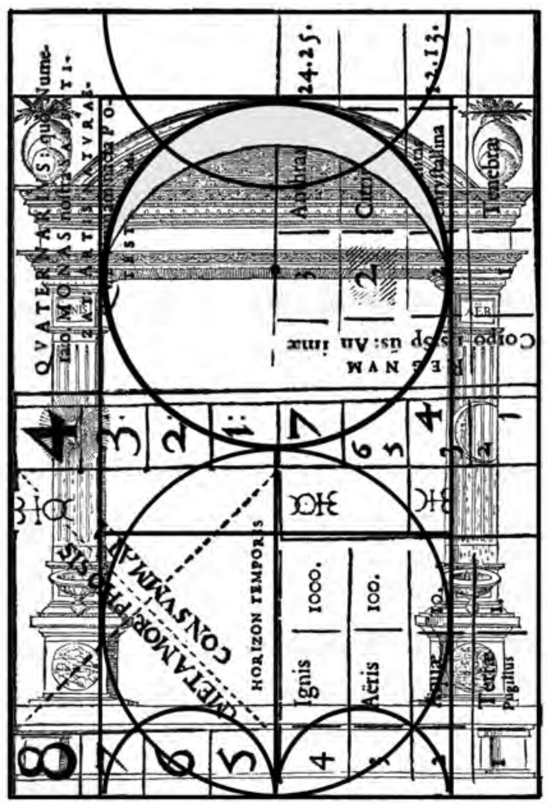
It shows that the Sun circle of the Monas symbol perfectly coinciding with the “ballooned” part of the “ballooned Creation” chart.

The horizontal arm of the Monas symbol’s cross is the same width as the “extended Creation” chart.

And the “feet” of the downturned “horns of Aries” fits snugly in the bottom corners.

With these pairings in mind, take another look at the four pairings superimposed and you can start to see all these interrelations.

(To minimize confusion in this version of the composite, I’ve eliminated the words and numbers from the “balooned Creation” chart, leaving just its two circles its gray “ballooned” shape and its outline.)



This graphic illuminates the very heart of the *Monas Hieroglyphica*.
It visually summarizes the key mathematical relationships, all of which ultimately derive from that famous quaternary: one, two, three and four.
Note particularly that the *centerpoint of the Sun circle* is the exact same height as the top of the columns (or the bottom of the entablature).

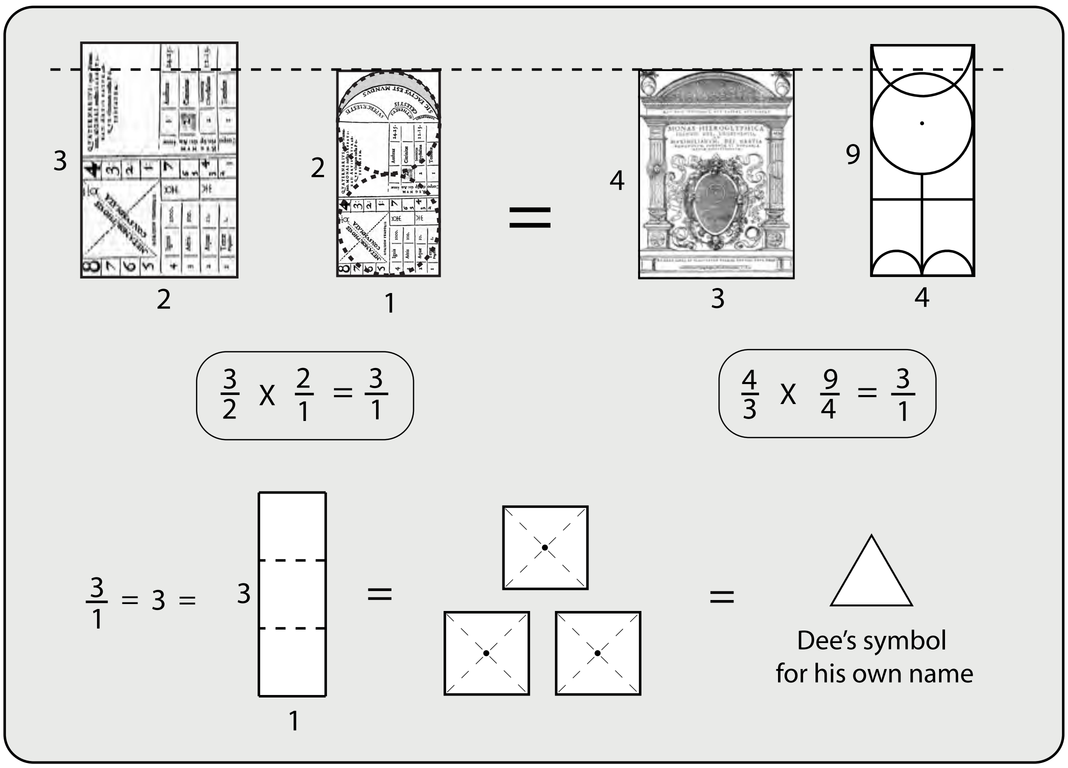
Suddenly some very interesting numbers appear!

Let’s further analyze the mathematics of this “equation of shapes.”

The proportions of the shapes on the left multiply to 6/2
and those on the right multipy to 36/12,
both of which are equivalent to 3/1 or simply 3.

This “3” might be seen as a stack of 3 squares.

Even better, these 3 squares might be arranged in a triangle,
with their centerpoints describing an equilateral triangle,
the shape Dee adopted for his own name.

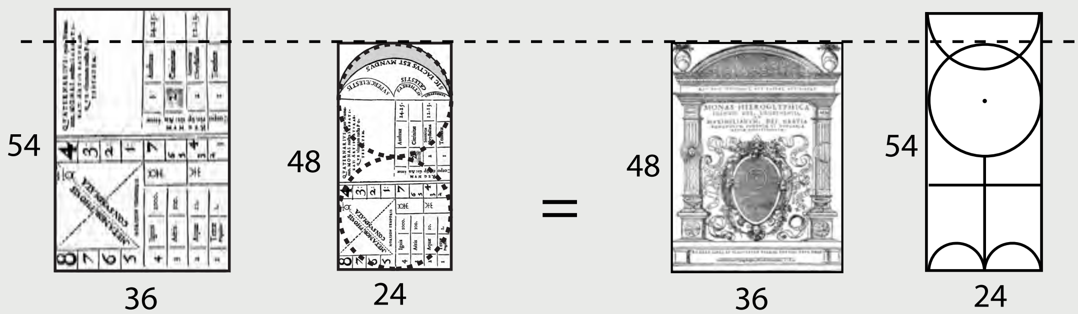


When the shapes are seen in terms of Dee’s grid squares,
a whole new set of doors opens up!

Now, each side of the equation multiplies to this interesting fraction **2592/864**.

On the surface this is simply a fancy way of expressing 3/1,
but cosmically these two large numbers are **very significant.**

Do you know why?



$$\frac{54}{36} \times \frac{48}{24} = \frac{2592}{864} \quad = \quad \frac{48}{36} \times \frac{54}{24} = \frac{2592}{864}$$

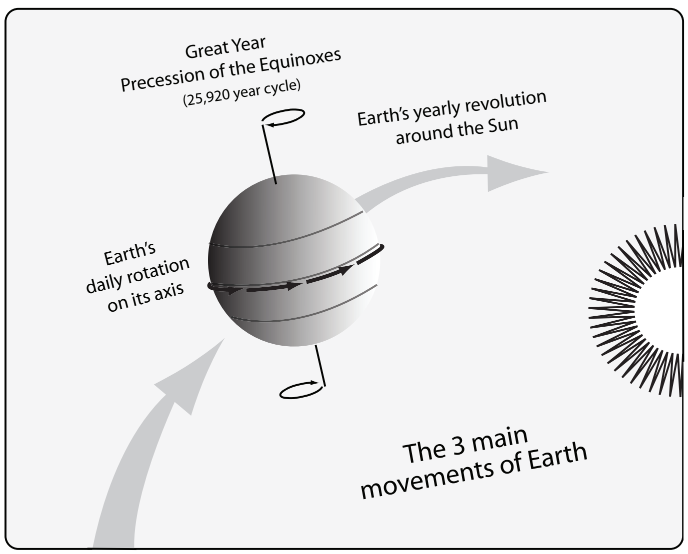
$$\frac{2592}{864} = \frac{3}{1}$$

I'll give you a hint.

They have to do with TIME.

And to understand how we mark time,
we must understand how our earth-sphere
moves in relation to the sun and the stars.

How 2592 and 864 relate to Precession and to the Yugas



The earth has 3 main dance moves.

The fastest is its twirling around its polar axis.
This spin is what makes the sun appear
to rise in the morning and set in the evening.

The second fastest motion is its orbiting around the sun.

A year might not sound very fast, but it's a blink of the eye compared
to the third motion, which is called the Precession of the Equinoxes.
This takes over 25 thousand years!

When you give a top a fast spin it rotates smoothly on its axis for a while.
But when it slows down its axis starts to wobble a little, then a bit more,
then finally the top keels over and scurries along the floor.


Spinning top (going fast)


Wobbling top (starting to slow down)

The Precession of the Equinoxes is just this type of “wobble.”
But don’t worry, the Earth is **not** a top slowing down on the verge of spinning out into space.

Though we generally call the Earth “round,” it isn’t absolutely spherical.
It’s got a beer belly. It bulges outward around the equator.
The combination of this this slight irregularity and sun’s gravitational pull and
creates a “torque” that makes the earth “wobble.”

In order to observe this “third motion,” the other two motions must be taken out of the equation.
The “rotation” motion can be eliminated by always observing the sun **at the same time of day**.
Astronomers generally pick the moment the sun breaks over the horizon at dawn.

The “revolution” motion can be removed from the equation
by studying the sunrise on **one specific day of the year.**
Astronomers usually choose the day of the the spring equinox (the first of Aries)..

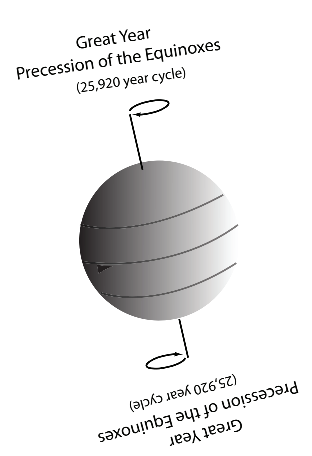
If the earth didn’t have this “wobble” movement,
every year at this equinox dawn the sun would appear
in front of the same cluster of stars that form a “back drop” to the sunrise scene.

But this doesn’t happen!

The “back drop” scene shifts by 1 degree about every 72 years.
This means the scene “shift” is 30° or one “zodiacal sign” every 2160 years.

So the scene shifts through all of the 12 zodiacal signs in 25,920 years, or one “Great Year”
(2160 x 12 = 25920).

(Hey, 25920 is ten times 2592.
That’s interesting.)



This wobble effect causes the north pole to point to a different part
of the sky, describing a complete circle in the sky in 25920 years.

And of course, the same holds true for the south pole.

This north pole circuit and south pole circuit provide a clear way
of showing the “Precession of the Equinoxes effect.”

The Greeks were aware of this strange "third movement."

Hipparchus

The Greek astronomer **Hipparchus** (ca. 190 BC – ca. 120 BC) was the first to recognize this "third earth movement." He wrote a book on it called "*On the Displacement of the Solsticial and Equinoctial Points*." He recognized that this motion was happening, but didn't attribute it to earth wobble. (Indeed, he still believed the sun revolved around the earth.)

If you've ever witnessed the sun's first appearance at the crack of dawn, you know that once even a small segment of the sun is visible, you can't look at it because it is so blazingly bright. And even before it rises, the horizon sky is so light you can't see the stars in the "back drop." So how did Hipparchus do it?

Well, actually, Hipparchus discovered this "third movement" an entirely different way. A lunar eclipse provided a clue. Hipparchus knew that the shadow that covered the moon was the earth's shadow, so at that moment, the moon, earth and sun were in a perfectly straight line. While observing a lunar eclipse, Hipparchus determined that the distance between the center of the Moon and a particularly bright "backdrop" star called *Spica* was 8 degrees.

He also had data from two of his predecessors, Timocharis and Aristillus, who had witnessed a lunar eclipse 169 years earlier. At that time Spica was only 6 degrees from the center of the moon. The whole tableau of the fixed stars had "shifted" by 2 degrees!

Much of what is known about Hipparchus comes from Ptolemy, who estimated that this "third" movement was about 1 degree every 100 years. Ptolemy's work was studied by the Islamic astronomers.

One such Arab was Ibn al-Shātir of Damascus (ca. 1320–1375) who estimated this third movement to be 1 degree every 70 Persian years. (George Saliba, *A History of Arabic Astronomy* page 241, n. 8).

Studying all these early sources and making his own observations Nicholaus Copernicus (1473–1543) determined the shift was 1 degree in 72 years, or 25,920 years for the "full cycle."

(Thomsa McEvilley, *The Shape of Thought: Comparative studies in Greek and Indian Philosophies,* p. 78-9).

His 1543 *De revolutionibus orbium coelestium* is the first time that precession was correctly attributed to the wobble of the Earth's axis. (But it wasn't until around 1687 Isaac Newton figured out that the wobble was a result of gravitational forces.)

John Dee knew as much about the precession of the Equinoxes as any Renaissance astronomer

John Dee had a copy of Copernicus' book in his library.
He even cites Copernicus in Axiom 67 of his *Propadeumata Aphoristica*.

(Roberts and Watson, book number 220).

In Axiom 75, Dee describes this third movement, explaining why the fixed stars:
"**However, all of the fixed stars are subject to an extremely slow Movement to the East, along the Ecliptic**."

(Dee, *Propaedeumata Aphoristica*, Aphorism 75).

Dee correctly notes that the fixed stars move in an "easterly direction."
This means the equinox point moves "backwards" through the signs
(for example,…Taurus, Aries, Pisces, Aquarius…)
compared with the way the Sun annually passes before the zodiac
(…Aquarius, Pisces, Aries, Taurus…).

This is why its called the **Pre**cession of The Equinox
("pre" meaning "before" in time, order or place.)

From around 4000 BC to 2000 BC, the Spring sun rose
in front of the backdrop of the stars in **Taurus**.

From around 2000 BC to 1 BC, it rose in **Aries**.

From 1 AD to the present day, it has risen in **Pisces**.

In a few years, the equinox sunrise
will appear against the backdrop of **Aquarius**.
(The lyrics "This is the dawning of the Age of Aquarius..."
celebrated this event in the 1960's musical "Hair."
As the constellations vary in width, it's hard to pinpoint exact dates)

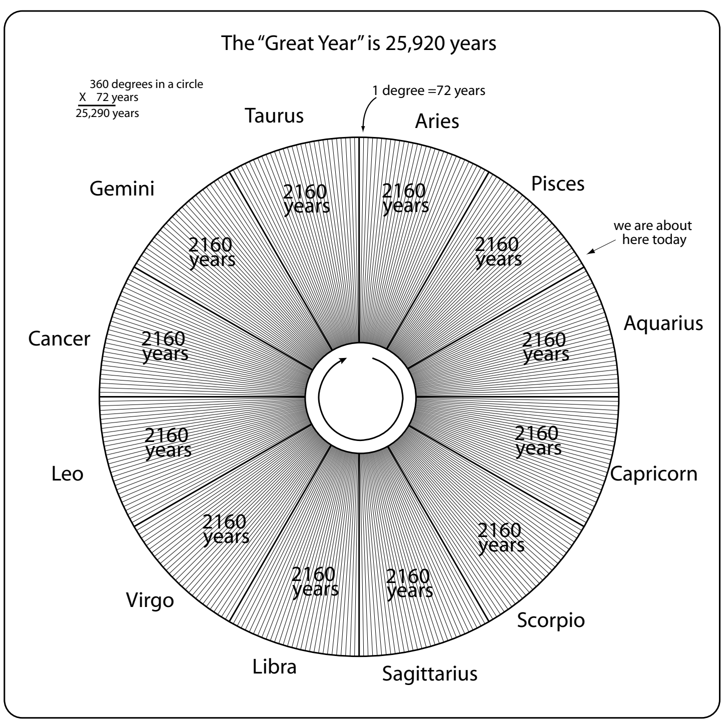
The 12 parts of the Great Year reveal special numbers.

Let's picture the Great Year as a circle divided into 360 degrees.

Each degree represents 72 years, so every 30 degrees is a new "Zodiacal Month" of 2160 years.
(72 x 30 = 2160)

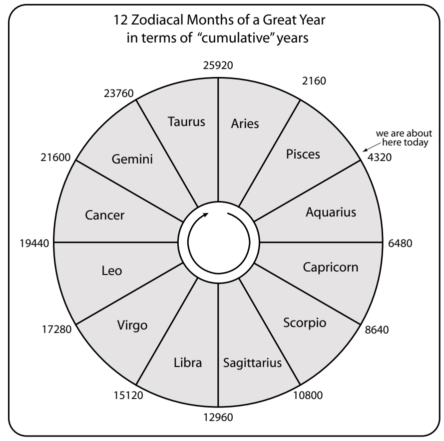
Twelve of these Zodiacal pie sections make the full 25,920 years in the Great Year.
(12 x 2160 = 25,920)

I've marked approximately where we are today,
still in the Piscean Age, but verging on the Age of Aquarius.



Next, let's look at this circular picture of a Great Year in terms of the number of years in each of the 12 Zodiacal months.

(Aries had 2160 years, but by the end of Pisces it will be twice that, or 4320 years).

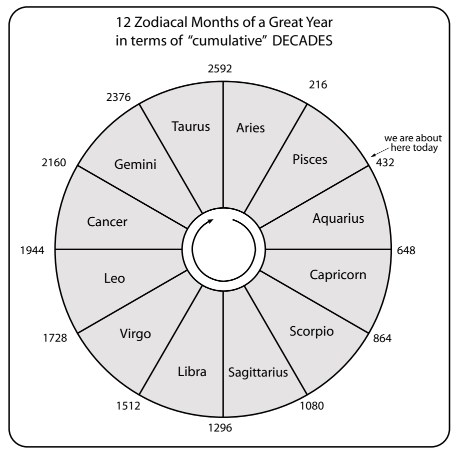


As each of these numbers of years ends in a zero, it's useful to this analysis to simply see them in terms of DECADES.

Look what "pops up."

Those two numbers, 2592 and 864!

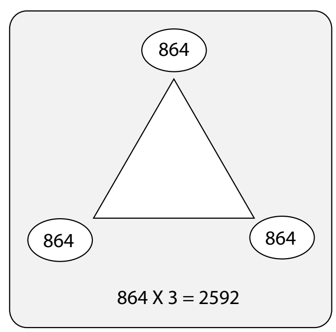
(as 216 x 12 = 2592
and
216 x 4 = 864).



This is simply another way of seeing that 2592 and 864 are in a
3 to 1 proportion.

Here are the three 864's totaling 2592.

(illustrated with Dee's "triangle" name)



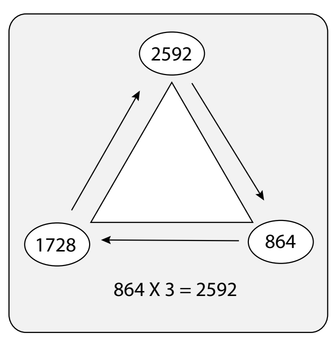
Or "cumulatively,"
they proceed from 864, to 1728, to 2592.

Mathematicians will recognize 1728 as 12 cubed.

This is the number that Marcello Ficino felt was Plato's "Nuptual number."
Dee felt it was special too.

The Title Page of the *Monas* has exactly 1720 grid squares

(as 48 x36=1728)



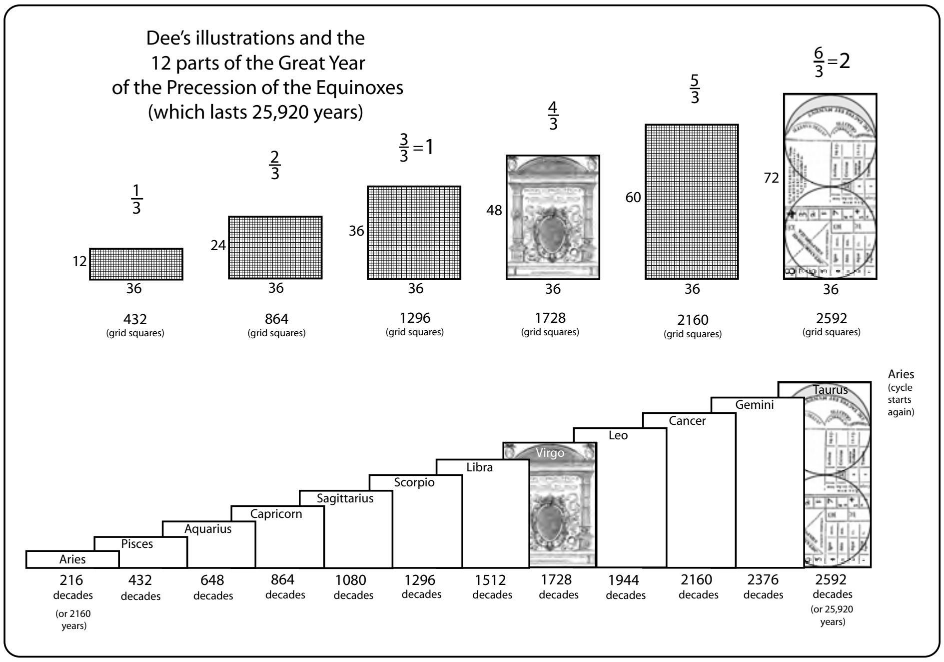
864, 1728 and 2952 in Dee's illustrations.

When two of Dee's illustrations are analyzed in terms of grid squares,
the same numbers appear!
In this diagram, the bottom row shows Capricorn, Virgo, and Taurus
as being 864, 1728 and 2592 decades respectively.

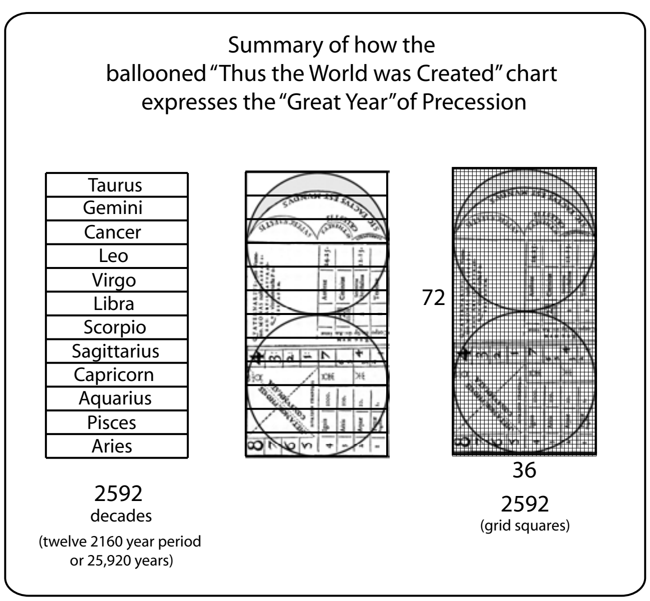
The top row of the diagram shows how "one half of the Title Page"
(up to the middle of the columns) is exactly **864** grid squares. (24 x 36= 864)

The full Title Page is **1728** grid squares. (48 x 36=1728)

And the full "ballooned 360" Creation chart is **2592** grid squares. (36 x 72 = 2592)



Here's a summary of the 12 Zodiacal Months of the Great Year
superimposed on the "ballooned 360" Creation chart.



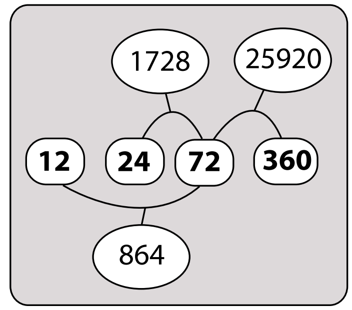
Why these numbers 864, 1728 and 25920 are so electrified

To most people 864, 1728, and 25920
might seem like random, run-of-the-mill numbers.

But if you have a sense of what Dee calls
the "dignity" of the Metamorphosis numbers,
you can see why these 3 innocent-looking numbers
must be electric with energy.

**The numbers 864, 1728, and 25920 are all products
of various combinations of Metamorphosis numbers!**

12 x 72 = 864,
24 x 72 = 1728,
72 x 360 = 25920.



And remember, each of
the Metamorphosis numbers
contains all the symmetry of the
preceding Metamorphosis numbers.
(Like 360 contains the symmetry of the simpler numbers 12, 24 , and 72).

The numbers 864 ,1728 ,and 25920 are *energized*
because the are products of *energized* numbers.

More "synchrony" seen in 864.

Another very special relationship between
the number 864 and the first 4 Metamorphosis numbers,
has to do with its transpalindromic mate, 468.

The sum of 12 + 24+ 72 + 360 equals 468.

12 + 24 + 72 + 360= 468
468 Ⱥ 864

Digging a little deeper, the difference
between 864 and 468 is 396.

And guess what!

The number 396 can be
found by adding together the
3 Metamorphosis numbers 12 + 24 + 360.

864– 468= 396
396=12+24+360
Thus,
12+12+24+24+72+360+360=864

In other words, 864 can be seen as
an additive string of Metamorphosis numbers.
(12 + 12 + 24 + 24 + 72 + 360 + 360)

(Note that 396 is a member
of the 99 Wave, as 4 x 99= 396.
This is a glimpse of that *synchrony*
between Consummata and Metamorphosis.)

A peek "under the hood" of 25920.

To further analyze 25920, let's pare it down (by a factor of 10) to its less-unwieldy relative, 2592.
The number 2592 still seems a random, typical number.
But, a closer look "under the hood" provides clues as to why it's so powerful.

Being 72 times 36, 2592 must be special.

The number **72** is a Metamorphosis number and a member of the 9 wave.

The number **36** is not a Metamorphosis number, but its special is special for several reasons. It's half of 72. It's a member of the 9 Wave. It's one tenth of the Metamorphosis number 360. It's the sum of the Metamorphosis numbers 12 and 24.

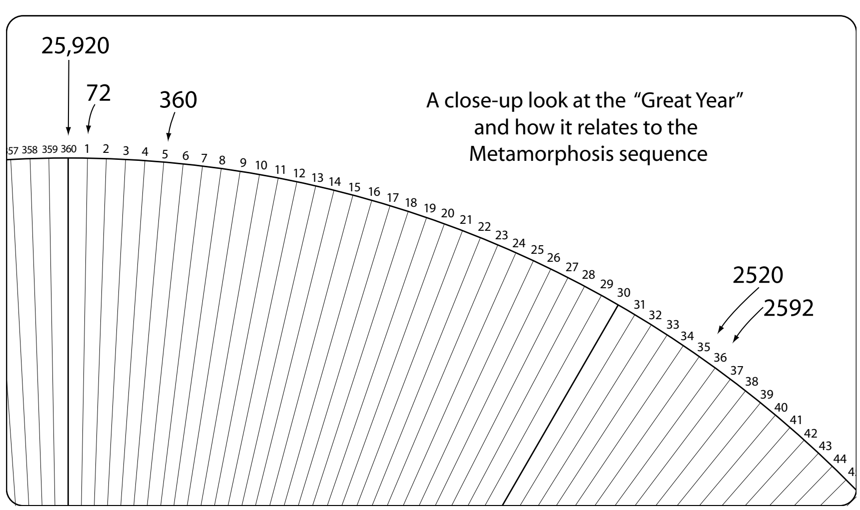
Type 36 times 72 into a hand calculator
and 2592 promptly appears.
But, multiply it using a pencil
and another special number "pops up."
Dee's magistral number, 252, appears
as the "7 times 36" part of this multiplication.

Actually, the way long multiplication works,
there is an "understood" zero in there,
making it really 2520.
(a *very* special Metamorphosis number)
This also reveals the fact that 2592 is actually
the sum of two Metamorphosis numbers (72 + 2520).

$$\begin{array}{r} 36 \\ \times\ 72 \\ \hline 72 \\ 252\ \\ \hline 2592 \end{array} \qquad \begin{array}{r} 36 \\ \times\ 72 \\ \hline 72 \\ 2520 \\ \hline 2592 \end{array}$$

This can be graphically seen by looking at a close-up view
of the beginning part of the Great Year Wheel.
(To simplify this 25920 Wheel, I've only shown 72 year increments, so there are 360 of them.)

Observe that 2520 years is "one notch" away (or 72 degrees) from 2592.
(I've also shown Metamorphosis number 360, which falls at the five degree mark.)



864 and 2592 in Hinduism.

It seems as though Dee learned of these two wonderful numbers
864 and 2592 from Copernicus' conclusions about the Great Year.
But there's a tradition that goes back way before Copernicus that involves these numbers.
This tradition started thousands of years in a land far,far away from Europe:

India

In the ancient tradition of Hinduism, 864 and 2592 relate to TIME.
Not time in seconds, minutes, hours, or even days, but in years.
Really, really long periods of years.

Let's start with a very brief overview of Hinduism and some of its main texts.

From 2000 BC to 1200 BC the Aryan peoples who entered India
from Persia practiced the Vedic religion.

Around 1500 BC, they composed the four **Vedas**, their primary religious texts.
Hinduism (which comes form the Persian word Hind, meaning India) evolved from Vedism.

From 800 BC–200 BC, the Hindus wrote treatises expounding upon the Vedas
called the **Upanishads**, which in Sanskrit means "sitting at the feet of a master."

Starting around 400 AD, the sacred writings of Hindu folklore and legend
were called the "**Puranas**," which means "ancient legend."

Around 1000 AD, the great Arab scholar **Al-Biruni** (973 AD – 1048 AD)
journeyed to India, learned Sanskrit, translated some of the Puranas,
and wrote an extensive account of Hindu philosophy and cosmography.
Al-Biruni's work introduced the Hindu Time Cycles to the Muslim World,
and eventually to Europe.

According to the Puranas, the world goes through a continuous cycle of long epochs.
We're not talking decades, centuries, or even millennia here.
For example, a "**Brahma Day**" is **3,110,400,000,000** years long.
That's 3 trillion, 110 billion, 400 million years long!
(Brahma is the creator God along with Vishnu and Shiva)

If you think that is long, Hindu tradition has it that the earth is in its 51st Brahma year,
making it over 150 trillion years old.

There are 12 "Brahma Months in each "Brahma Year."
So (dividing 3,111,400,000,000 years by 12),
there are **259,200,000,000** years in a "Brahma Month."
Ignoring all those zeros, here we have Dee's "hidden" number **2592.**

Furthermore, there are 30 "Brahma Day and Nights" in a Brahma month.
So (dividing 259,200,000,000 by 30),
there are **8,640,000,000** in a "Brahma Day and Night."
Again, ignoring the zeros, we have **864**, the other one of Dee's "hidden" numbers.

The Yugas in Hindu Timekeeping

The shortest Yuga is the ***Kali Yuga***,which lasts for 432,000 years.
(The consonant K in *Kali* apparently derives from the word *Eka* meaning "one" or "one part.")

The next longest Yuga is the ***Dvapara Yuga***, lasting 864,000 years.
(There's Dee's number 864 again.)
(Dvapara comes from the word *dva* meaning "two" or "two parts,"
as it's twice as long as the Kali Yuga)

Next, is the ***Treta Yuga*** of 1,296,000, as it is the length of 3 Kali Yugas
(Treta comes from the word *tre* or *tri* meaning "three," as 432 x 3 = 1296).

Finally, the ***Krita Yuga*** is 1,728,000 years long.
(The word Krita has the same consonants, (R and T) that are in the word *Chatur*, meaning "four."
Krita Yuga is also sometimes called Satya Yuga.) (Wikipedia:Yuga).

Hindu Timekeeping
(in Years)

Kali Yuga	432,000
Dvapara Yuga	864,000
Treta Yuga	1,296,000
Krita Yuga	1,728,000

Incidentally, right now we are apparently in a Kali Yuga period,
which the ancient scriptures say started on February 16, 3102 BC.

As about 5 thousand years have passed since then,
so there's still a lot of Kali Yuga time left – about 427,000 years!

Many historians believe that these large numbers found in the
ancient chronologies should be taken symbolically and not literally.

The ancient mathematicians didn't use decimal points
or fractions, but instead used very large whole numbers.
This makes the unit quantity so small that there was more
precision when comparing them with the large whole numbers.

Also, many historians believe that the 4 Yugas correspond
to the 4 Ages written about by ancient Greek authors like Hesiod (around 800 BC).

The longest age (Krita) is associated with the Golden Age,
next the Silver Age, then the Bronze Age, then the Iron Age.

(Note: These are ***not*** the archeologists' Bronze Age and Iron Age
which relate to eras when these particular metals came into regular use.)

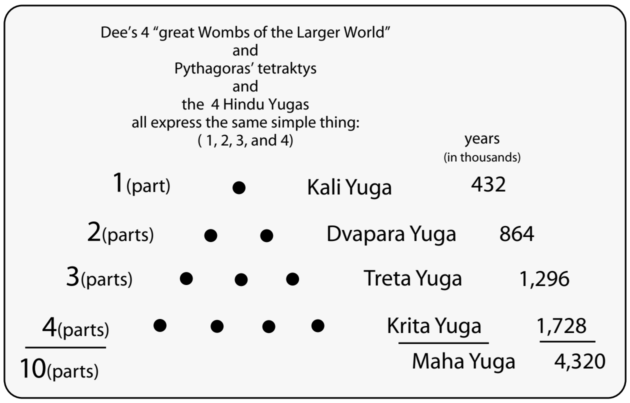
(Robert Bolton, *The Order of the Ages World History in the Light of a Universal Cosmogony,*
Ghent NY, Sophia Perennis, 2001, p. 64-5, 97, 215-217.)

The proportions of the Yugas to each other.

It's obvious that the four Yugas are in the 1:2:3:4 proportion to each other.
Dee calls "1, 2, 3, 4" the "Four separate great Wombs of the Larger World."
(My interpretation of what Dee means in Axiom 18 of his *Propaedeumata Aphoristica*).

In the *Monas*, Dee calls "1, 2, 3, 4" the "Pythagorean Quaternary."
This is the Pythagoras' tetraktys. "1, 2, 3, 4" add up to 10.

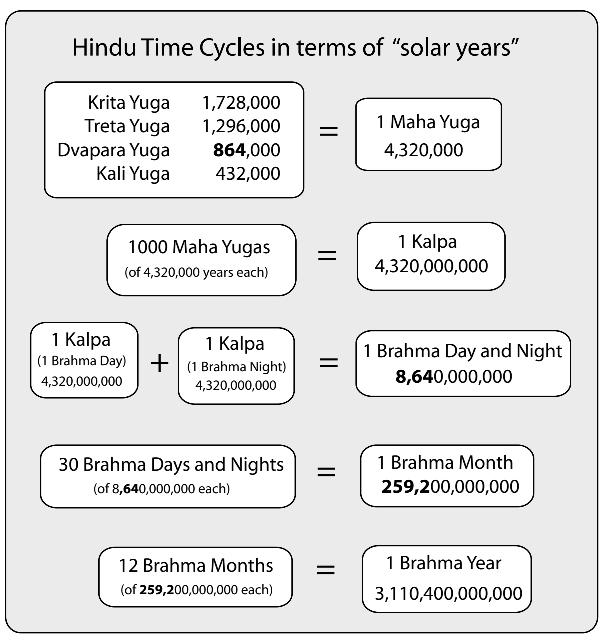
The Hindus called the total of these four Yugas a ***Maha Yuga***, or 4,320,000 years.
Ignoring the zeros, you can see how the "10" part Maha Yuga (432)
is "return to 1," a Kali Yuga (also 432).



As if over 4 million years isn't enough, the Hindus called
1000 of these Maha Yugas a "Kalpa" or 4,320,000,000 years.

A Kalpa is 1 Brahma Day, but when combined with 1 Brahma Night,
it makes the "Brahma Day and Night" that I mentioned earlier, with 8,640,000,000 years.

All these terms and years get confusing, so I've summarized them in this chart:



The Hindus used an alternative accounting of these cycles in terms of "**deva years**."
As one "deva year" equals 360 solar years,
the numbers in the accounting became a little more "user friendly."

Hindu Time Cycles in "deva years"

1deva year = 360 years

Krita Yuga	4,800
Treta Yuga	3,600
Dvapara Yuga	2,400
Kali Yuga	1,200

= 1 Maha Yuga 12,000

1000 Maha Yugas (of 12,000 each) = 1 Kalpa 12,000,000

1 Kalpa (1 Brahma Day) 12,000,000 + 1 Kalpa (1 Brahma Night) 12,000,000 = 1 Brahma Day and Night 24,000,000

30 Brahma Days and Nights (of 24,000,000 each) = 1 Brahma Month 720,000,000

12 Brahma Months (of 720,000,000 each) = 1 Brahma Year **8,64**0,000,000

In "deva years,"
a Kali Yuga is 12 hundred,
a Dvapara Yuga is 24 hundred,
Treta Yuga is 36 hundred,
and a Krita Yuga is 48 hundred.
Sound familiar?

The Hindu Yugas in deva years and solar years

1200 deva years	x	360 solar years per deva year	=	432,000 solar years
2400 deva years	x	360 solar years per deva year	=	864,000 solar years
3600 deva years	x	360 solar years per deva year	=	1,296,000 solar years
4800 deva years	x	360 solar years per deva year	=	1,728,000 solar years
12,000 deva years	x	360 solar years per deva year	=	4,320,000 solar years

The "Yuga Numbers" relate to Dee's Title Page grid.

This chart compares the numbers of the "deva years" and the "solar years" of the Yugas with the "height" and "grid square areas"of various sections of the Title page.

The results are exactly the same!

Dee's Title page grid expresses the Yuga Numbers!

(When I use the term Yuga Numbers here, I'm "tossing out" the zeros
The "deva years are seen in terms of "hundreds," like 1200 becomes 12.
The "solar years" are seen in terms of "thousands," like 432,000 becomes 432.)

Yugas

	deva years	solar years
Kali	**12**00	**432**,000
Dvapara	**24**00	**864**,000
Treta	**3600**	**1,296**,000
Krita	**48**00	**1,728**,000

Dee's Title Page

	hieght in grid squares		total grid squares
1 quarter of the Title Page	**12**	 36	**432**
2 quarters of the Title Page	**24**	 36	**864**
3 quarters of the Title Page	**36**	 36	**1296**
4 quarters or Full Title Page	**48**	 36	**1728**

This "quartering" of the Title Page is not arbitrary.
Dee highlights the 1/4, 1/2, and 3/4 "heights" with important features of the architecture.
(the bottom of the column, mid-column, and the top of the column, respectively).

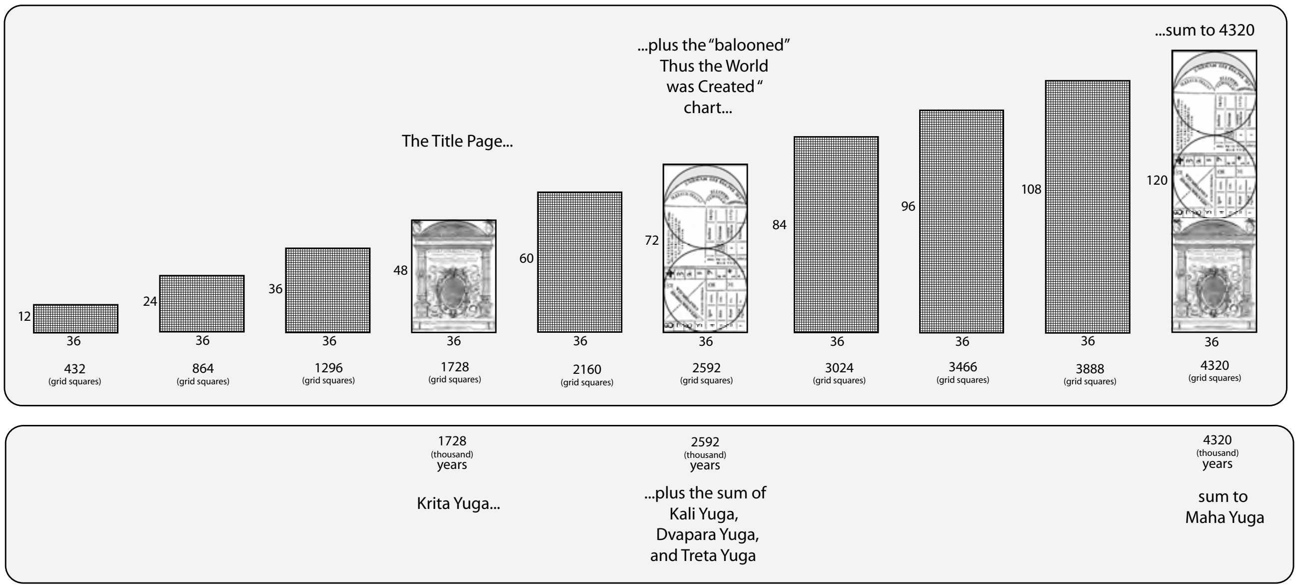
As you might have deduced,
Dee's Title Page expresses the Yuga Numbers
because the width of the Title Page is 36 grid squares,
which is similar (by a factor of ten) to the 360 years in a "deva year."

(Even though 36 is not a Metamorphosis number, its close relative 360 is, and its "double," 72, is as well.
Thus, 36 is still a "key number" in this way of looking at numbers known to the ancient Hindus, John Dee, and Bob Marshall.)

Here's another way Dee's illustrations relate to the Yugas.

The grid of the Title Page (36 x 48 = 1728)
plus the "ballooned Creation" chart (36 x 72 = 2592)
sum to **4320**.

Similarly, the 4 Yugas sum to **4320**.
(Also, Kali, plus Dvapara, plus Treta equals 2592 solar years.)



So you can see how Dee's innocent looking Title page
is like a"measuring stick" that can integrate the Yugas seen as
either "solar years"or as "deva years"

Let's take another look at those *really long* Hindu time cycles
expressed in terms of deva years.

A Maha Yuga is **12** thousand deva years,
a Kalpa is **12** million deva years,
a Brahma Day and Night is **24** million deva years,
a Brahma Month is **720** million deva years,
and a Brahma Year is **8,640** million deva years.

Even in this "deva year" accounting," Dee's "hidden" number **864** turns up again!
And the other results all relate to the first three Metamorphosis numbers, **12**, **24**, and **72**.

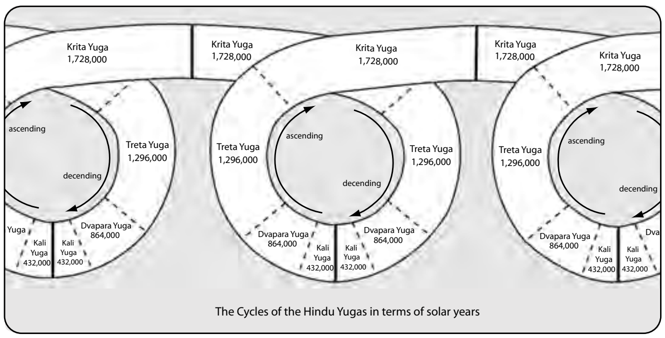
The Hindu cycles exhibit “retrocity” or “oppositeness”

We’ve seen that the Hindus felt
that a Brahma Day (or a Kalpa, 4,320,000,000 years)
needed a Brahma Night (another Kalpa of 4,320,000,000)
to make a complete whole or a “Brahma Day and Night” (of 8,640,000,000 years).

This appears to be an expression of “retrocity,”
just as Dee’s Sun needs the Moon to be complete.

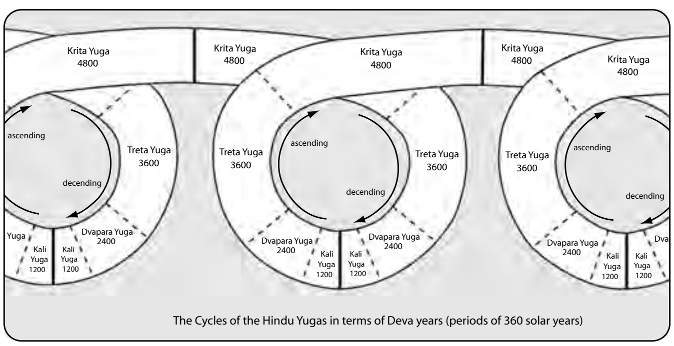
Also, **within** each of these Kalpas, there is another Hindu time-keeping pattern,
It also appears to be an expression of “retrocity.”

Every “**descending**” sequence of Krita, Treta, Dvapara, and Kali
is followed by an “**ascending**” sequence of Kali, Dvapara, Treta, and Krita.
The cycle continues endlessly.



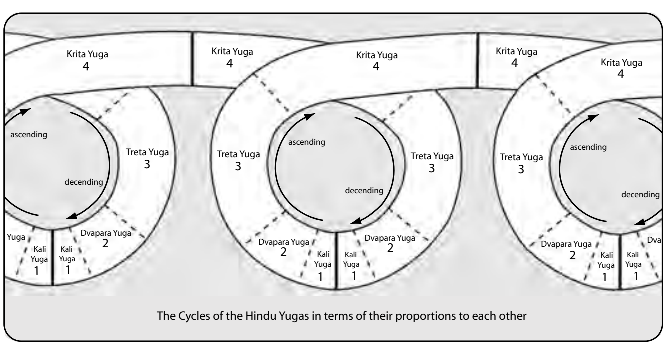
The Cycles of the Hindu Yugas in terms of solar years

This first chart shows these looping cycles in terms of solar years.



The Cycles of the Hindu Yugas in terms of Deva years (periods of 360 solar years)

This next chart shows the cycles in terms of “deva years” (of 360 solar years each).



The Cycles of the Hindu Yugas in terms of their proportions to each other

And this final chart shows the cycles
in their simplest proportions
(…4, 3, 2, 1, 1, 2, 3, 4,…).

Does this sound familiar?

It’s reminiscent of
Bucky’s “+4, –4, octave” rhythm
found in Base Ten numbering.

The Yugas are all of a different lengths of time.
To simplify, let's take a look at the cycling
using an "**average length**" of the 4 Yugas.

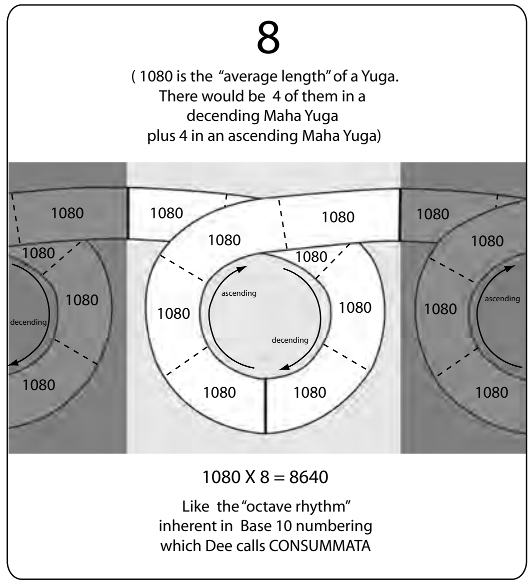
Krita		Treta		Dvapara		Kali		Kali		Dvapara		Treta		Krita	
1728	+	1296	+	864	+	432	+	432	+	864	+	1296	+	1728	= 8640

The "average" size of these differently-wsized Yugas is

$$\frac{8640}{8} = 1080$$

1080 + 1080 + 1080 + 1080 + 1080 + 1080 + 1080 + 1080 = 8640

Adding all 8 Yugas in a cycle makes 8640.
Then dividing by 8, reveals that
the "average Yuga length is 1080.
Seen as one place value less,
this is an expression of
"8 times the very sacred Hindu
number 108 equals 864."
The number 108 dancing in an octave rhythm,
It's a beautiful sight to behold.



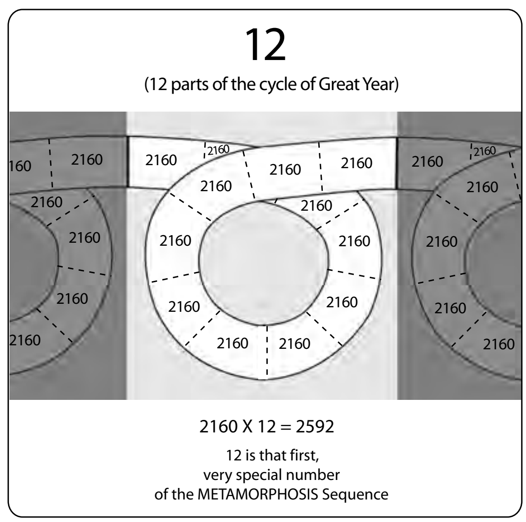
Let's picture the "Great Year of the Precession of
the Equinoxes" in a similar, cyclical way.
We don't need to find an average
because all the Zodiacal Months
are the same length (2160 years).

2160 + 2160 + 2160 + 2160 + 2160 + 2160 + 2160 + 2160 + 2160 + 2160 + 2160 + 2160 = 25920

Each "zodiacal month"
of a Great Year of the
Procession of the Equinoxes
is 2160 solar years

$$\frac{25920}{12} = 2160$$

The fact that there are 12 parts to this cycle
makes it reminiscent of the Metamorphosis
sequence that starts at 12.



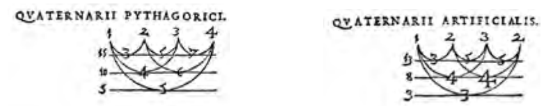
I call it "reminiscent" because this cycling
is not a full description of
the Metamorphosis sequence.
(For example, after the three "cycles of 12," 36 "Zodiacal
Months" have passed, but 36 is not a Metamorphosis number.)

Similarly, the 8-part rhythm cycling
of the Yugas shown above is only
"reminiscent" of Consummata.
The octave rhythm shown above lacks that
"null 9" pause between cycles of octaves.

But don't think for a moment that the "9 wave" is not involved with of all these numbers!

As proof that its involved, simply multiply that "null 9" times the multiplicative results of Dee's Pythagorean Quaternary (24) and of his Artificial Quaternary (12).

The results are 216 and 108, relatives of 2160 and 1080, the pieces we just cut these two cycles into!



1 x 2 x 3 x 4 = 24

24 x "null 9" = **216**

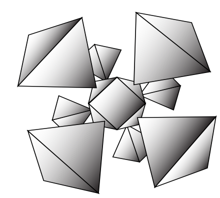
1 x 2 x 3 x 2 = 12

12 x "null 9" = **108**

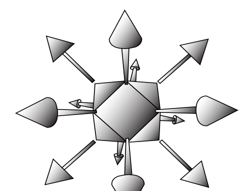
That null 9 is definitely involved in all these synchronous numbers.

We've already seen another "reminiscence" of "octavity" and "twelveness" in the field of 3-D Geometry.

The cuboctahedron, seen as either "**8** tip to tip tetrahedra" or "**12** vertices," is a similar echo of Consummata and Metamorphosis.



The 8 tetrahedra of a cuboctahedron



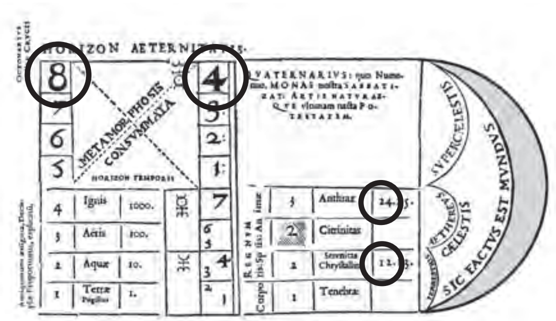
A cuboctahedron has 12 vectors

As we've seen, Dee references these "number concepts" in his "Thus the World Was Created" chart.

In the octave in the upper-left quadrant the 4 and 8 are in **bold**.

The 12 and 24 are **prominently** visible in the lower-right quadrant.

(And more cryptically in the proportions of the circle segments labeled "Terrestrial" and "Aetheric Celestial.")



Playing with the "Yuga Numbers" and the "Great Year Numbers."

The best way to get a feel for the many interrelationships of these recurring numbers is to have some fun and "*play*" with them for a while.

The following charts of the multiples of 108 and 216 provide a good overview. More profound insights can be gleaned by studying Bob Marshall's "108 Wheel" (60 spirals of numbers around a circle with 108 divisions, going up to 6840.)

The Yuga numbers 432, 864,1296, and 1728 can be seen as 108 times 4, 8, 12, and 16.

The "Yuga numbers" are all multiples of the sacred Hindu number 108

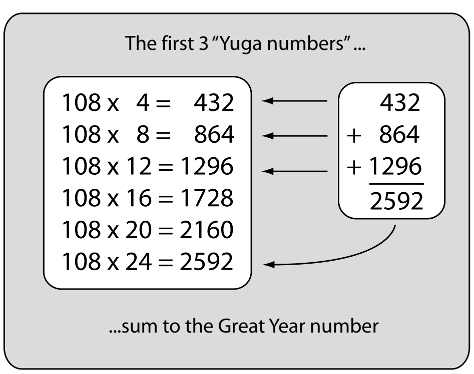
108	X	1	=	108
108	X	2	=	216
108	X	3	=	324
108	X	4	=	**432**
108	X	5	=	540
108	X	6	=	648
108	X	7	=	756
108	X	8	=	**864**
108	X	9	=	972
108	X	10	=	1080
108	X	11	=	1188
108	X	12	=	**1296**
108	X	13	=	1404
108	X	14	=	1512
108	X	15	=	1620
108	X	16	=	**1728**
108	X	17	=	1836
108	X	18	=	1944
108	X	19	=	2052
108	X	20	=	2160
108	X	21	=	2268
108	X	22	=	2376
108	X	23	=	2484
108	X	24	=	**2592**

This chart of the multiples of 216 gives the 12 cumulative results for the 12 "Zodiacal Months of the Great Year." Prominent in this list are the 4 "Yuga numbers," products of 216 with 2, 4, 6, and 8.

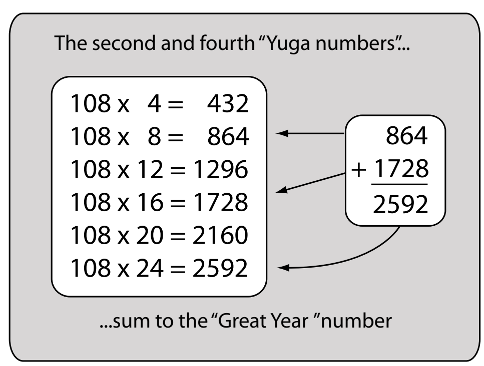
The "Yuga numbers" are also "Precession of the Equinox numbers" (that is, multiples of 216)

216	X	1	=	216
216	X	2	=	**432**
216	X	3	=	648
216	X	4	=	**864**
216	X	5	=	1080
216	X	6	=	**1296**
216	X	7	=	1512
216	X	8	=	**1728**
216	X	9	=	1944
216	X	10	=	2160
216	X	11	=	2376
216	X	12	=	**2592**

We've seen how the first 3 "Yuga numbers" sum to the "Great Year number."

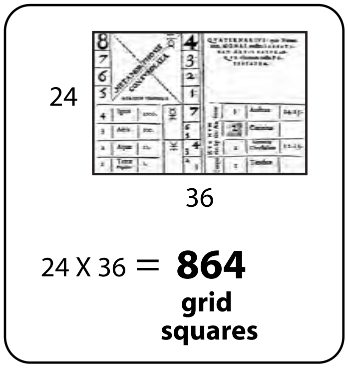


The 108 and 216 charts above make it easy to see why the second and fourth "Yuga numbers" also sum to the "Great Year number."

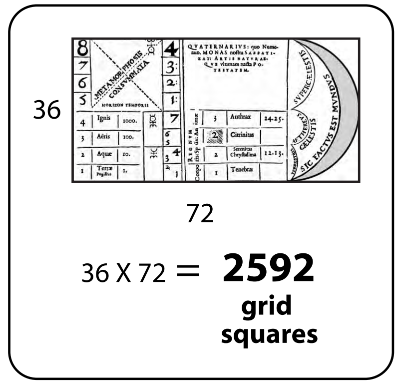


Dee wove the numbers 864 and 2592 into the fabric of his chart.

The "rectangular part" of the chart might be seen as 24 grid squares tall by 36 wide, making for a total of 864 grid squares.



The "ballooned 360" version of the chart might be seen as 36 grid squares tall by 72 wide making it 2592 grid squares.



(That's a lot of grid squares for a small chart that's only about 5 inches wide in the printed book.
I'm not suggesting Dee actually drew so fine a grid on his working copies.
However, it's quite easy to do it "conceptually" with simple mathematics.)

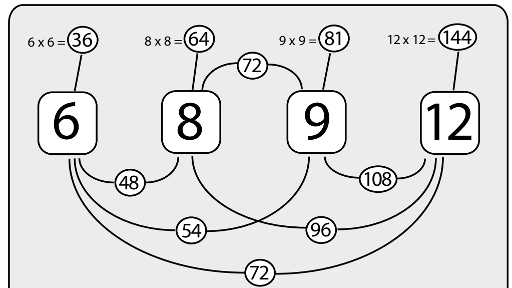
2592 even pops up in relation to the "most perfect proportion", 6:8:9:12.

The numbers of Nicomachus and Boethius' "greatest and most perfect symphony" (6, 8, 9,12) also integrate with 2592.

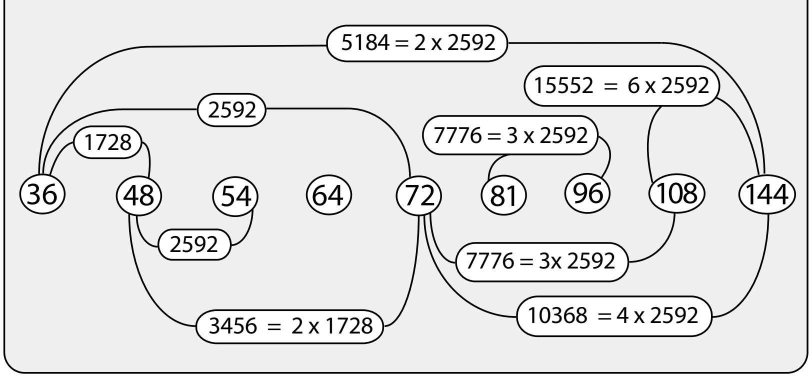
This first chart shows the products of pairs of those numbers.

Some of these products multiplied together make multiples of 2592

(like 48 times 54 equals 2592).



Multiplication of the numbers in Boethius' "greatest and most perfect symphony"



Some of the multiplications of the results of the previous diagram that relate to "Yuga numbers" and the "Great Year number"

Playing with 6's instead of 12's.

Instead of using 12 to analyze these numbers, let's use its good friend, 6.

The result of "6 cubed" is 216, the "Zodiacal Month number."
The product of 6 x 6 x 6 x 6 is 1296, the "Treta Yuga number."

6 = 6
6 x 6 = 36
6 x 6 x 6 = 216
6 x 6 x 6 x 6 =1296

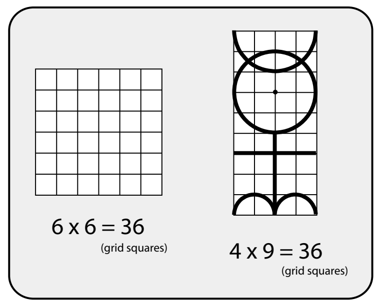
When this result is doubled, it makes 2592, the "Great Year number."

1296 x 2 = 2592

Here is another (sort of "Babylonian sexagesimal") way to look at 1296.

60 x 60 x 60 x 60 = 1,296.000
500 cycles of the 25920 Great Year = 1,296,000

The result of "6 squared" is 36.
Not only is this a tenth of that great number 360, but its also the number of grid squares in the 9 to 4 grid of the Monas symbol.

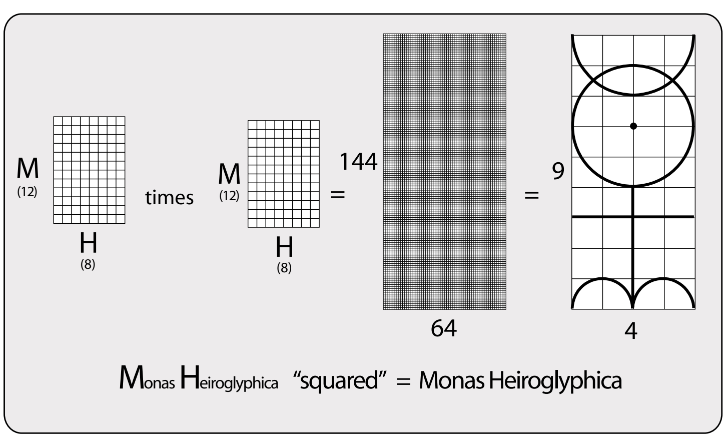


The M and H of Monas Hieroglyphica.

One playful way to see this 3/2 x 3/2 = 9/4 relationship is with the two beginning letters of the words *Monas Hieroglyphica*.

The Letter M is the 12th Latin letter and H is the 8th Latin Letter.
Two rectangles that are M by H, (or 12 high by 8 wide) multiply to the proportion of the Monas symbol.

(Multiplying letters does sound strange, but remember how many numbers Dee creatively derived from the letter X, like 2, 3, 4, 7, 8, 21, 100, 25, 2500.)

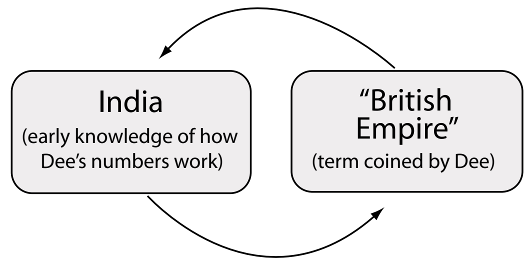


Empirical Irony with regards to the Sages of the Ages

There's a touch of irony in the idea that some of Dee's understanding of numbers ultimately derived from ancient Indian sages.

Dee coined the phrase the "British Empire."

About 150 years after Dee died, India became part of the British Empire, and remained so for about 2 centuries (from around 1750 to 1949).

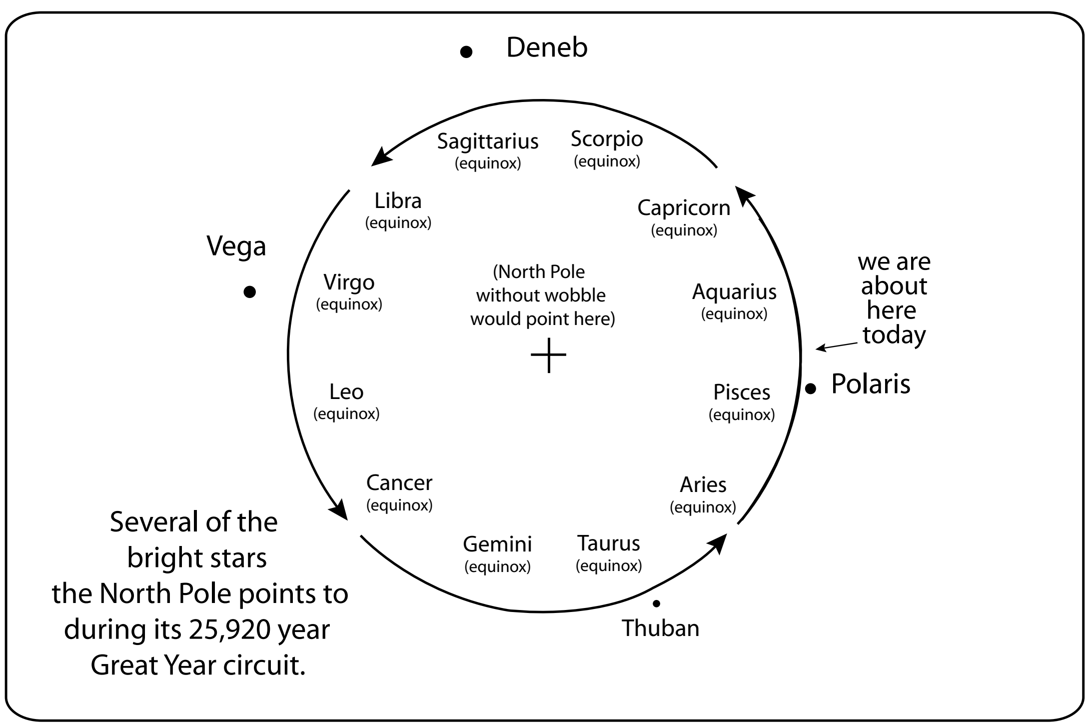


Anus – Annulus – Annus

All this integration with the "Great **Year**" and 25,920 with the illustrations in the Monas make it seem as though that key Latin Word "Anus" probably refers not only to "Anulus," meaning a "ring" (the Gold Ring of Gyges), but also "Annus" the Latin word for "**year**".
Having triple meanings (or more) for the same clue would not be uncharacteristic of Dee.

The Great Year and the North Pole.

The effect of the "Earth's "wobble" is that an imaginary extended axis of the North Pole "points" to different parts of the sky in a slowly moving, 25,290 year circle.
Currently the North Pole points to the star **Polaris**.
In 3000 BC, when the Spring sunrise was in Taurus, the North Pole pointed at **Thuban**.
In 10,000 AD, the North Pole will point close to the very bright star **Deneb**.



Polaris, at the tip end of the Little Dipper's handle is sometimes difficult to locate.

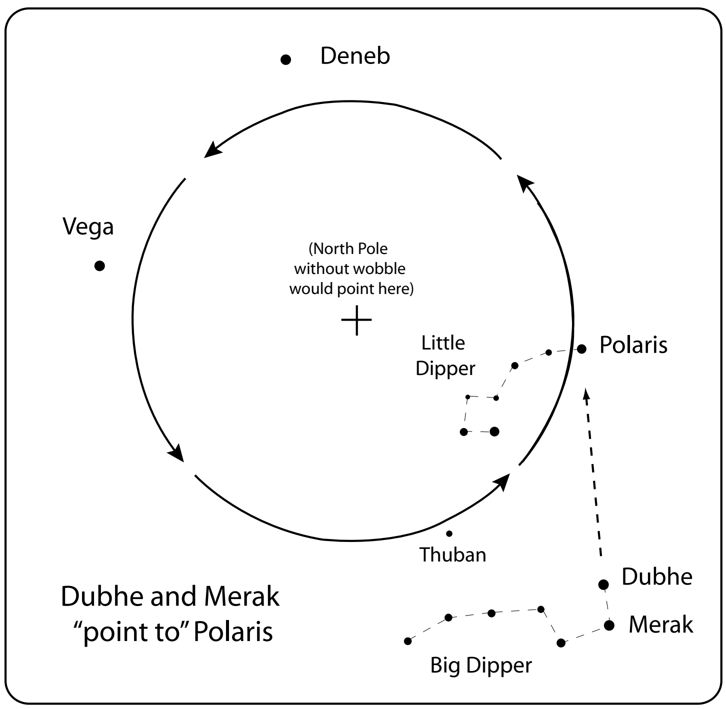
Over the centuries, astronomers and navigators have used
the very obvious **Big Dipper** as a guide to locating Polaris.

The handle of the Big Dipper is actually the tail of a large constellation
called Ursa Major, The Great Bear (as it resembles a walking bear.)

The two brightest stars in the Big Dipper are the two on the outer edge of the Dipper's cup.

At the bottom corner of this cup is the bright star Merak.
Above it, on the upper lip of the cup, is the very bright start **Dubhe** (rhymes with tubby).
Dubhe is als prominent because it's the only orange colored star
out of the 7 main stars that comprise the Big Dipper.

And as every good Boy Scout or Girl Scout knows, an imaginary
line from Merak through Dubhe points very closely to Polaris.



The bright star "Duhbe" and the astronomy of the Tower at Newport.

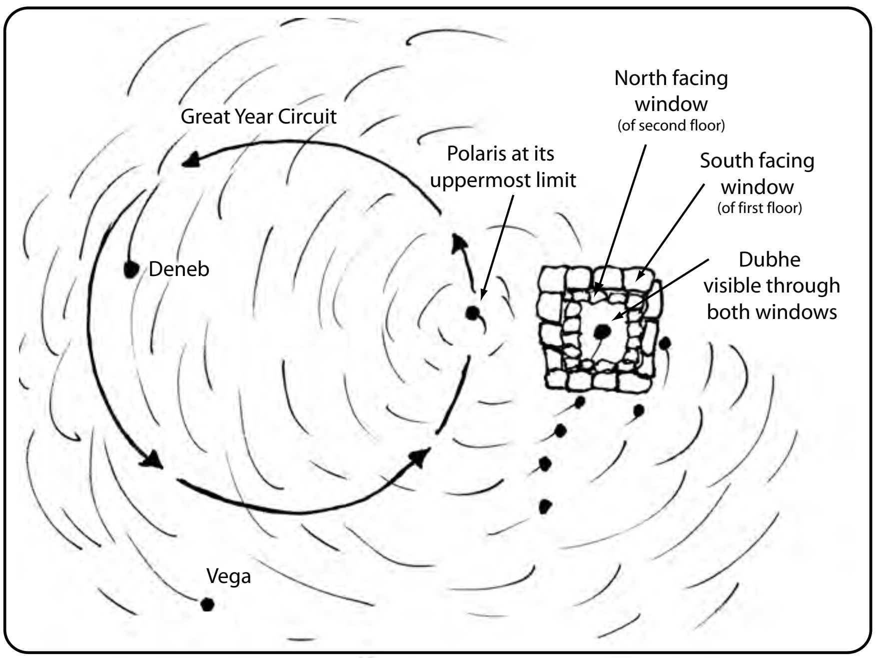
My reason for explaining this "North Pole pointing"
is that it corresponds with an astronomical alignment
that astronomer Bill Penhallow observed at the Tower in Newport.
First, it should be noted that although we call Polaris the "pole star," it's not *exactly* north.
You can see from my illustrations that it's just off to the side of the circle of precession.

Nowdays, Polaris is about 1 degree away from the north celestial pole.
So if you took a long time exposure photo of this area of the northern sky,
Polaris will not be exactly at the center of the concentric circular star trails.

Polaris itself makes a small loop around the exact center.

One way astronomers refine the position of this "moving star"
is to only consider its "upper culmination."
As we look up at the sky, this is the "uppermost" point
of Polaris' small circuit around true north.

Well, here's what Penhallow discovered:
When Polaris is at its upper culmination, the bright star Dubhe
would be visible through two of the windows of the Tower,
namely, the southern window (in the first floor room)
and the northern window (in the second floor room).
(This line of sight must requires a hole in the wooden second floor, which I shall discuss later.)



But the interesting thing is that Penhallow calculated that such a sighting
would occur only during the period from 1200 AD to 1600 AD.

After that, the sky would have shifted and Dubhe would
not make an appearance through the 2 windows.

This Dubhe alignment is not visible today
nor would it have been visible to the
early Rhode Island colonial settlers in the mid-1600's.)
(William Penhallow, *The Newport Tower from Arnold to Zeno,* p. 38-9)

So among other things, the Tower is aligned with a very visible clue
relating to the Precession of the Equinoxes, the long, slow "Third movement."

The idea of making two windows align with "bright Dubhe" would be just the type of feature
that John Dee would have wanted to include in his cosmically aligned, harmonious Tower.

In his *Preface to Euclid*, in the Art of Architecture, Dee writes:

"Likewise, by Perspective, The Lights of the heavens,
are well-led in the buildings, **from certain quarters of the world**…
As for Astronomy, theArchitect must know East, West, South, and North,
and the design of the heavens, the Equinox, the Solstice, and the course of the stars,
Anyone who lacks knowledge of these matters will be unable to understand the Art of Horology."
(Dee, *Preface to Euclid*, p. diij verso, emphasis mine)

This "Northerly alignment" it is similar
to other alignments of the "Lights of the Heavens" in the Tower.

For example, the "Winter Solstice alignment" (through the South window and the West window)
and the "Lunar Minor alignment" (through the Northeast window and the West window).

Remember, Dee was an expert horologist and loved to study
how **time** related to the movement of the Earth, Sun, Moon and Stars.
He wrote an extensive treatise on Calendar Reform for Elizabeth in 1583.

He refers to **time** in several places in the Monas:
The category "Tempus" in the Artificial Quarternary chart, The Horizon of Time,
the word HORARUM (one of the Extra-Large Letter words),
and the "center" word in his "Third Letter" to John Gwynn is the word "tyme."

Is time a line or a circle?

Growing up, I though of time as linear.
Birth, youth, school, work, retire, death was a linear thing.
The rows on my calendar formed a long row of days.
The 50's, 60's, 70's, 80's, 90's, 00's and now the 10's, seemed linear.
In school, I studied the "time lines" of historical events.

But many of the ancient scholars saw time differently.
They saw it as circular – as a series of cycles that fit into larger cycles,
which fit into even larger cycles...

We determine **time** from the movement of the Earth
with respect to the Sun, Moon, and Stars.
The "3 movements" of Earth,
rotation, revolution, and
Procession of the Equinoxes
are all **circles**.
Time is wheels within wheels.

One way to get a sense of the **circularity of time**
is to stare at the second hand of your watch
for a couple of minutes.

Stare at your watch long enough
and Days and Nights can be
seen as **circles** as well.

A year can be seen as a **circle** of
"Jan, Feb, Mar, ... months" or Zodiacal months.

The Great Year and the Hindu Yugas are even larger **circles.**

Time is curcular.

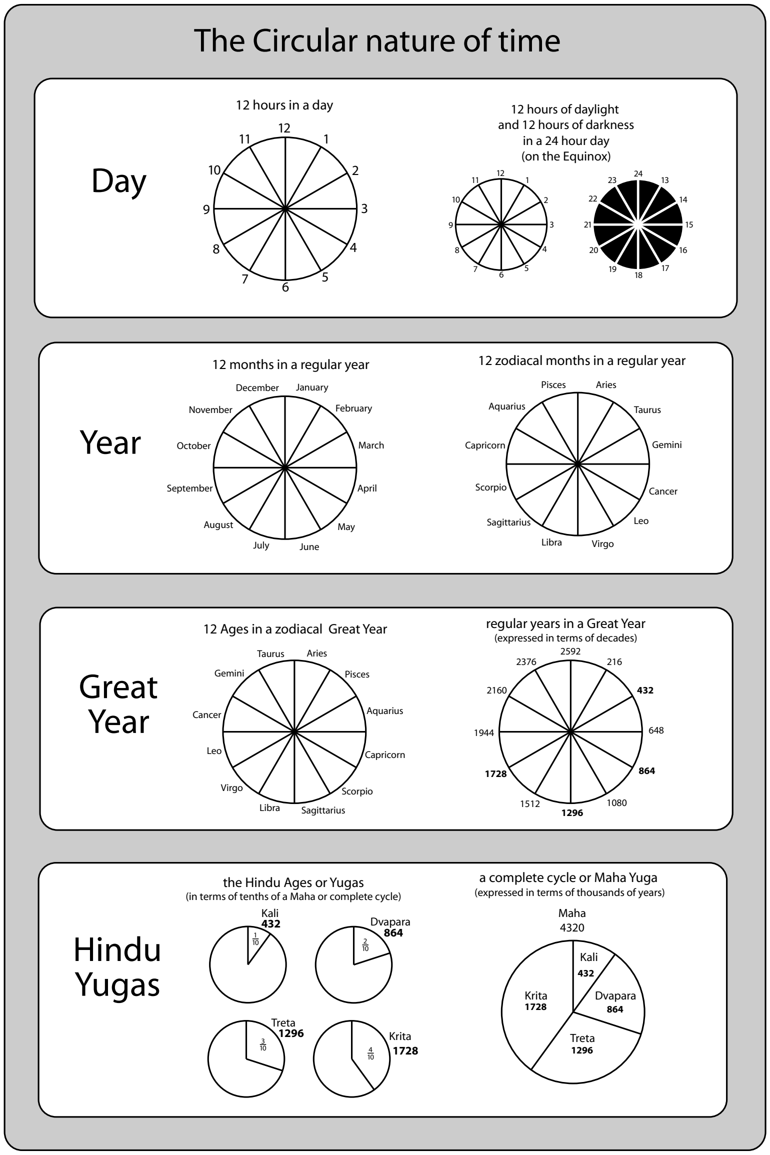
The great Art Critic Thomas McEvilley discusses "circular time" in his
The Shape of Ancient Thought: Comparative Studies in Greek and Indian Philosophies.
He spent 30 years (from 1970-2000) doing research for his insightful book.

"This circular view of time is part of the shape of ancient thought and one of the major differences between ancient and modern attitudes. Most Indian and Greek philosophers taught such a view. In the early stages of the Greek tradition, versions of that are found in the works of or attributed to Hesiod, Pythagoras, Anaximander, Anaximenes, Heraclitus, Diogenes of Apollonia, Xenophanes, and Plato–a not to mention later schools like the Stoics. In India, it was the standard view of the Hindu, Buddhist, and Jain philosophers. It was the most widespread, indeed the normal or ordinary, view of time among ancient philosophers in both Greece and India.

Judeo-Christian-Islamic tradition has featured a linear view of time; while the seasonal simplicity of the fertility calendar is echoed in the recurrence of holy days, time over all is conceived as a straight-line segment which began at a certain time (Creation) and will end at a certain time (the Last Judgment). This view of time seems to have originated with Zoroaster and has since come to dominate the three western religions whose view of time grew essentially from Zoroastrian origins: Judaism, Christianity, and Islam. In the secularized West, a linear view still holds–usually as an expression of the idea of ongoing scientific progress–but without clear enunciation of its beginning and end.

Most cultures have held the view that time is better described by a circling or spiraling line in which the repetition of some events is emphasized rather than the difference of others. In Greece, for example, both the Orphics and Empedocles and in publicly is hand in India both the Buddhists and the Jains described time as a revolving wheel."

(McEvilley, p. 69)



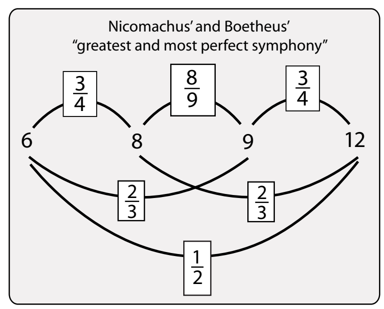
Seeing time as *cycles within cycles* makes the idea that numbers are *cycles within cycles* more reasonable to accept.

Both Consummata and Metamorphosis are comprised of *cycles within cycles*.

Dee saw both time and number as circular.
The circularity can be seen in the numbers hidden in the text
of the *Monas*, indeed, even in the Sun circle of the Monas Symbol,.
Dee also embedded this idea of circularity in the John DeeTower.
Not only is it physically “round,” but its plan incorporates
numbers that express cyclings (like 8 and 12).
But most importantly, by incorporating alignments
of the “Lights of the heavens,” the Tower
becomes an instrument for seeing the
circular nature of
time.

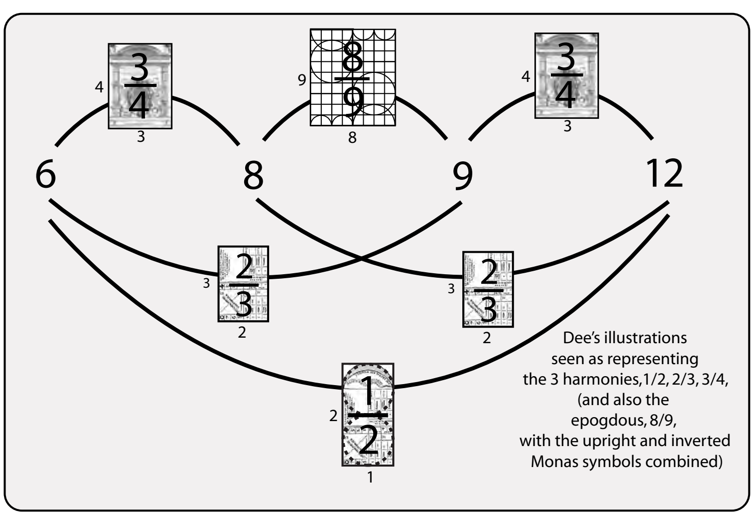
DEE'S 4 ILLUSTRATIONS ARE "SELF-REFERENTIAL"

To review, this is a representation of Nichomachus'and Boetheus' "greatest and most perfect symphony" using our "modern" expression of fractions for the various proportions.



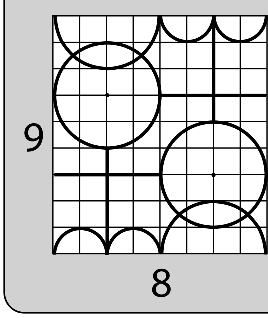
We've seen how 3 of Dee's illustrations are drawn in the various proportions 1/2, 2/3, and 3/4. But the fourth illustration, the Monas symbol, is 4/9, which doesn't quite match up the "epogdous" 8/9.

As Dee discusses the upright and the inverted Monas symbol in his text, I decided it might be acceptable to "pair them up side-by-side" to make the "epogdous" proportion. (as 4/9 +4/9 =8/9)



One clue that this might very well be what Dee had in mind is that the new "combined shape" now has an area of 72 grid squares (9 x 8 = 72).

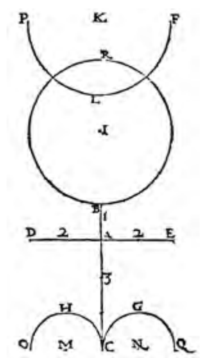
72 grid squares



This is a key number in Dee's cosmology, not simply as a Metamorphosis number, but also as the number he associates with the "SUPERCELESTIAL"–the realm of the 72 Angels.

Being aware of all the correspondences between 36, 72, and 108 and other Metamorphosis numbers, I decided to explore the grid of the Monas symbol in more depth.

In Dee's geometric construction of the Monas symbol in Theorem 23, all of the 17 points
(which he labeled using the letters A through R)
coincide with the lines of a 4-unit wide by 9-unit tall grid.

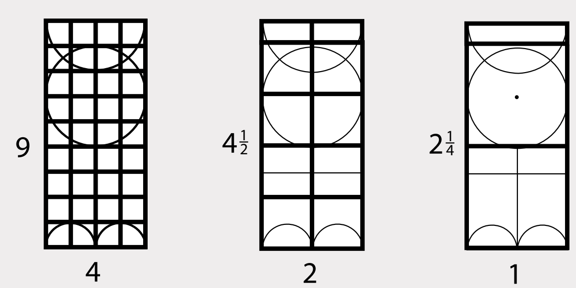


Dee's geometric construction of the Monas symbol from Theorem 23

While finer grids could be used (like a grid 8-units wide by 18-units tall), no larger size grids will fit withou
t leaving a fraction "left over."

For example, we might put on a 2-unit wide grid by 4 1/2-unit tall grid, or even a 1-unit grid by 2 1/4-unit tall grid, but both of these grids involve numbers which are not whole numbers.

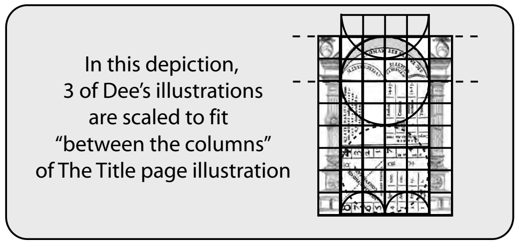
Possible grids for the Monas symbol



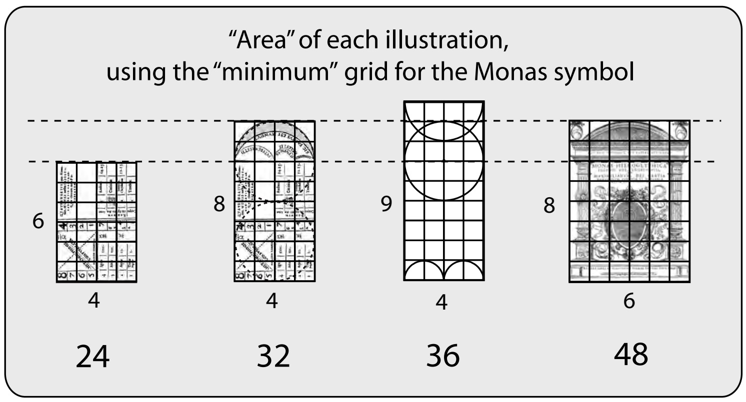
(this one involves whole numbers)

(these both involve a number which is *not* a whole number)

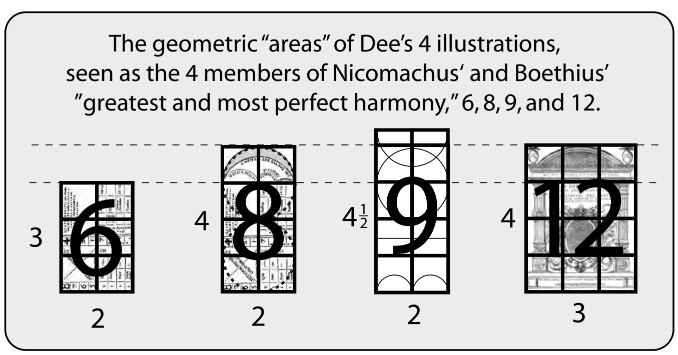
In this analysis, I have scaled 3 of Dee's illustrations so that
they fit "between the columns" of the Title page architecture.



I applied grids to all of the illustrations using the same sized grid I used for the Monas symbol.
Multiplying the height times the width, the "areas" of these illustrations are 24, 32, 36, and 48 (grid squares).



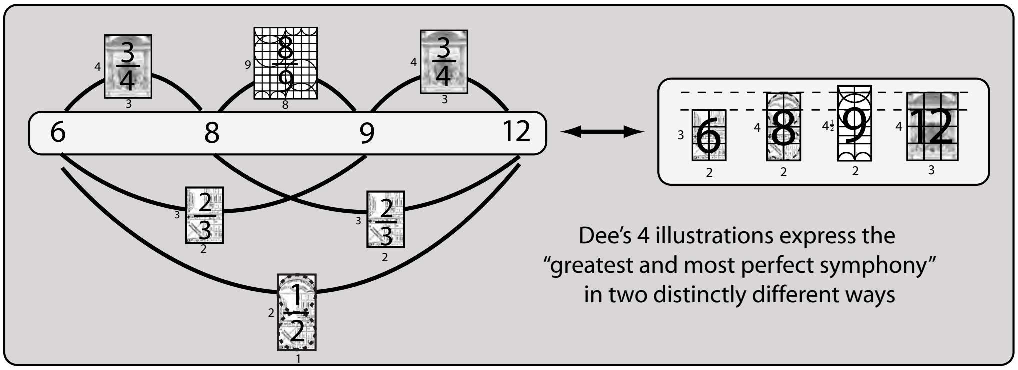
Trying to get to the bottom over what was going on,
I noticed that all these numbers were divisible by 4.
When I "reduced" them all, I was rewarded with ***a pleasant surprise!***



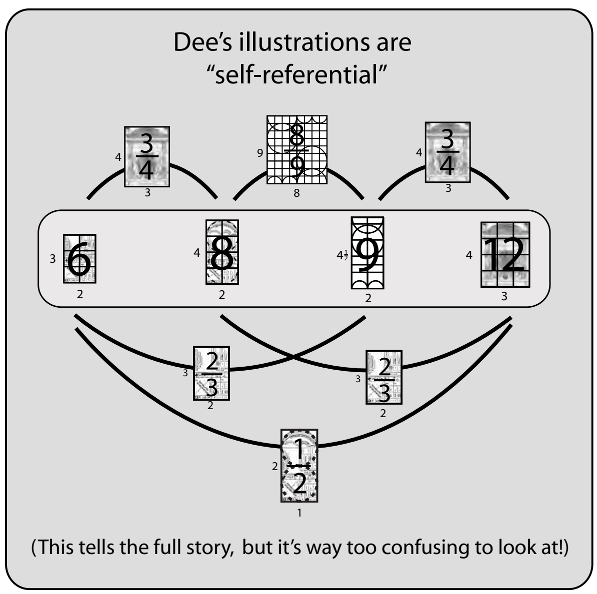
The result was 6, 8, 9, and 12.
The four numbers that Nicomachus and Boetheus praised as
"the greatest and most perfect harmony."
It appeared as though I had come across the core reason
why Dee proportioned the four illustrations the way he did.

What Dee has done here is pretty amazing.
He has used the proportions of his illustrations to express "the greatest and most perfect symphony," 6, 8, 9, and 12 in ***two distinctly different ways***.

One involves the proportions of the sides and
the other involves the proportions of the areas.



We could put both of these distinctly different methods together in one chart,
but personally I find it confusing to look at and hard to read.
(It's much easier to contemplate the two methods on the chart above.)



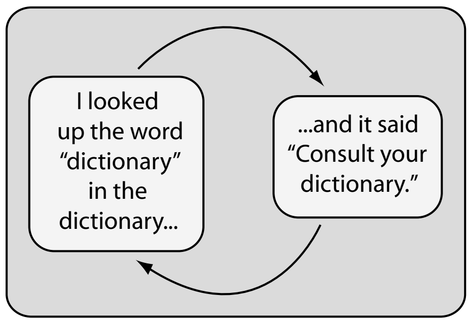
Dee has made his 4 illustrations "self referential."
This is theme in logic in which something "refers to itself."
Dee definitely wouldn't have used that term, but he was
certainly familiar with what the concept of a "paradox."

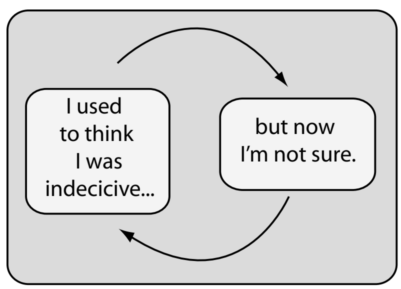
He had even invented a special geometer's measuring tool
called a "Paradoxal Compass" for the Muscovy Company navigators.

When sailors were headed for a destination which was other than that a Great Circle,
the shortest route was often a gently curving line.
The rate of curve is more significant the closer the ship is to the poles.
This was a very important tool for those exploring the Northeast Passage.
As a curved course is not what one might expect,
he called it a "paradoxal" route. (This word later morphed into our more familiar paradoxical.)

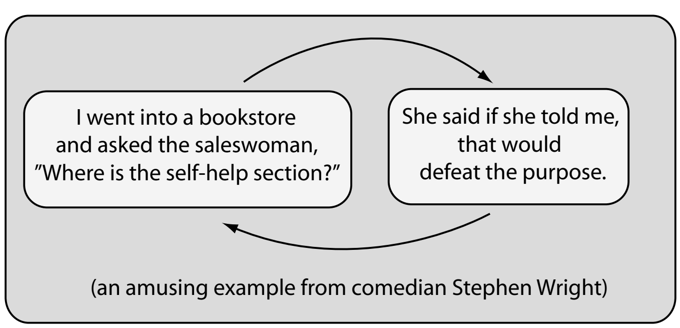
In philosophy, the idea of a paradox goes back to Eubilides of Miletus, (around 350 BC),
who reportedly said,
"**A man says that he is lying. Is what he says true or false**?"

This "Liar's Paradox" and other self-referential statements
have been discussed by many philosophers throughout history.





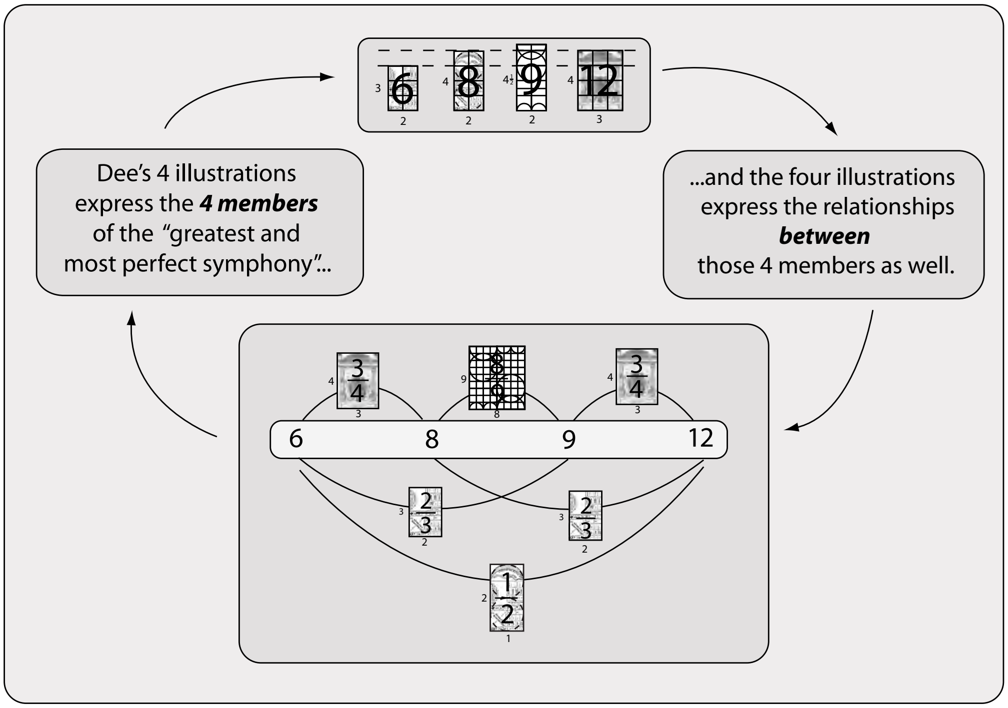
Here are some modern-day examples which I show
with arrows to indicate their "circular form."
The examples are all "self reflexive" (reflex means to bend or fold back)
or to use a more modern term, "self reflective."



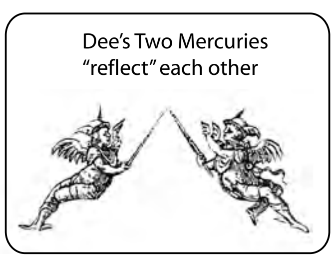
Returning to the four illustrations,let's look Dee's amazing geometrical "self reference" in terms of "Forma Circulata."

The ***4 individual shape proportions*** are
in those same for proportions ***to each other***.

Conversely, the proportions of the 4 shapes ***to each other***
are the ***4 individual shape proportions.***



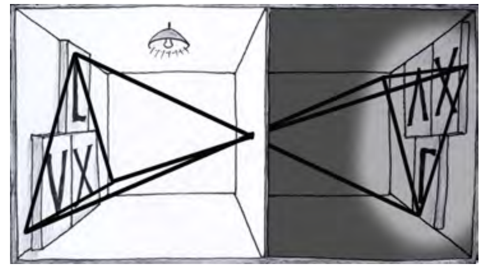
This idea of self-reflection is a main (though cryptic) theme of the Monas.
The Moon is a "reflection" of the Sun (figuratively as well as literally;
a full moon is really just a "full" sun reflection.)
On the Title Page this self-reflection is expressed not only
by the symmetrical design of the architecture and its features,
but also by Dee's depiction of the Roman god Mercury.
It's not one Mercury
It's two identical Mercuries, mirroring each other's gesture.



In the physical world, self-reflection
can be seen as a “mirror.” But an even better
expression of self-reflection is the camera obscura.

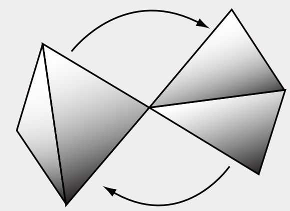
Everything “outside,” every little detail, down to its
color and shading, is “reflected” on the inside wall.

In his advice to Opticians, in his *Letter to Maximillian*,
Dee cryptically describes Alberti’s conceptualization
of how vision works, using the term “*Forma Circulata*.”



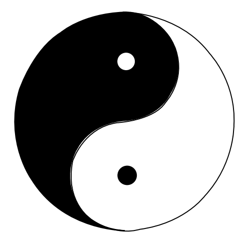
Geometrically, the most “economical” 3-D expression
of “self reflection” or “Forma Circulata”
is 2 tip to tip tetrahedra (the Bulky bowtie) .

In 3-D geometry,
“Two tip to tip tetrahedra”
make the most “economical”
expression of
“self reference”
or “forma Circulata



The ideas of “Forma Circulata”
and “self-reflection” are basically the same thing.

This can be seen in the Ouroboros,
which is devouring its own tail,
or in the two interweaving sides
of the Yin-Yang symbol.



Ouroboros
from
Synosius
by Theodoros Pelecanos
(1478)

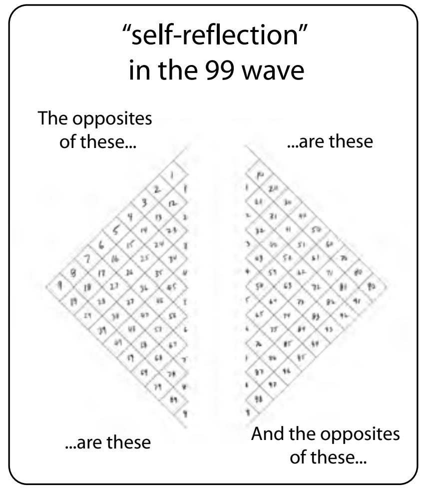
Dee’s discovery (and later, Marshall’s) was that
“self reflection” or “circular form” can be seen in number!

Special finite groupings of numbers are “wheels”
or “circles” or “self-reflections” within themselves.
Furthermore these wheels are part of larger wheels,
which also have perfect “circularity” or “self-reflectiveness.”

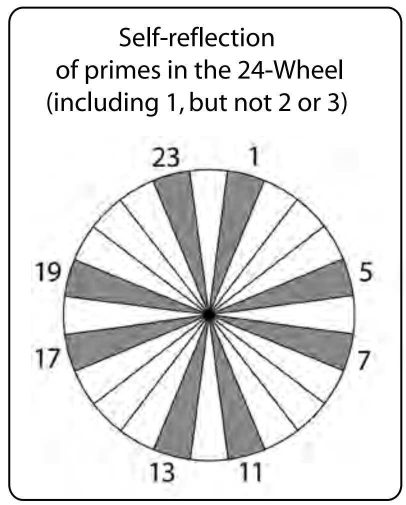
These finite groupings come in two flavors: Consummata and Metamorphosis

(The 9 Wave/11 Wave, 99 Wave, 1089 Wave and the Holotomes 12, 24, 72, 360, 2520…)

In Comsummata, we've seen that the
single and double-digit numbers
show perfect self-reflection.
The "opposites" of all the digits on the
left side can be found on the right side.
And the "opposites" of all the digits on the
right side can be found on the left side.
A mirror.
But also a circle.



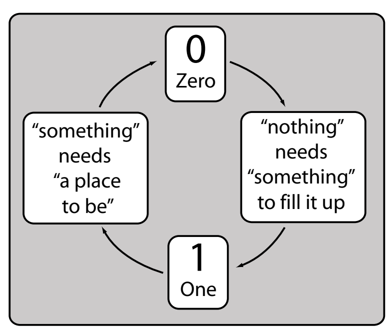
There is self-reflection in Metamorphosis,
for example the 24 Wheel.
(As explained earlier in this demonstration
of what I call the "24-Wheel," the "special numbers" 2 and 3
are not included as primes, however 1 is included)
The shaded areas (primes) are seen as
"reflections" of other primes, but it's
clear that the whole thing is "circular."



Here is a circular depiction of what Marshall
calls of the root of all "retrocity":
the relationship between Zero and One.

The void is but an empty void
unless it has something in it.
What it contains is "one"
(the all, everything, the universe).

At the same time, this "everything" needs a place to be.
The void accommodates it perfectly!
Zero and One are the ultimate expression
of self-reflectiveness or "Forma Circulata."

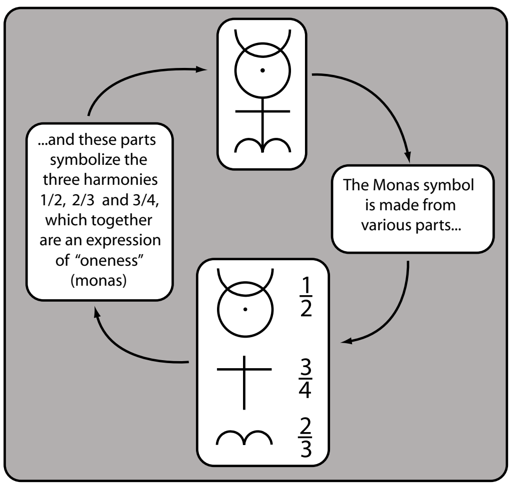


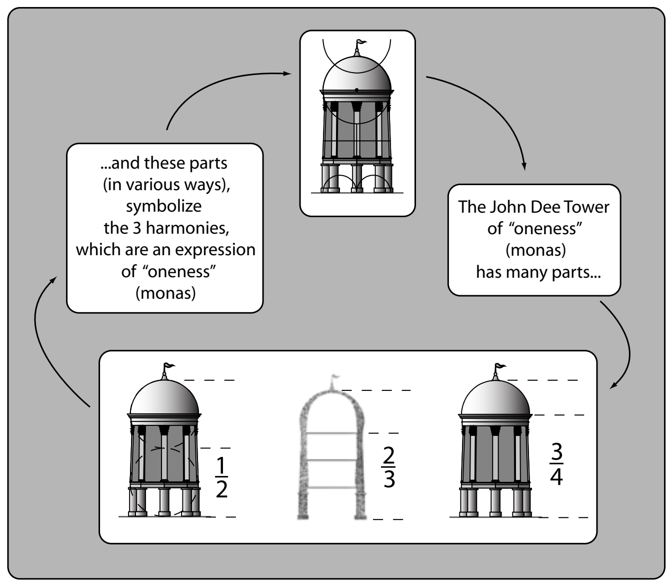
We have also seen how the Monas symbol itself is "self reflective."
As a whole, it is a symbol of "oneness" (monas),
however it is made from several "parts."

The Sun and Moon expressed the 1:2 proportion.
The Cross of Elements expresses the 3:4 proportion.
The Aries symbol expresses the 2:3 proportion.

In Dee's cosmology, (as we have seen in Axiom 18 of the *Propaedeumata Aphoristica*)
these 3 harmonies ***combined*** are also an expression of "oneness."

The parts express to the whole.
As Dee puts it, "nothing else can be added or taken away."
It is perfect and complete.





With all this "self-reflection" and "Forma Circulata"
in mind, let's look at Dee's Tower.
The whole Tower expresses "oneness"– just like the Monas symbol.

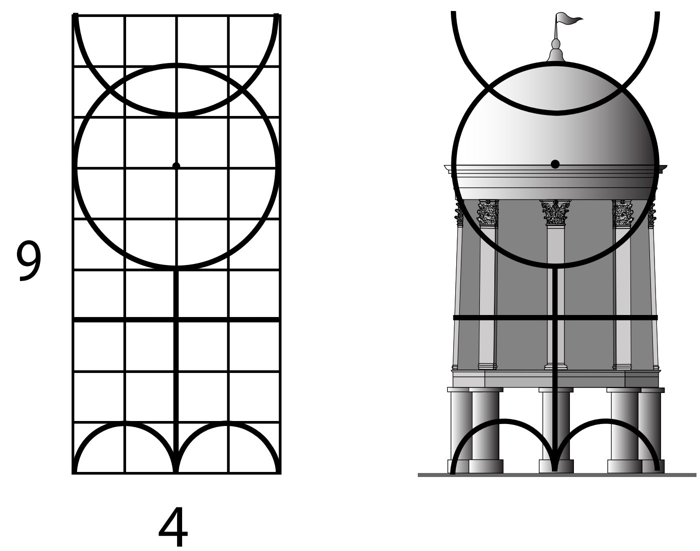
Hidden in the overall plan are two circles, thus expressing the **1:2** proportion.

Internally, the level of dome room floor is at 2/3 of the height of the whole
stone-and-mortar part of the Tower, thus expressing the **2:3** proportion.

Externally, the dome height counts for 1/4 of the Tower,
leaving 3/4 of the Tower below, an expression of **3:4**.
(Another 3/4 expression is the
12-foot-tall pillar section:16-foot-tall dome room.)

The shapes of Dee's 4 illustrations.
The "parts" of the Monas symbol.
The design of the Tower.
They are all singing the same song:
"the greatest and most perfect symphony."

Oh, one last thing.
Add the 6-foot finial to the top, and the 2:1 (height to width) Tower
becomes a 4:9 (height to width) Tower,
thus corresponding perfectly with Monas symbol.



In other words, the Tower and the Monas symbol are the same thing.

They were both designed based the 3 harmonies 1/2, 2/3, 3/4.

And those harmonies are based on the simple story of

1, 2, 3, 4.

The Maxim in Theorem 10 is a mathematical equation

In Theorem 10, after introducing the Aries symbol,
Dee illustrates the 4 parts of the Monas symbol, then
delivers an perplexing "maxim" involving all 4 parts.

The word "maxim" is an appropriate word to describe Dee's declaration.
Nowadays the word maxim means "a short, pithy statement that expresses
a general truth." It derives from the Latin phrase *maxima propositio*
meaning "the largest or most important proposition."

Dee emphasizes this maxim by writing it in ALL CAPITOL LETTERS,
including some letters that are super –duper CAPITALIZED.
He also emphasizes it by writing a whole sentence just to introduce it.

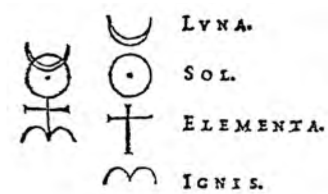
(The idea that "maxim" can be found in "Maximilian" no doubt occurred to Dee, but he didn't go there, nor will I.)

THEOREM 10

The (Sharp, Pointed) symbol of the Zodiacal Division of Aries, used by Astronomers; is quite well known to everyone.

It is also well  known that this is the place in the heavens where the Fiery Triplicity Begins. Thus, we shall add the Astronomical sign of the Aries (in the Practice of this MONAD) to signify that the aid of fire is required.

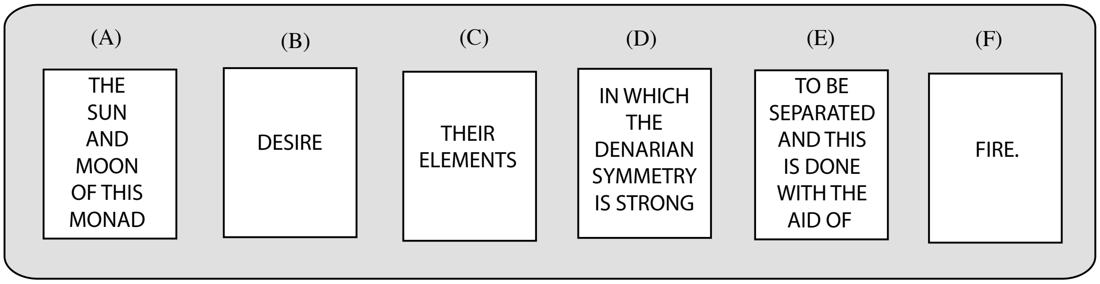
We can summarize this hieroglyphical consideration of our Monad in our hierglyphical statement:



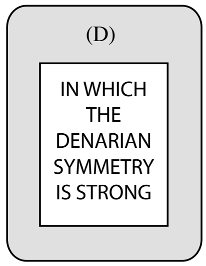
THE SUN AND MOON OF THIS MONAD DESIRE THEIR ELEMENTS, IN WHICH THE DENARIAN SYMMETRY IS STRONG, TO BE SEPARATED, AND THIS IS DONE WITH THE AID OF FIRE.

(Dee uses the Latin word MINISTERIO, which I have translated here as"aid,"
in the sense of "to serve, to promote or to further."
In Latin, a "minister" is "a servant, an attendant, or accomplice."
These words derive from the Latin word "minus,"meaning "less.")

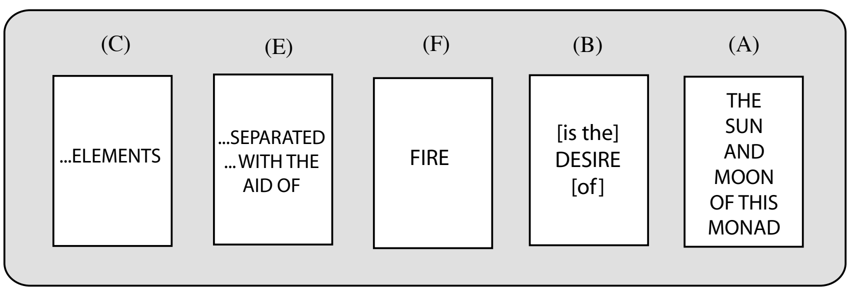
To make sense of what he is saying, let's break his maxim into six sections (labled A through F), and then perform a little "separatio and conjunctio" on them.



To help make sense of this maxim, allow me to temporarily remove this descriptive clause...

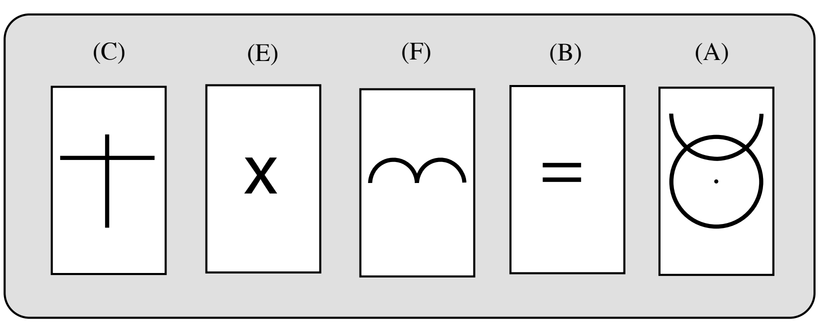


...and then shuffle the other pieces around a bit to make this new sentence, which is still in keeping with Dee' original sentence:



Let's look at "SEPARATED WITH THE AID OF" as if it is an expression of "MULTIPLICATION."

And "DESIRE"as an expression of "EQUALS."



Next, let's simply replace those parts with their respective numerical "harmonies."
Suddenly what sounded like grand, cosmological, philosophical statement
turns out to be a simple Fifth Grade mathematics equation!

(C)	(E)	(F)	(B)	(A)
$\frac{3}{4}$	x	$\frac{2}{3}$	=	$\frac{1}{2}$

But, to Dee, this was much more than a simple equation.

To Dee it demonstrated the elegant interrelatedness
of the 3 "greatest and most perfect harmonies."

To Dee it expressed the inrerrelationship of the
"primary, secondary, and tertiary productions"
of the self-refectiveness of zero-one that is inherent
in the "four separate Great Wombs of the Larger World," (1, 2, 3, 4).

And its not ***just*** this equation.
Remember, this equation
can be expressed in ***12 different ways***.

(A graphic exposition of these 12 equations was shown in an earlier chapter.)

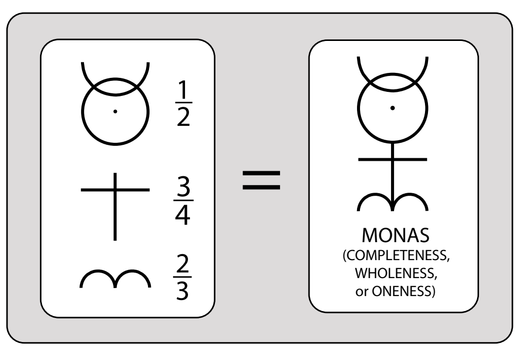
12 variations
of the interesting
interrelationship
of the 3 harmonies

$\frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$	$\frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$
$\frac{2}{1} \times \frac{2}{3} = \frac{4}{3}$	$\frac{2}{3} \times \frac{2}{1} = \frac{4}{3}$
$\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$	$\frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$
$\frac{3}{2} \times \frac{4}{3} = \frac{2}{1}$	$\frac{4}{3} \times \frac{3}{2} = \frac{2}{1}$
$\frac{3}{4} \times \frac{2}{1} = \frac{3}{2}$	$\frac{2}{1} \times \frac{3}{4} = \frac{3}{2}$
$\frac{4}{3} \times \frac{1}{2} = \frac{2}{3}$	$\frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$

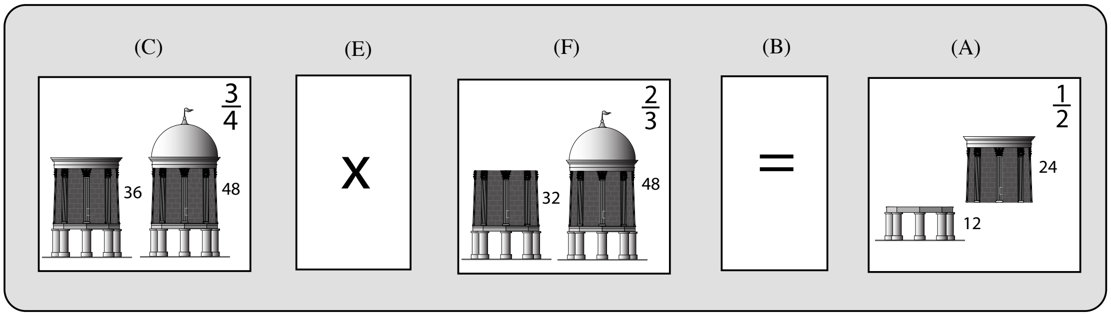
Dee's maxim in Theorem 10 refers to this equation

Here's what Dee is getting at:

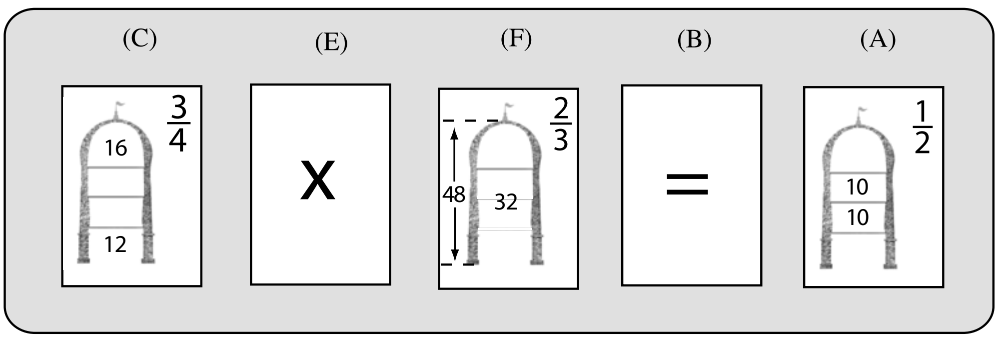
All the parts of the Monas symbol
are intrinsically interrelated, and
when all 3 are "grouped together,"
they make the completeness,wholeness,
or oneness (MONAS) of the full symbol.



As Dee imbued his Tower design with the mathematical concepts of the Monas Hieroglyphica, we might write the equation using the Tower's expressions of the 3 harmonies. For example, here are some expressions of 1/2, 2/3, and 3/4 found on the ***exterior*** design of the Tower.



Or, here are some of those 3 harmonies found in the ***interior*** design of the Tower.



The maxim in Theorem 10, the Monas symbol, the John Dee Tower
and Axiom 18 of Dee's *Propaedeumata Aphoristica*
are all ***singing the same mathematical song***.

Next, let's return to that section of the maxim which I previously removed.
I felt comfortable removing it because it is what grammarians
call a "relative clause," also known as a "which clause."

As Bruce Ross Larson explains in his book *Stunning Sentences,*
"set off by commas, the *which* clause can be left out
without disrupting the meaning of the main clause."
(Bruce Ross Larson, *Stunning Sentences*, N.Y., Norton, 1999)

(D)

IN WHICH THE DENARIAN SYMMETRY IS STRONG

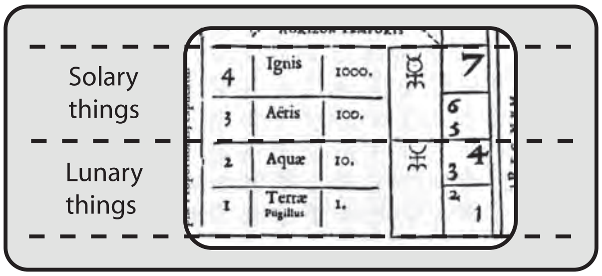
It’s actually a little unclear what this “*which clause*” refers to.
It seems to refer to the “Elements.”

But, by by using the possesive pronoun “their” in the term “their Elements,”
Dee is suggesting that the “Elements” belong to the “Sun and Moon.”

A quick glance at the “Thus the World Was Created” chart clarifies Dee’s intent.

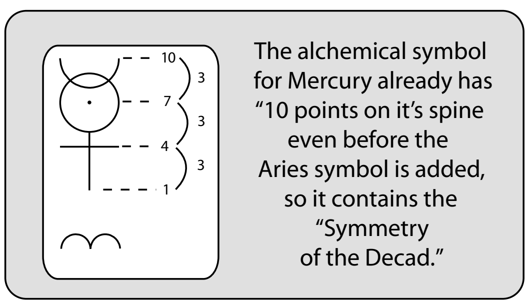
The top two elements listed, Fire and Air, are Solary things.
The bottom two, Water and Earth, are Lunary things.

So, in this sense the Elements ***do*** “belong” to the Sun and the Moon.

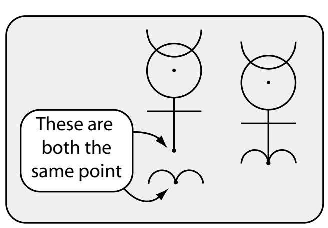


But, in another sense they are separate,
as Dee capitalizes the word “Elements” within extra large E.
Also, in his illustration of the parts of the Monas symbol,
the Cross of the Elements is quite separate from the Sun and Moon.

Dee seems to be suggesting that the “Denarian Symmetry”
can be seen when the “Sun, Moon and Elements” are combined,
even before the Aries symbol comes into the picture.



Indeed, the full height of the Monas symbol’s spine is already in place,
exhibiting the Symmetry of the Decad.
When the Aries symbol is added, the spine still remains the same length.
(The centerpoint of the Aries symbol coincides with the bottom-most point of the Cross of the Elements symbol.)



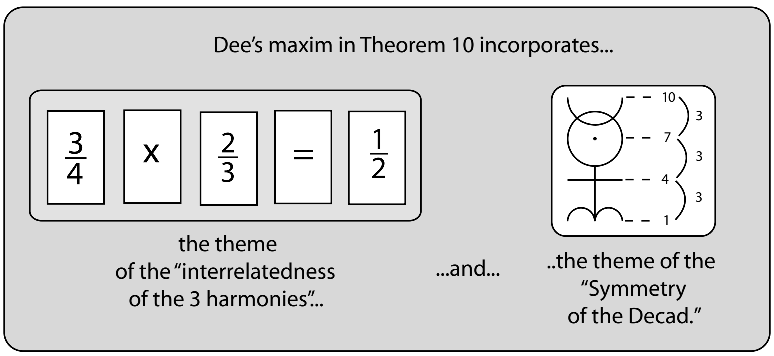
If the Monas symbol as seen as an artificial "little man" or
homunculus, the best description of this point might be "Anus."
(Dee's cryptic word next to the "Engraved 2")

In his *Letter to Maximilian*, Dee describes the Aries symbol:

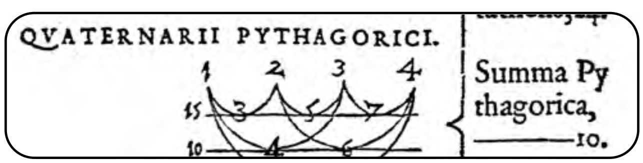
"**Unique… Hieroglyphic Character" as "MERCURY… (fortified by a sharp point)**."

Dee's word for sharp is "acumine" which is the point of a spear,
the barb of thorn, or the stinger of a bee.

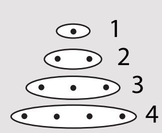
To summarize, Dee includes two important mathematical concepts in his maxim.
He **seems** to be obfuscating (clouding) his intent by intermingling
these two seemingly unrelated ideas in the same sentence.
But not really.



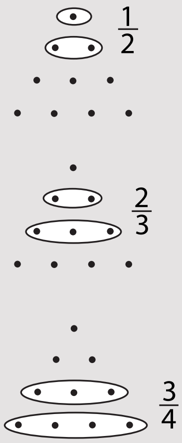
Dee actually saw these two ideas as ***very interrelated***,
as can be seen by contemplating the "additive result"
of his Pythagorean Quaternary of Theorem 23.



Let's look at the wise Pythagoras' graphic depiction of this "Quaternary", his beloved tetraktys.
In it, we only can we easily find the "four great Wombs of the Larger World," (1, 2, 3, and 4),

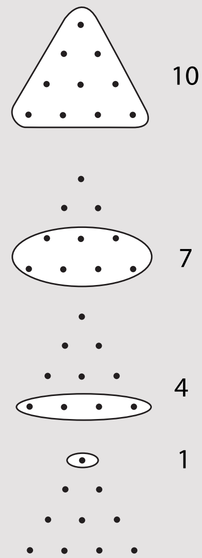


The tetraktys as "the 4 great Wombs of the Larger World", 1, 2, 3, and 4



The tetraktys as the 3 harmonies

But its also very easy to find
also the 3 harmonies (1/2, 2/3, and 3/4),



The tetraktys as the key members of the "Symmetry of the Decad"

And it's also easy to find the key members
of that Symmetry of the Decad!

Over 2 millenia ago, Pythagoras recognized that 10 was integrally related to "1, 2, 3, and 4."

Dee saw this as well,as he tells us in Theorem 23,

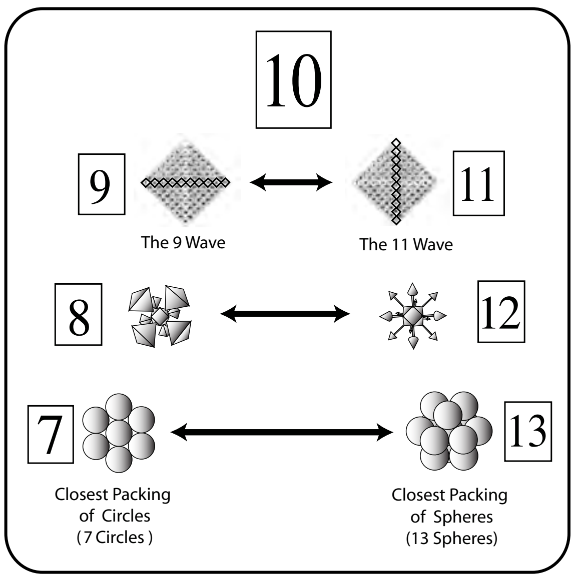
"**So we give here one Reason, above all others**
(which, together with this whole new art, we divulge from the first time)
why the QUATERNARY, as well as the DENARY impose,
for the common good, certain limits in Numeration."

Remember, Ten isn't just a great number simply
because we use it in our Base Ten numbering system.
Dee clarifies this (very cryptically) in his "Third Letter to John Gwynn."

Ten plays a role as the important pivot-point between 9 and 11 (the 9 Wave and the 11 Wave).

Ten is the pivot between 8 (the octave of Consummata)
and 12 (the "docena" of Metamorphosis),
(both key numbers in the cuboctahedron).

Ten is the pivot between 7 (closest-packing-of-circles)
and 13 (closest-packing-of-spheres).



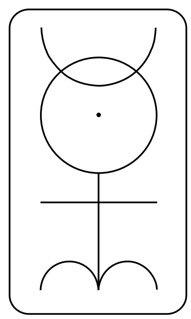
One confirming clue that Dee had all this in mind is that he
chose to put this ***maxima propositio*** in **Theorem 10**, or we might say:

To conclude, remember how Dee glorifies the number Ten
in his advice to Arithmeticians, in his *Letter to Maximillian*:

"Will he not be filled with the greatest admiration by this most Subtle yet General Evaluating Rule: that the strength and intrinsic VALUE of the ONE THING, purported by others to be Chaos, is primarily explained (beyond any Arithmetical doubt) by the number 10.

(Dee, *Monas*, p.5 verso)

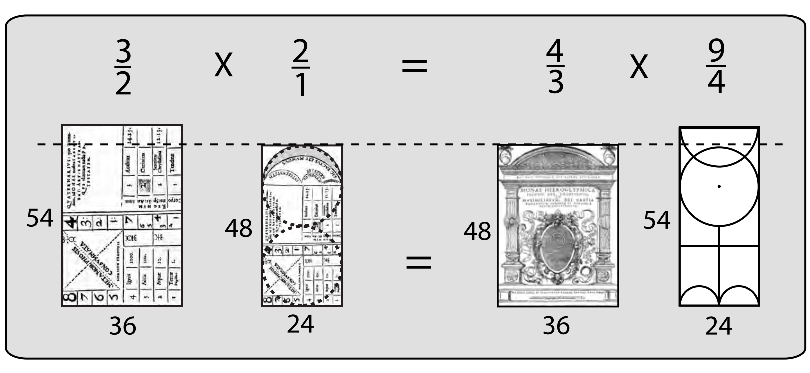
The John Dee Tower is the Monas symbol



Upon first realizing that John Dee designed the Tower,
I suspected that his design was based upon his beloved Monas symbol.

My feelings were reinforced as I began to understand
the mathematical meaning of the *Monas Hieroglyphica*.

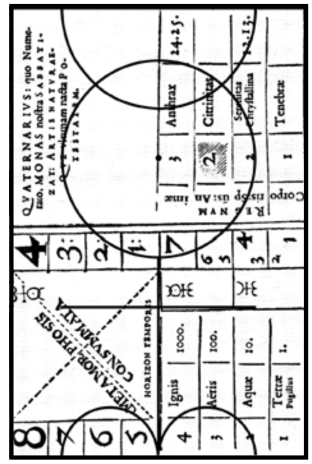
I became more convinced once the interrelationships
of the proportions of the four illustrations became clear.



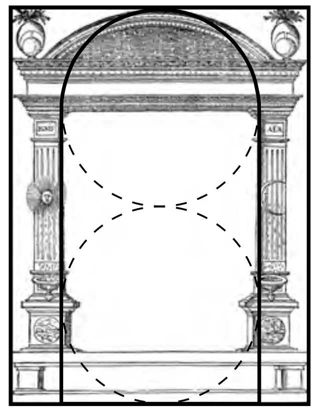
In this “visual equation,”
two of the shapes are relatively tall (54 units)
and two are relatively short (48 units).

The superimposition of the two taller shapes
involves the Monas symbol.

We'll eventually return to the Monas symbol,
but first let's examine the shorter pair.
(Which includes the Title page
with it's interesting architecture.)



This pairing (of the shorter shapes) includes
the "extended Creation chart,"
which is essentially just the shape of the Monas
symbol without the horns of the Moon.

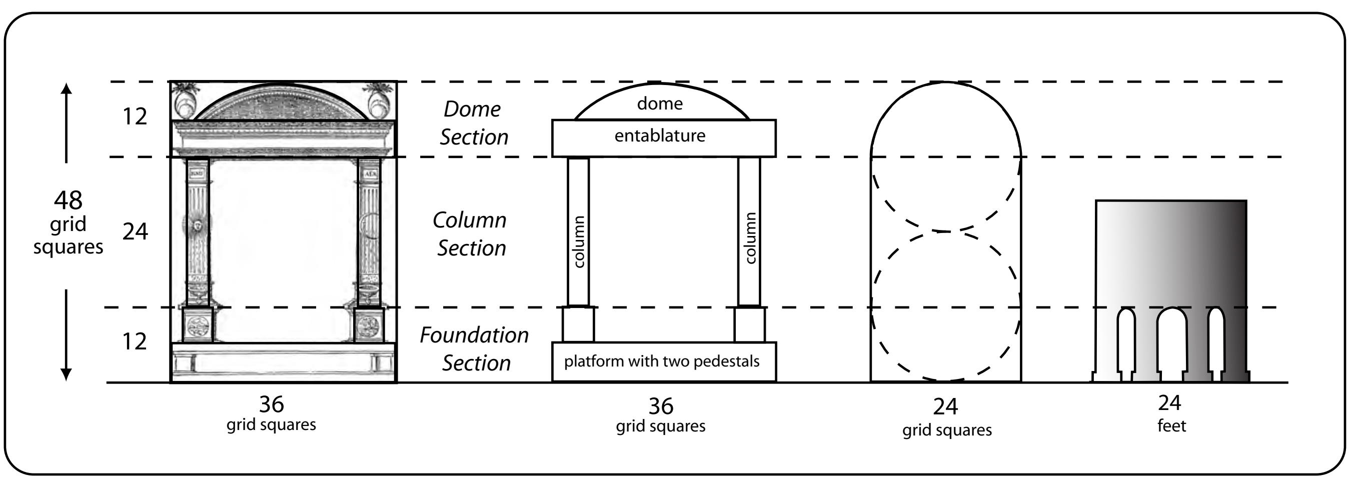


I've divided the architecture of the Title page into three sections.

The "**Foundation Section**", which includes the platform (6 units tall)
and the pedestals (6 units tall), is 12 units tall in total.

Next the "**Column Section**" is 24 units tall.

And at the top, the "**Dome Section**" includes an entablature (6 units tall)
and the Dome (6 units tall), making it 12 units tall.

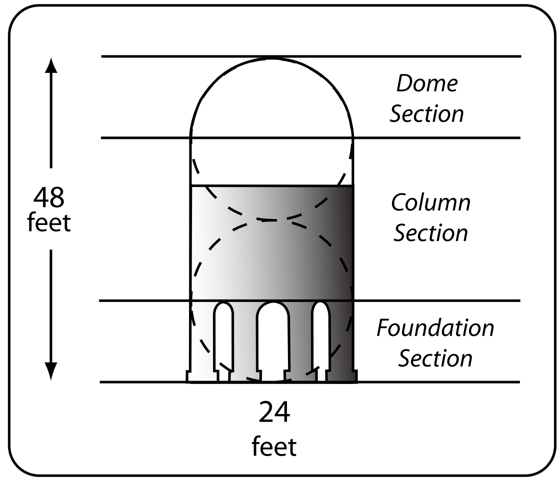


How do these three sections relate to the Tower as it exists today?

First of all, I'm basing my analysis on the premise that Dee used the simplest design scale possible:

1 grid square of the 48 x 36 grid square Title page
equals
1 foot of the Tower's height or width.

The "**Foundation Section**" extends approximately up to the tops of the arches of the Tower.



The exterior width of the tower is "on average" 24 feet.

I use the term "on average" for several reasons.
The diameter of the cylindrical part of the Tower,
where it sits on top of the pillars, is indeed 24 feet.
But the Tower "tapers inward" as it goes upwards.
(This is discussed in depth in another chapter)

Because, the eight pillars jut outwards with respect to the upper cylinder,
So the foundation diameter (measured from the outer edges of two opposing pillars)
is more than 24 feet (actually about 24 1/2 feet).

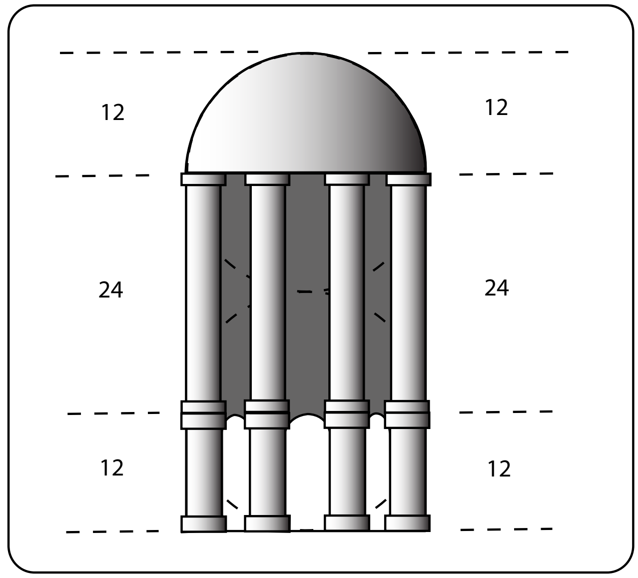
For this analysis of the "big picture" of the Tower design
(and knowing how much Dee loved the numbers 12 and 24)
let's assume that 24 feet was intended.

Also note that the eight pillars are each shown with drums at theie bottoms.
About 6 inches of these drums is exposed above ground today,
but archaeologists have found that they were originally about 1 1/2 feet tall.

The tower has not sunk.
Over the centuries, every time Touro Park was redesigned,
new dirt was hauled in to help with the plantings.
About a foot of soil has accumulated.

Let's start with a hypothetical design.
Here, I've drawn a 12-foot "Foundation Section"
upon which rests a 24-foot "Column Section,"
upon which rests a 12-foot "Dome Section."

(Only four of the eight pillars are visible from this viewpoint.
Also the width between the pillars seems irregular,
but is as how they look to the eye from this viewpoint.)



Obviously ther could not be full columns where I have indicated,
as even today the stone and mortar cylinder of the Tower occupies that area.

Instead, I envision that Dee used **pilasters**, flattened columns
that project from a facade by about one quarter of their width.

The Greeks and Romans used pilasters and
Leon Battista Alberti reintroduced them in Renaissance times.
From a distance, they give the appearance of a strong supporting
column, but they are actually just ornamental, not structural elements.

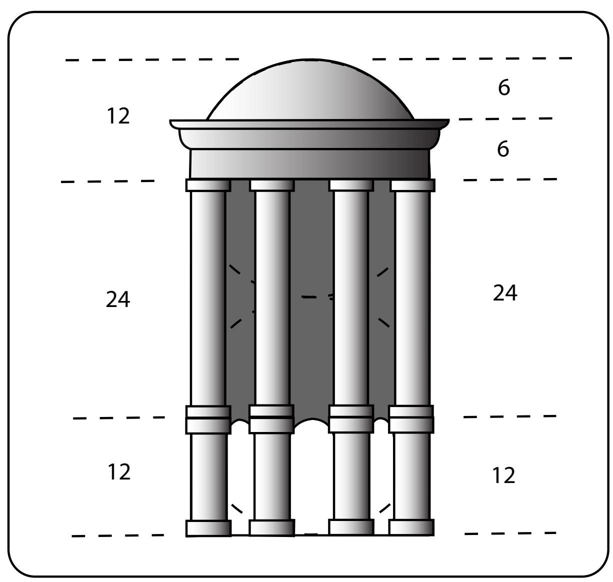
Theres a major problem with this simple rendition.
In classical architecture, columns are generally surmounted
by a horizontal lintel called an "entablature,"
especially if they are to hold a large element like a dome.
(This term entablature is based on the word "table,"
which is a horizontal plank resting on legs which are like columns.)

So, let's put the 6-unit entablature from
Dee's Title page on top of the columns.

Note that I've kept the height the same
(6 units), but reduced the width by a third
(from 36 units on the Title page
to 24 units on the Tower).

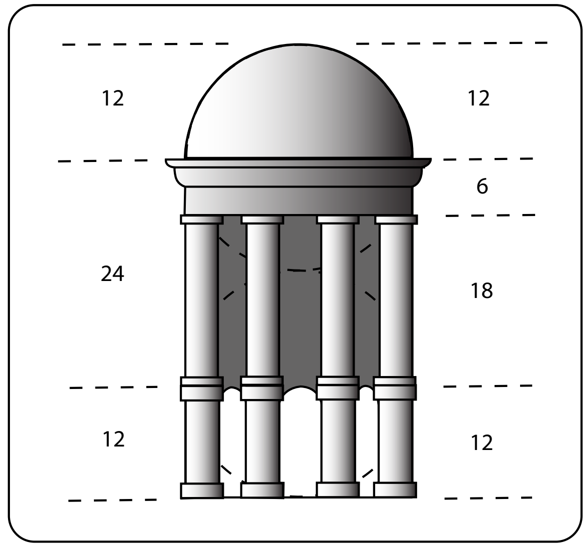
The "Dome Section" here somewhat
resembles the dome of the Title Page,
but on this narrower width, it doesn't
look very aesthetically pleasing.

The entablature seems way too bulky and
the dome looks like a kid's beanie cap.

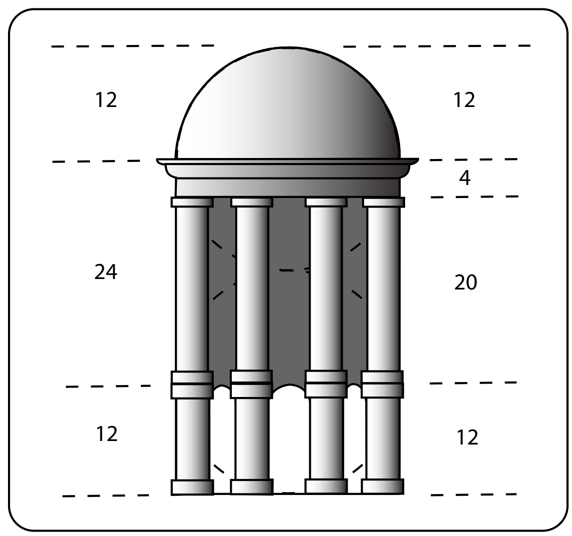


So let’s try lowering the 6-unit entablature down into the Column Section.

Now the dome looks much more majestic, but the new columns seem too short, making the entablature seen even more massive.



As we previously reduced the width of the entablature of the Title page by a third (from 36 units to 24 units), reduce its height by a third as well (from 6 units to 4 units).



Now this “feels” to be an appropriate size for the dome! But as this enablature is now in the Column Section, the columns are now only 20 feet tall.

But there is still another “architectural” problem remaining.

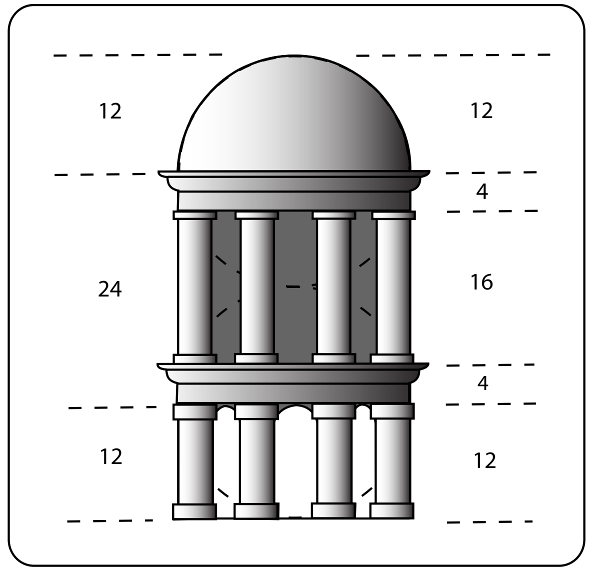
Classical columns don’t generally sit directly on top of pillars. Below the columns there should be another entablature (which spans the pillars).

So let's "duplicate" the top enablature
and place it on top of the pillars.

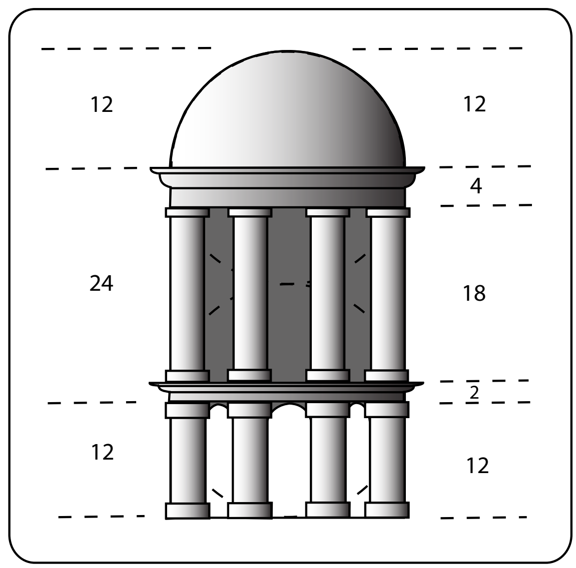
This leads to a symmetrical design
(the numbers on the right are 12, 4, 16, 4, 12),
but the columns have gotten even shorter
(16 feet tall).

An important characteristic of in entablatures
is that their height is dependent upon the height of
the supports beneath them.

It's not likely that Dee would put
a 4-foot-tall entablature on ***both***
the 12-foot-tall pillars
and a 16-foot-tall columns.



Reducing the height of the pillar entablature
from 4 feet to 2 feet is a step
in the right direction, but the
"pillar entablature: pillar" proportion
(2:12, which is 1:6)
is ***not*** equal to the
"column entablature: column" proportion
(4:18, which is 2:9).



But the more significant problem with this design is that it
doesn't correspond with what remains of the tower today.

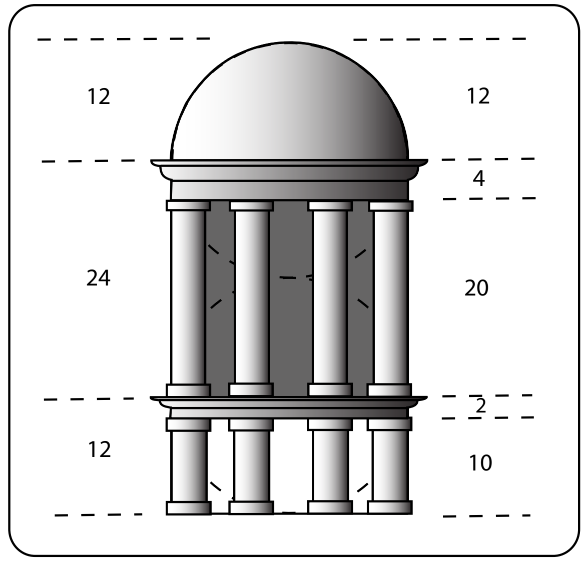
About 10 feet above the original ground level are slate ledges,
small projections that clearly define the tops of the pillars.

This problem can be easily solved by simply moving the (2 foot-tall) pillar entablature downwards by 2 feet (from the Column Section down into the Foundation Section).

Now everything seems to click!

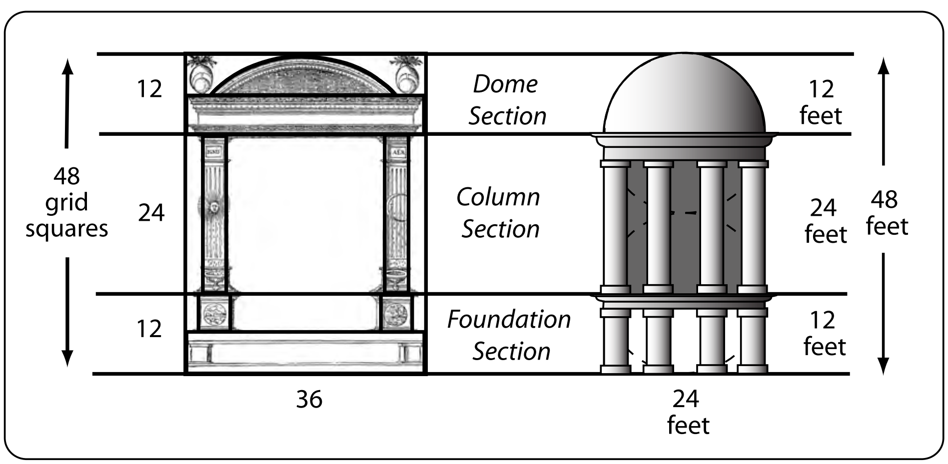
The "pillar entablature: pillar" proportion (2:10, which is 1:5) is now exactly equal to the "column entablature : column" proportion (4:20, which is also 1:5)

Another, even more revealing feature of this design is that the two entablatures are in the same proportion (2:4) as the "pillars : columns" (10:20). They are each in the important 1:2 proportion.



The preceding analysis is about the "big picture" of Dee's design. The columns were probably more slender than the thick pillars. Also, I have only shown generic-looking capitals and bases. The actual capitals and bases were, no doubt, much more ornate.

The important thing to see is how this Tower design relates to the two circles (which I've kept in the background as dotted lines). This design solution is chock full of Dee's much lauded numbers 12 and 24. The diameter of each circle (24) is the height of the Column Section (24). The radius of each circle (12) is the height of both the Foundation Section (12) and the Dome Section (12).



This side-by-side comparison shows how the Foundation, Column, and Dome sections of the Title page are found in the Tower design.

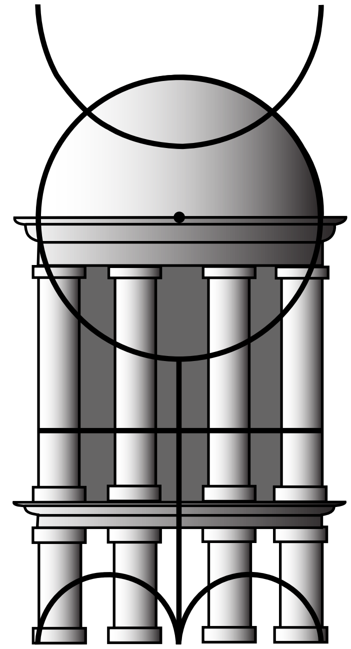
When the Monas symbol is superimposed on this
Tower design, “horns”of the Moon project
above the height if theTower.

This situation is resolved by simply adding
a finial or a small spire on top of the dome.

Vitruvius called for a small finial on domed circular
temples and most classical domes seem to have them.

Finials are the visual “frosting on the cake,”
the “dotting of an i,”
the culminating gesture to the heavens.

A dome without a finial is but a silo.



It’s hard to say what this top finial exactly looked like,

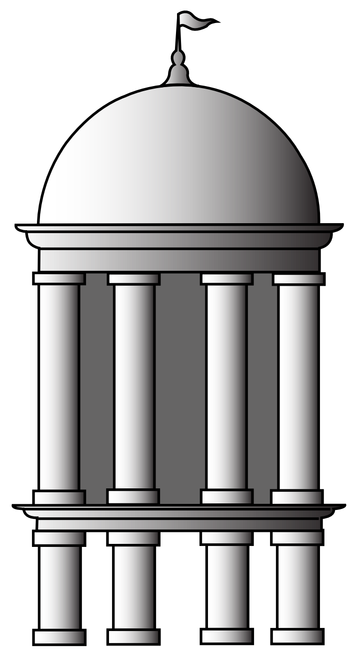
It could have been a thick short spire or even a thin flagpole.

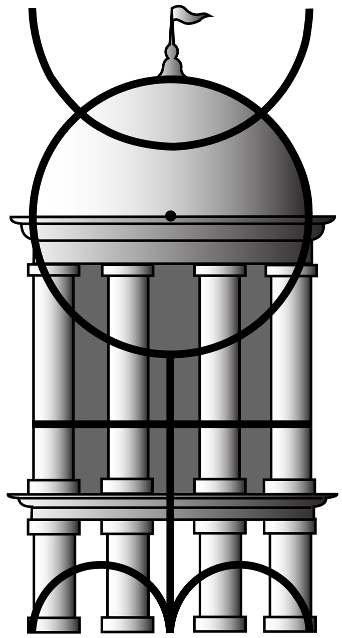
But more likely it had an ornate base
and rose to a “sharp, stable point.”

It could have held a flag or pennant
honoring blessed Queen Elizabeth.

Or possibly a weathervane
(like the Tower of the Winds in Athens),

Whatever the finial looked like it,
would have undoubtedly been 6 feet tall,
the difference between 54 feet and 48 feet.





The fact that the finial's relatively thin profile is so different
from the rest of the solid stone-and-mortar structure
accentuates its role as an expression of the "epogdous"
(which literally means "containing a whole and 1/8).

The whole, in this case, is the 48-foot-tall stone-and-mortar structure
and that extra "eighth" is the 6-foot-tall finial.

To summarize, with but a few minor changes,
it's easy to see how the architectural details of the Title Page,
combined with the proportions of some of the illustrations,
can morph into a harmonious looking building.

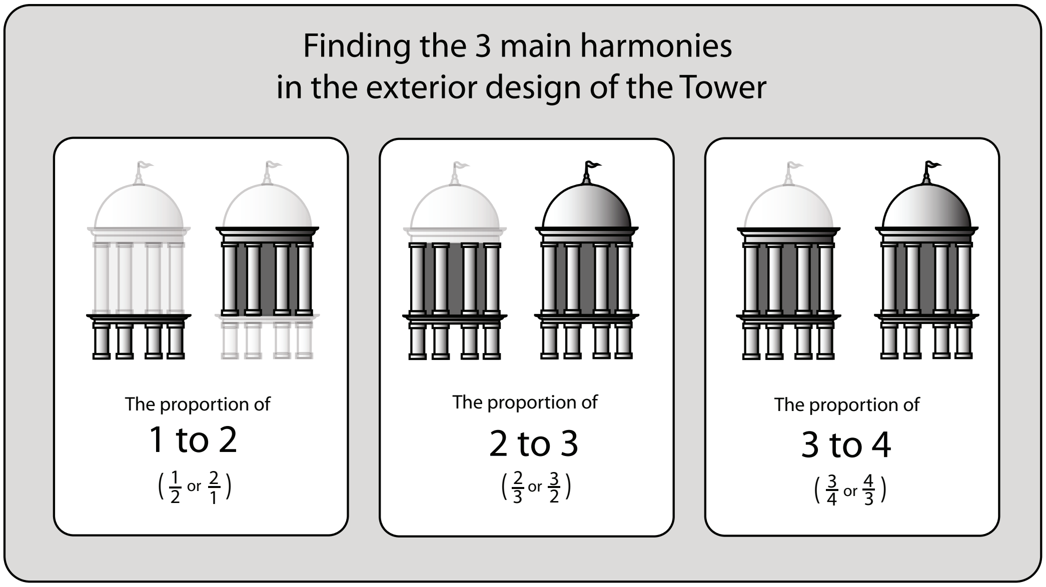
Looking at the "28-foot-tall" stone-and-mortar structure
that stands in Touro Park today,
it's hard to envision that it was once 48 feet tall.

It's also hard to envision the eight pilasters, the two entablatures, and the dome.

But trust me, if the original tower was a man in a tuxedo,
today we're only seeing him in his underwear,
and we're only seeing his lower half.

The Tower sings a harmonious song.

The 3 main harmonies (1/2, 2/3, 3/4) can be seen
by comparing various parts of the Tower design.



The Foundation section : Column section
is in a 1:2 proportion
(as is the Dome section : Column section)

The Foundation plus only the columns : the whole stone-and-mortar structure
is the 2:3 proportion.

The Foundation section plus the whole Column section : the whole structure
is in the 3:4 proportion.

The tower visually sing a song of maximissmo harmonisio!

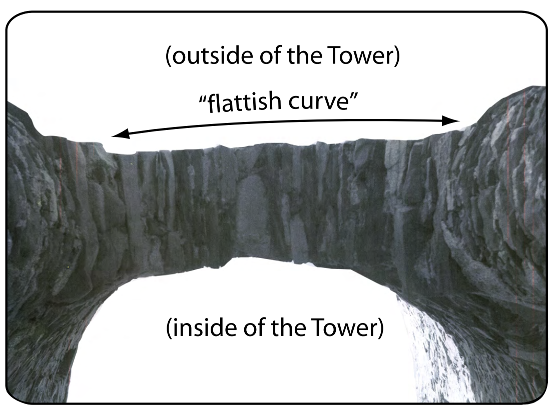
The Tower Tapers

Is the exterior diameter of the Tower actually 24 feet?

The answer that question is ultimately "yes."
But there are many places that it is definitely **not** 24 feet in diameter.
What's going on?

The eight pillars form an octagon, but the upper part of the tower is a round cylinder.
The architectural challenge becomes:
How do you make an octagon morph into a circle?
The answer is: gradually.
Let's start at the eight arches, which rest on the octagon of eight pillars.

John Howland Rowe, in his 1884 *Rowe Report* points out that
"**making an arch on the circle, as at Newport, it is necessary to turn the arch on two radii, the one vertical (that of the arch itself) and the other horizontal (that of the circular building)**."
(Means, p.9)



The "vertical turn" gives the arch its strength.
But the "horizontal turn" is tricky.
If it bows out too much, the integrity of the arch will be jeopardized.

The design solution was to make a slight bow,
resulting in what Rowe calls "**a flattish curve**."

This photo, taken from the ground looking straight up into an arch,
shows that "flattish curve" on the exterior of the building.

The pillar entablature was a 24-foot diameter octagon

The width of the Tower (when measured from the outside of any pillar to the outside of the pillar which is opposite it) is on average, 24 feet 6 inches. Thus, a precise 24 foot diameter would be 3 inches "inwards" on each side.

This bird's-eye-view cross-section demonstrates simple way that the 24-foot diameter might be accommodated.

The dark circle, (which is below everything), is one of the 3-foot diameter pillars.

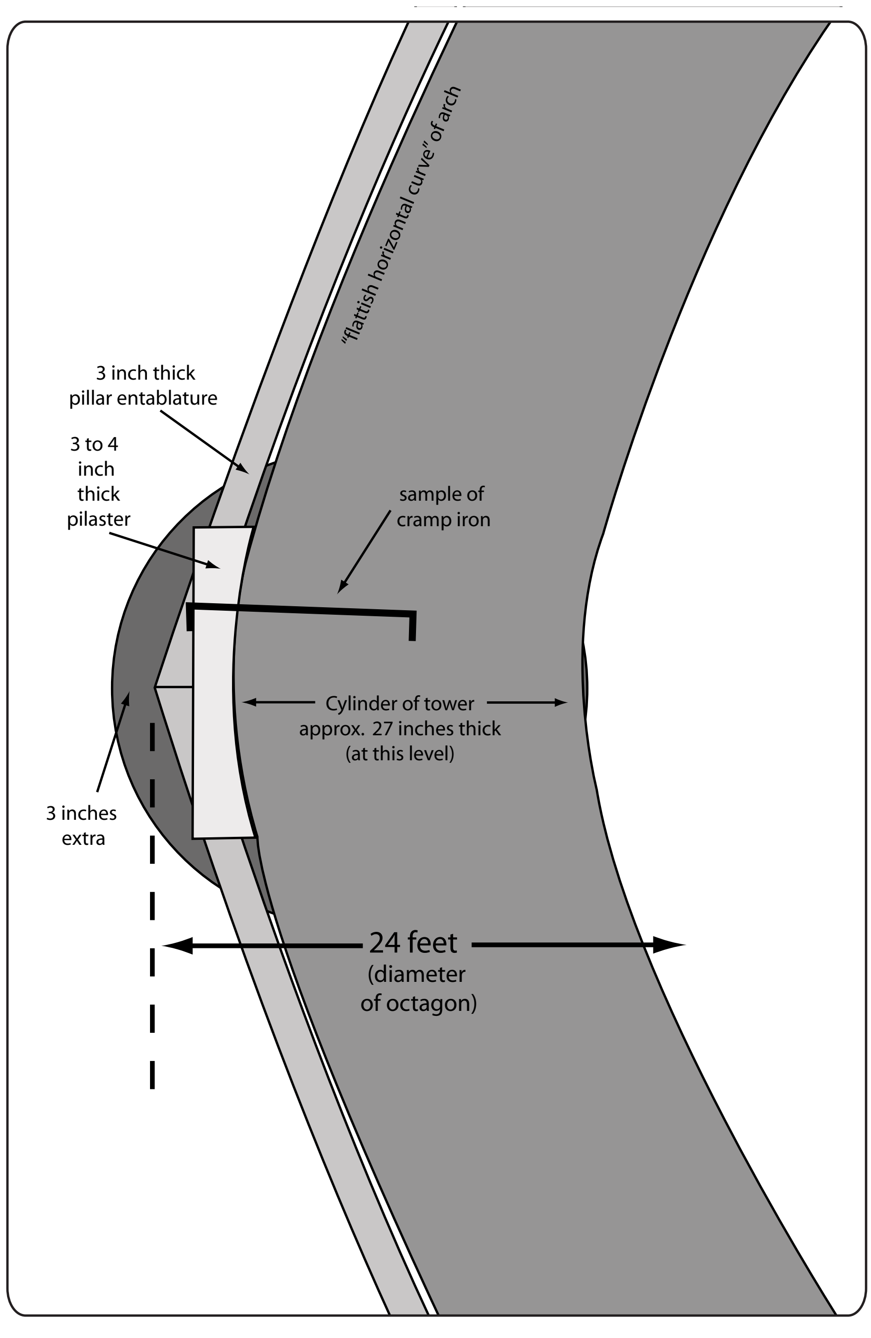
The large curved section is the thickness of the tower wall. You can see the approximately 8-9 inches of the slate shelf on top of the pillars.

(But this drawing does not include the additional 2-3 inches of that "shelf" that juts out over the edge of the pillar because I measured the 24 1/2 feet from pillar exterior to pillar exterior.)

I envision that the pillar entablature was made from eight pieces of solid lumber each 2 feet tall by 9'2" by 3 inches thick.

Wrapped around the tower, their ends would meet on top of each pillar making a 24 feet diameter octagon whose perimeter was 73' 6."

(Perhaps they were slightly bowed to accommodate the "flattish, horizontal curve" of the arch")



Resting on top of the entablature is the cross-section of a pilaster (shown in light gray). The back face of the 20-foot tall x 2-foot wide pilaster,which is 3 or 4 inches thick, might have been gouged out with an adze to accommodate the curvature of the cylinder. They could easily be held on with “cramp irons”, metal bars that had been pre-installed at appriopriate places in the masonry as the Tower was being built. Once the protruding cramp irons were finally bent over, the pilaster would fit the cylinder snugly, with very little of its weight actually being supported by the pillar entablature beneath it.

The Case of the Tapering Tower

After the Tower has morphed from an octagonal plan into a circular plan there is yet another morphing. The tower tapers. As a tower rises (after approximately the height of the west window) it tapers inwards; in other words, its diameter gradually decreases.

The tapering is so gradual it seems like it might simply be caused by parallax, the “visual illusion” of tapering. When you photograph a building who sides you know are parallel and those sides aren’t parallel to the sides of the frame in the viewfinder, that’s parallax. When the same building is seen without looking through the viewfinder, the parallax isn’t as evident because our eye/mind knows that the sides are straight and parallel.

A fellow NEARA member, Steve Volukas, showed me a way to clearly see the taper of the tower: by comparing it to something that *is* vertical. In Touro Park, just to the south of the Tower is a statue of Reverend Channing which rests on a large concrete base. From a low position, one can visually align the edge of the concrete with the Tower in the background and the taper of the Tower is much more clearly evident.

To measure this taper I rented to 40-foot extension ladders and bought long surveyor’s measuring tape. After getting permission from the Newport Parks Department, my photography assistant John Tavares and I wrapped the measuring tape around the Tower at various heights.

The first measurement was done 15 feet above the original ground level. This is just above the “approximately 16 sided” area, at about the level of the sill of the West window. This, the widest part of the cylinder, measured 74 feet in circumference. This means it is 23’ 6”in diameter (or 6 inches less than a 24-foot diameter).

This chart shows four other circumference measurements made at different heights. The top measurement (71’4”) translates into a 22’8” diameter.

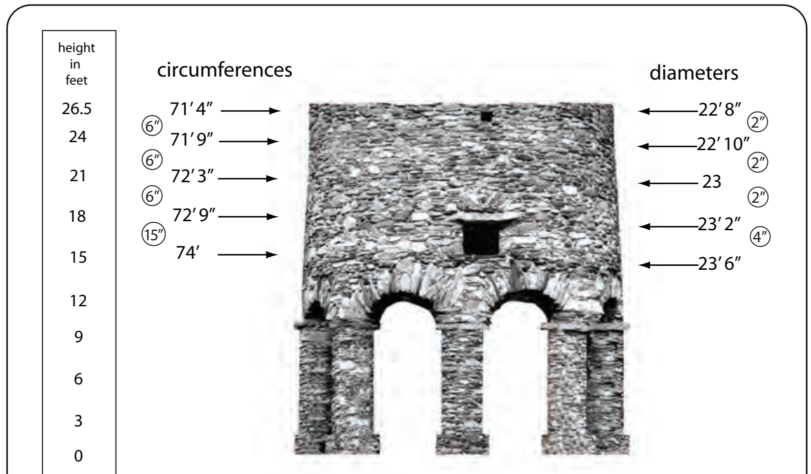
This means that the diameter of the tower has diminished by 10 inches when compared to the first measurement.

And this means that the cylinder of the Tower tapers inward by 5 inches on each side (over a span of about 12 feet of height). This is significant enough to be seen by the eye.

The numbers that I have encircled here show the “rate of tapering.”

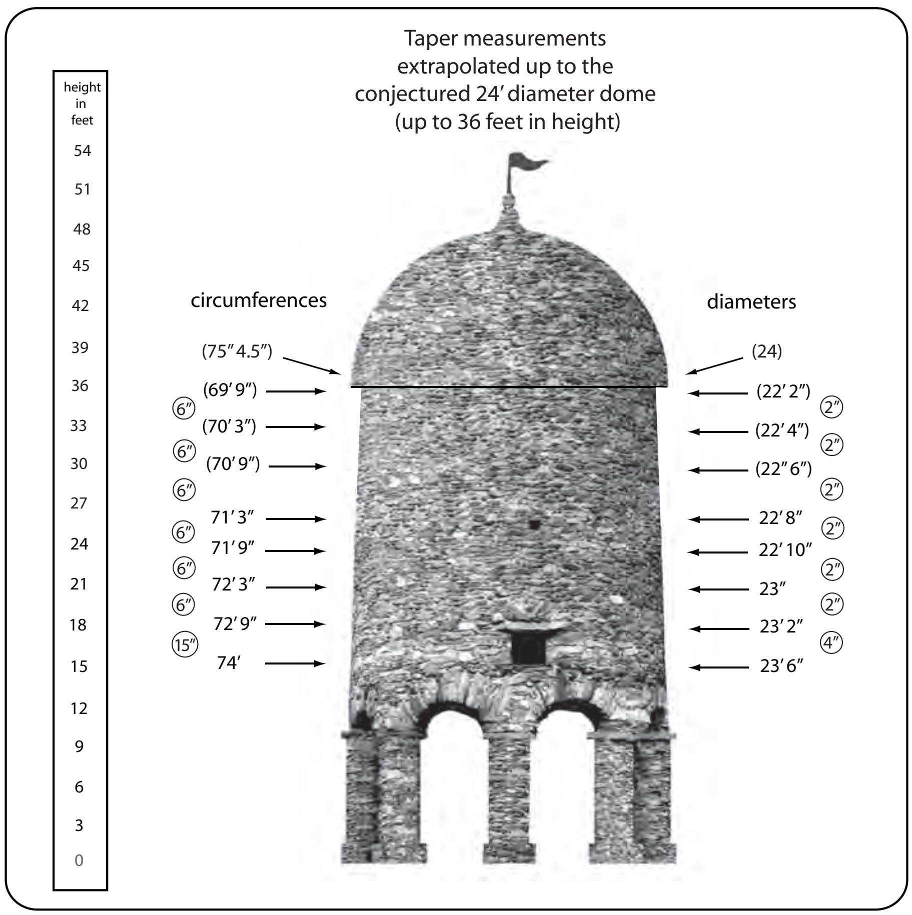
The taper is greatest at the lowest measured level of the cylinder, where it tapers 4 inches (over a span of 3 feet of height).

Upward from there, it tapers at a rate of about 2 inches for every 3 feet of height.



Tower as it exists today

Assuming that the taper continued at the same rate, I extrapolated more measurements up to the 36 feet of height where I assert there was a 24-foot diameter dome.



One might think that the dome should also be 22' 2"
so that it sits nicely on top of the tapering tower.
But knowing Dee, and the *Monas*, I'll guarantee you that there was no way
he would **not** have a 24-foot diameter dome (with its 12 foot radius).

A 22' 2" diameter dome (with its 11' 1"radius) would throw off
of the harmonious proportion of the whole Monas-symbol-proportioned tower.
Dee would not have deviated from the overall plan of "two circles, one on top of the other."

OK then Jim, then how do you explain the 20-inch discrepancy? (24 feet minus 22' 2")
That means that the dome overhangs both sides of the tower by 10 inches!
Did the heavy stone-and-mortar dome simply **float** there atop the Tower?

One idea is that the pilaster entablature was very substantial and
stuck out at least 10 inches to help support the overhang.
But this is not plausable because the pilaster entablature was not structural.
It was only attached to the Tower with clamp irons.

An entablature like this which is 4 feet tall would be hard to attach if it
was 2 or 3 or 4 inches thick, never mind having the top part of it
being 10 inches thick, and supporting a weighty dome.
This is not a reasonable solution, but it does bring up an important clue.

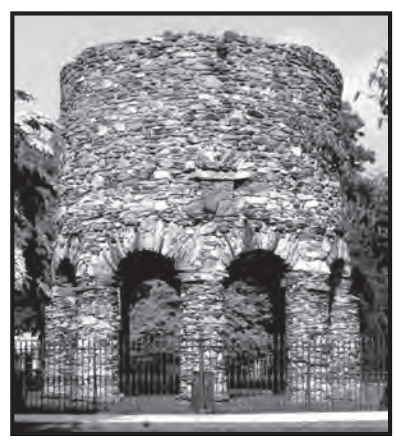
A 4-foot tall entablature, which is simply a decorative facade,
is tall enough to ***hide*** an "outward tapering" of the tower
(needed to bring the diameter 24 feet).

The following series of illustrations explains how this would not only be feasible, but probable.
First, let me explain that there are two possible ways to view the Tower
in order to maintain "visual symmetry" with regards to the pillars.
One way is to view it centered on an arch, and the other is to view it centered on a pillar.

Viewpoint:
centered
on an arch

In my previous analysis,
I used to the simpler
"centered-on-an arch viewpoint"
(where four pillars are visible and
four are hidden behind them).

For this analysis, I'm using the
"centered-on -a-pillar viewpoint."
(Seeing five pillars makes my
illustrations easier to visualize)



Viewpoint:
centered
on a column

It’s hard to get back far enough from the
Tower to photograph it completely “flat-on.”

So digitally adjusted my photo of the Tower
to compensate for that photographic parallax.

Also, in silhouetting it,
I also made the **drums** (feet of the pillars)
their full 18 inches in height
(the computer makes it a lot easier
than actually digging around
the Tower to expose them).

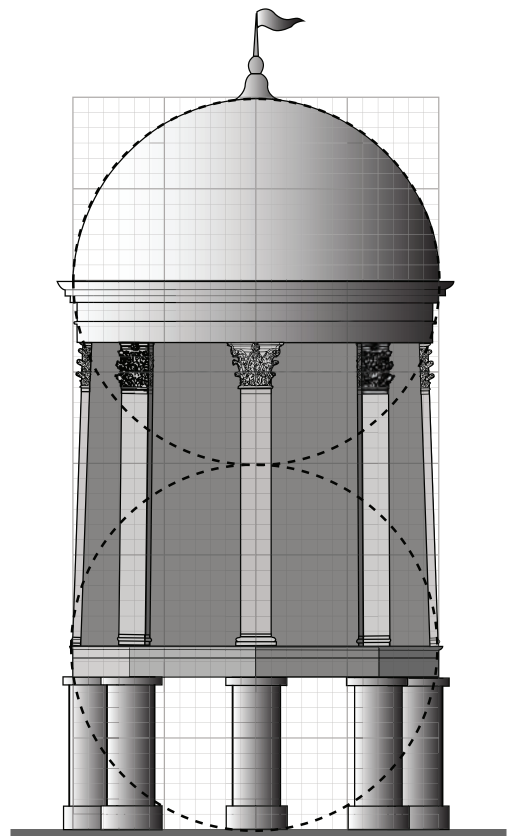
(silhouetted and retouched
to show drums,
and a “flat-on” perspective)

Here is what I think Dee’s side-view
plan of the Tower looked like.

You can see the tapering of the pilasters,
but the 24-foot diameter of the two circles
is still well-defined in two critical places:

12 feet above ground level
(where the pilasters sit on the pillar entablature)

and 36 feet above ground level
(where the dome sits on the pilaster entablature).



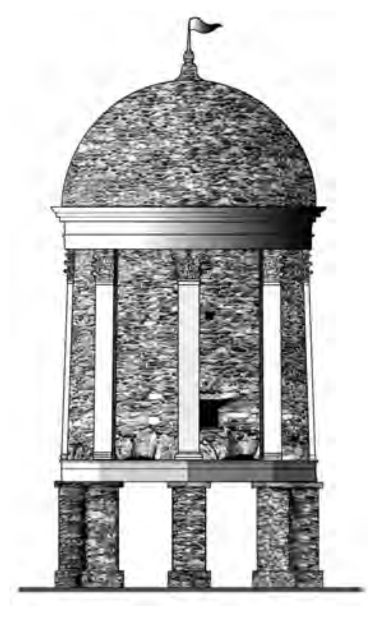
Now, let's superimpose the
"existing part of the Tower"
over that plan.

You can see how they
"taper" at the same rate.

As parts of the columns and
pillar entablature are hidden,
this illustration is a little hard to read.

Here is a "ghosted" superimposition.
But its still hard to read.

This combination is a lot clearer.
Notice how the pillar entablature hides
the graceful (but really only structural)
arches above the pillars.



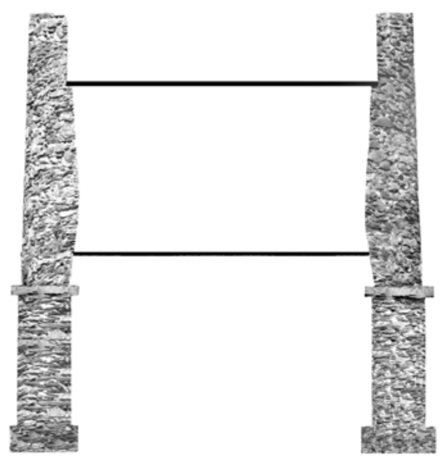
Next, I've eliminated the illustrated plan and "the "photoshopped in" the "missing" stonework of the Tower.

Naked like this, it looks like a minaret with an onion-shaped crown.

But the architectural feasibility of this idea can't really be determined without examining the ***thickness*** of the Tower.

Here's a cross-section of the existing tower with two floors in place.
(But not the floor supporting beams.)

Remember, the drums are 4 feet thick,
the pillars are 3 feet thick,
the lower part of the cylinder is about 3 feet thic
and the top is about 2 feet thick.
That's pretty substantial!



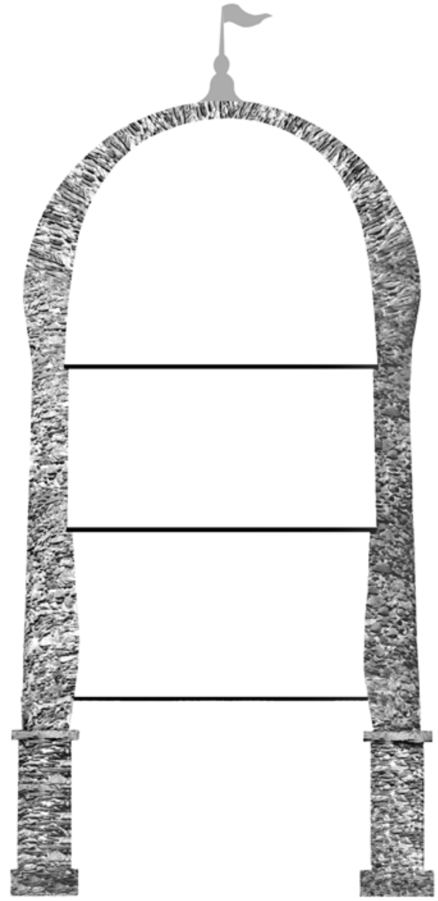
Cross-section of the existing Tower (showing the placement of the first and second floor)

Here is a “photoshopped”
cross-section of the
whole tower.

As the tower rises,
that 2 feet of wall thickness is
maintained up to a level of 32 feet,
then for over the next 4 feet of height,
the thickness widens to 2’10” inches (34 inches),

That “problem” of the “10-inch overhang” is ***gone***!

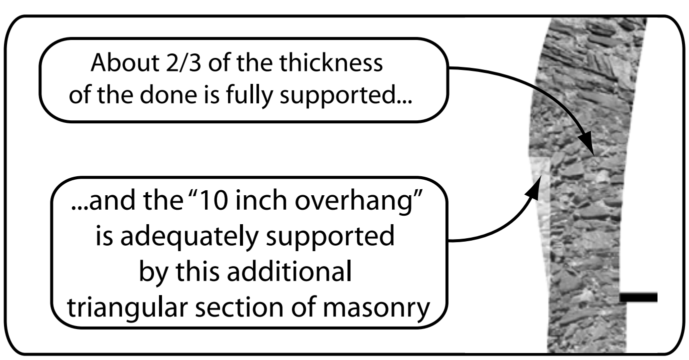
While it seems counterintuitive
to have a “thicker wall”
on top of a “thinner wall,”
because the Tower walls have **so much mass,**
it’s not of big problem here.



Remember, the tower is not simply “rocks on top of rocks.”
They are all cemented solidly together with mortar.

(An example of this splaying can be seen in the chimney caps of old chimneys.
They splay outward consideably, mainly to keep the rainwater dripping from
their edges away from the junction of the roof and chimney down below,
a spot which was prone to leaking.)

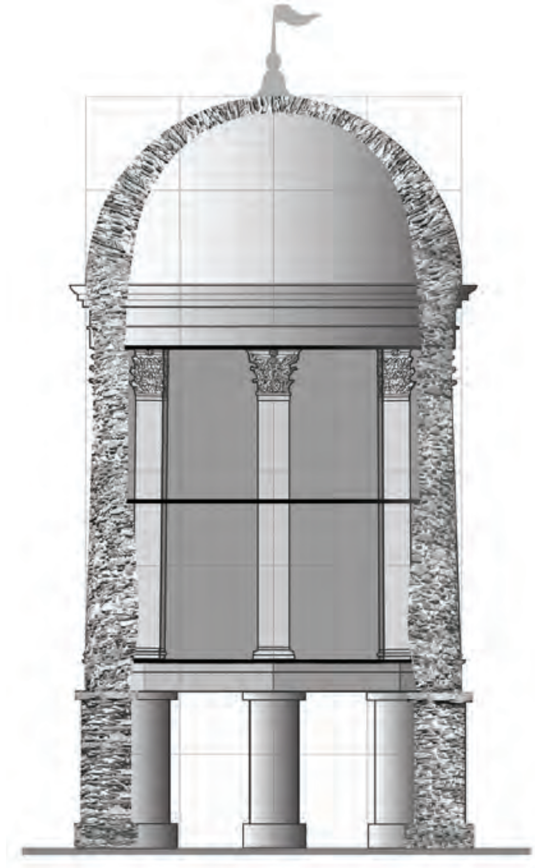
And remember this work was done by master Elizabethan Masons
who had previously had much greater challenges building castles,
towers and defense walls for members of the Queen’s court.



And the other advantage of this plan
is that the 4-foot-tall pillar entablature
isn't involved in the structure at all!

Like the pilasters, it could easily be held
on with cramp irons (L-shaped brackets).

Decorative moldings and details
would be simply thin strips of wood
or perhaps bas-relief plastering,
or even simply just painted on,
as it is ***way*** up there
(from 32-36 feet above the ground).



Here, I've merged that cross-section with
the plan to give a better indication of how
the **inside** of the tower relates to the **outside**.
Note the heights of the floors.

The first floor level is where
the pilasters sit on the pillar entablature.

The second floor is at the middle of the column.

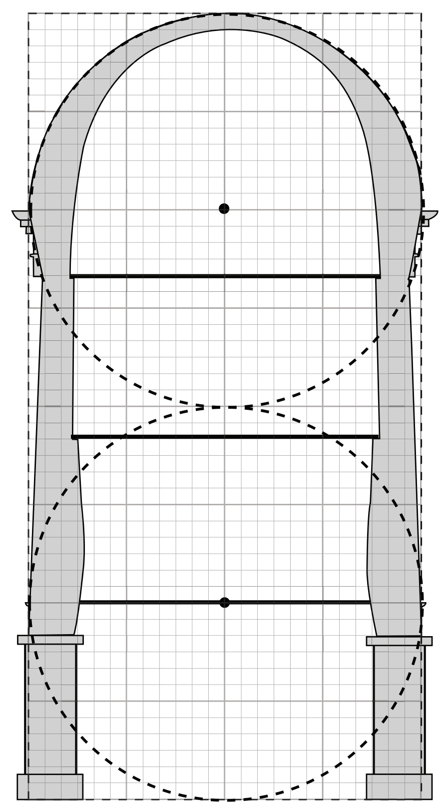
This means that the 10-foot-tall first-floor room plus the
10-foot-tall second-floor room relate harmoniously with
the 20-foot tall pilasters on the exterior.

The third floor level is where the tops
of the pilasters meet the pilaster entablature.

The 4-foot-tall pilaster entablature and the
12-foot-tall dome combine on the inside making
the 16-foot-tall dome room.

In other words, the apparent
"12, 12, 12, 12" plan of the **exterior** and
the "12, 10, 10, 16" plan of the **interior**
are not independent.
They're actually quite related.

Here's a simplified view showing the cross-section,
the two circles, and two grids:
a 48 x 24 grid showing actual feet
and a bolder 8 x 4 grid.



What about the Windows?

Though all you see when viewing the tower today is stonework,
I don't think **any** of it was meant to be seen as rough stonework.
It's not even the "skin" of the tower. It's more like the skeleton.

The skin of the tower would be the "pargeting," which, (most historians agree)
once covered the entire exterior and interior of the Tower.
One of the few traces of this plastering that has survived is on the inside of the Northwest pillar.
It's actually the same "coarse-grained" concrete that holds all the rocks together.

But the tower was not a "white elephant."
Even that skin wasn't meant to be "seen".

Some of the skin is covered by "clothing," that is, the pilasters and the entablatures.
But these weren't simply decorative adornments.
They were meant to look like sturdy Classical
features made from marble or quarried stone.
(Strong enough to support a grand dome.)

I think the entire building was disguised
so that none of it looked like stuccoed stonework.
It was all meant to look like cut stone.

This "seen" exoskeleton was not structural at all;
the "unseen" internal skeleton did all of the work.

Philip Means in his 1942 book Newport Tower reports that:
"**R.G. Hatfield, an experienced architect
who studied the tower with great care in 1879,
conjectured that the crude-looking bases of the pillars
and equally crude-seeming impost blocks
once were provided with neatly finished details wrought in plaster
so that they had the appearance of simple bases and capitals.**

This together with the then plastered shafts of the columns [pillars]
**made the pillars far more handsome and to some extent stronger looking
than they were after the plaster disappeared**."

(Means, p.17)

None of the historians over the centuries have suggested that there were eight pilasters
resting on the eight solid pillars. And for good reason.
These pilasters cover parts of some of the windows!
(I will resolve that issue momentarily.)

But none of the historians knew it was the crafty John Dee who designed the Tower.
(Though it must be mentioned that Means recommended that
"**the student who seeks new light on dark historical points
should study writers like Dr. John Dee a learned man of Queen Elizabeth's day
who had quaint notions about King Arthur
and his alleged conquests in lands remote from the British Isles**.")

(Means, p. 37, 221)

They would not have considered Dee's love of Roman and Greek architecture, his admirration of Vitruvius and Leon Battista Alberti.

They would not have been aware of Dee's penchant for being cryptic, making something appear to be something else.
(Like the whole *Monas Hieroglyphica* and in the well-hidden design plan that it contains).

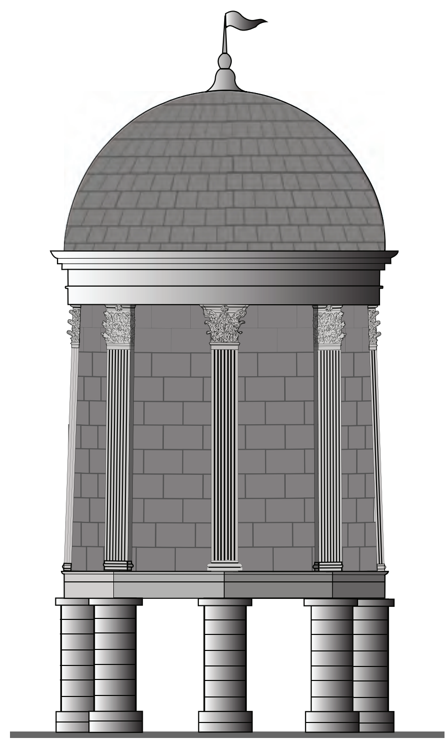
The historians would not have considered that Dee had seen many buildings with false exteriors in his travels through Europe.

Example of sgrafitto in Prague
(with patch missing to show plastering)

To make a plastered building look like cut stone many Renaissance architects used technique of called "**sgraffito**." From this term we get our modern-day word "graffiti," the "artwork"which adorns the walls of many cities.

The word sgraffito (which starts with that weird combination of letters "sg") ultimately derives from the Greek word *graphein*, meaning"to write."

Not only did they draw borders of the "faux cut stones" drawn onto walls, but they also painted dimensional sculpting, like "faux raised panels,"complete with "faux shading" to give it depth.
(Examples of sgraffito can still be seen in Europe today, especially in Bavaria, and around Prague)



The 8 **pillars** of the Tower, with their decorated drums and capitals, might have had lines drawn around them to make them look like finely-cut wheels of marble stacked on topof each other (the way the ancients actually constructed pillars and columns.)

And the **pilasters** probably had fluting, perhaps done in plaster bas-relief, but a least sgraffitoed on with some shading to give the grooves a sense of depth.

The **walls** of the cylinder between the pilasters were probably sgraffitoed to appear like neatly assembled large blocks of stone.

And the **dome** as well might have had lines painted on it to make it look like it was assembled from squared off blocks arching to a keystone (like ice blocks of an igloo). The pilasters and entablatures, though made from wood, would also be decorated to look like hand-chisled marble.

With this version of the handsomely detailed "skin" and exoskeleton"
and knowing the over all simple symmetry of the design,
it's easier to understand why the apparently random placed
and different sized windows were ***not*** meant to be seen.

They simply ***do not*** fit any of the symmetrical design.
But that's not to say that they aren't important.
Indeed, they are an essential part of the Tower.

It's just that they weren't meant to be seen by ***everyone***.

Just as Dee conceals many of his wonderful secrets in the *Monas* "from the vulgar,"
he hides his wonderful windows from anyone not willing to "be silent and learn."

It seems as though Dee had been burned by so many false accusations
in the paranoid Elizabethan times that kept his wisdom "close to his chest."

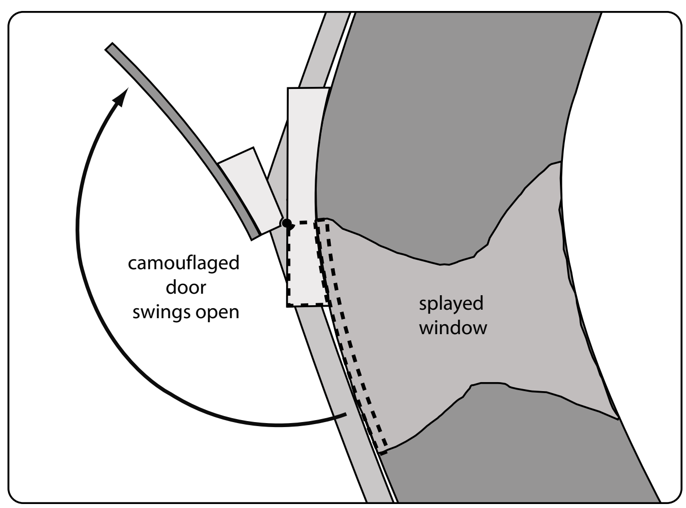
He yearned to tell the world, and indeed he did, through his written works,
but he disguised it all under a veil that requires work and study to see through.

Besides, it's difficult enough to get a Lunar Minor Moonrise alignment (northeast to southwest)
and a Winter Solstice Sunrise alignment (southeast to northwest)
through one common window (the West window), without also having to be concerned
with the 2-foot wide pilasters that were spaced regularly around the cylinder.

The windows were allowed to be where they
needed to be to make them function properly,
without concern for the symmetrical design of the pilasters.
(This accounts for their apparent randomness.)

The Windows could be camouflaged,
yet still be fully openable
in a very simple way.

This illustration, as seen from above,
shows what the camouflage covering
for the West window might have looked like.

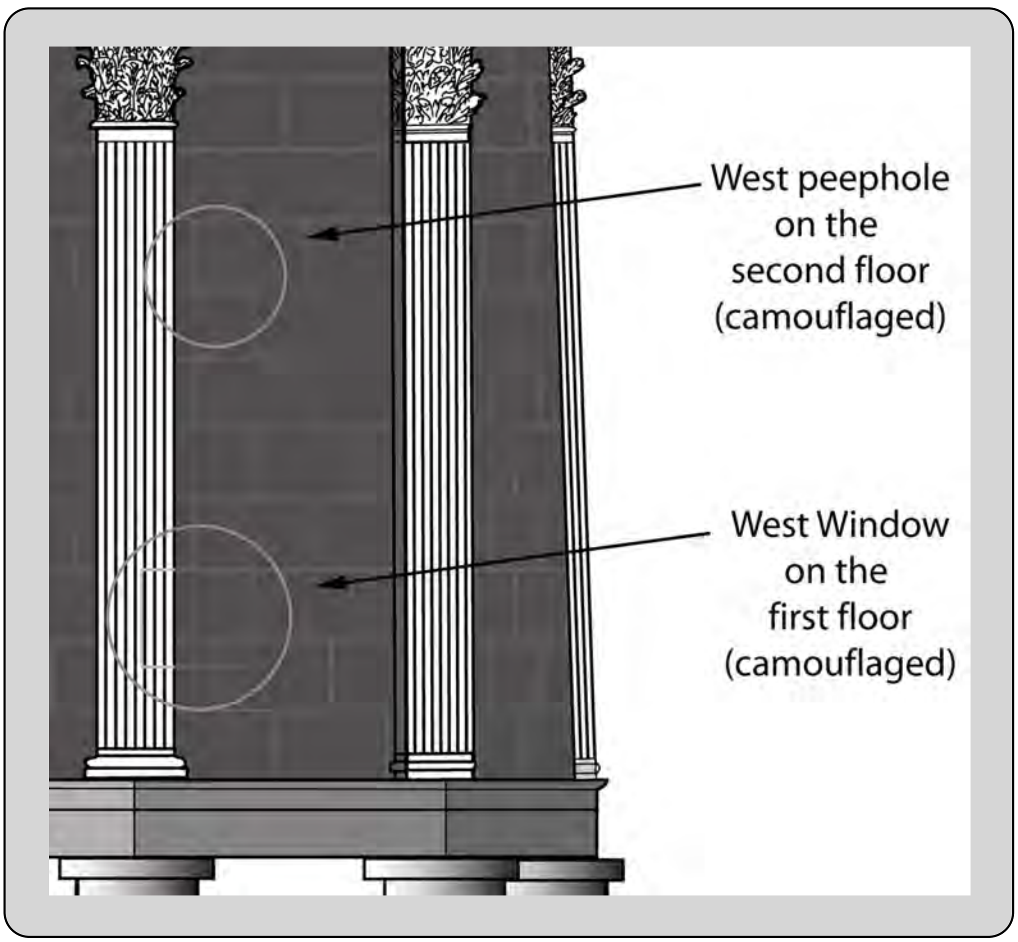


Whether it was part pilaster or part sgraffitoed wall,
the master carpenters could build a small door-like covering
over the window and put a hinge on it so it could be swung open.

The small seam around such a tight-fitting door would hardly
be visible from below once the Tower was "fully decorated."
(Just like the many subtle clues on the Title page
of the *Monas* are lost in all the "busy-ness")

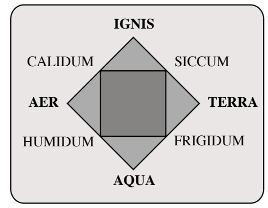
The small peepholes would be treated the same way,
whether they coincided with a column or not

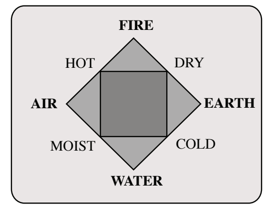
(On the second floor, one small peephole, seems to fall
smack dab in the middle of the southeast-facing pilaster.)



Decorations on the Tower

It's also probable that Dee also decorated his Tower
with clues relating his cosmological ideas.
The adornments drawn here are purely speculative,
but serve as examples as to what he might have done.
On the pillar entablature he might have written the names of the
four Elements (in Latin), interspersed with their for shared qualities.
(much like his Art of Graduation described in the *Preface to Euclid*)





Alternatively, he might have listed with a Quadrivium:
Arithmetic, Geometry, Music and Astronomy,
along with four other "derivative" Arts that he lists in the *Preface*
(for example, Perspective, Cosmography, Horometry and Zography.)

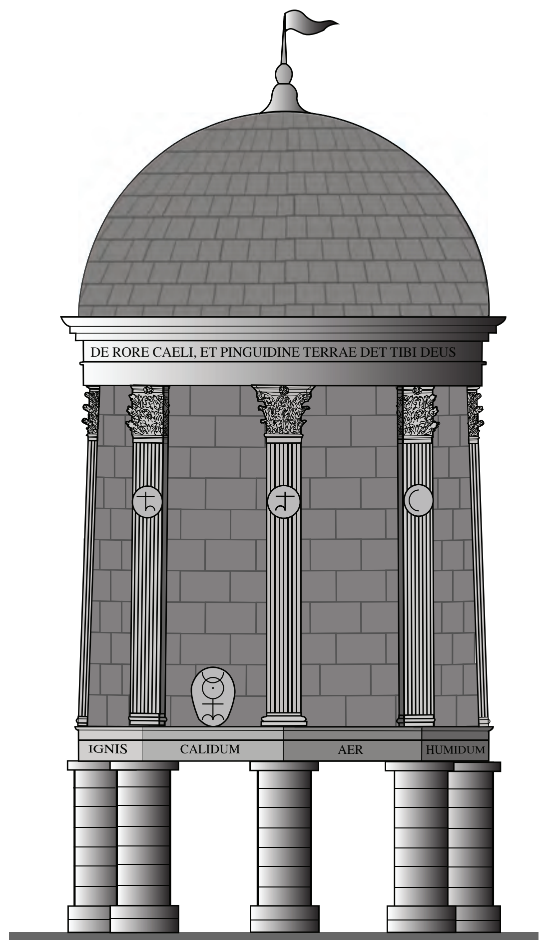
Given that the Tower is a monument to number, Dee
might have included the planetary symbols, which
he discussed in Theorems 12 and 13 of the *Monas*.

In this view you can only see 3 of them,
but each one represents a number.
As the Sun symbol and Solar Mercury symbol
are each represented by the number (seven),
perhaps there were two circular disks
that shared a pilaster.

A Lunar Mercury Planets symbol represents
the number eight, so perhaps it had its own pilaster.

The Solar Mercury Planets Symbol,
(which is a full Monas symbol)
represents the number nine,
but there are only eight pillars.
That's OK because the number 9
is that odd-ball "null" number anyway.

To represent it on the facade,
I've enclosed it in an egg shape
(as on the Title Page of the *Monas*)
and placed it just above the
northwest -west section
of the pillar entablature.



I’ve done that because that is the position of the two rocks which crudely resemble the shape of the Mona’s symbol. They can still be seen today.

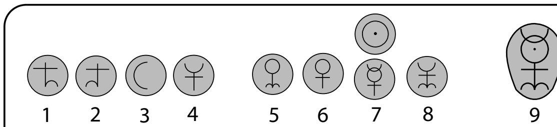
The round, red “Sun stone” with subtle glittering crystals sits on top of another rock with “shoulders” vaguely resembling a flat T-shirt. Perhaps the “Sunstone” was only partially plastered so its reddish color could be seen.
(But of course it must have had a small dot in its center.)

northwest-west arch
(exterior view of tower)

On the pillar entablature I have “faux-engraved” the quote from Genesis 27, which Dee used on the foundation of the Title Page. He might have placed some other bits of his *Monas* wisdom there (like “Intellect Judges Truth”) or even a statement about Queen Elizabeth, the “New Time” or the start of the “New Colony on the John Dee River”.

“Monas symbol”
on the tower

Here’s a summary of the symbols he might have used to decorate his ”Monas” Tower



Why did Dee even bother to taper the Tower?

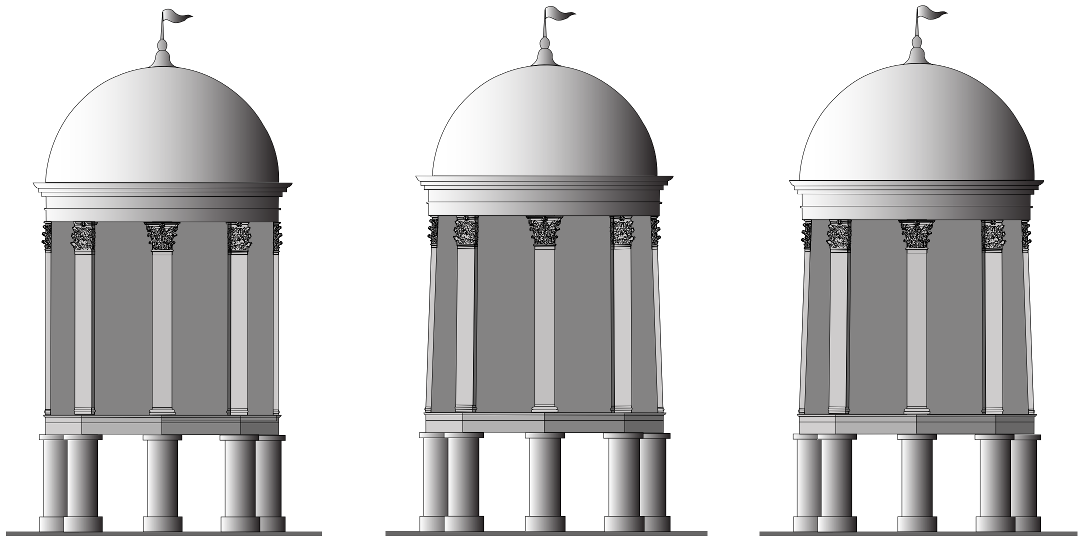
Why did he create this architectural challenge of fitting a 24-foot wide dome on top of a 22’ 2” cylinder? There are at least 3 good reasons.

First, it’s sexier. A tower that is in 2:1 proportion, 48 feet tall, can look monolithic and perhaps a little stiff. Here is a visual comparison of three different versions.

The first is what the Tower would look like with no tapering.

The second shows the tapering, but fit with a 22’ 2” diameter dome. The dome fits nicely but it looks rather small compared to the third version shown here, which has the full 24-foot diameter dome.

Which of the three designs do you find sexiest?



The second reason also has to do with the dome.
The finial might be the highest feature,
but it's nothing compared to the Dome itself.

The finial "enhances" the dome from above,
just like the drums, pillars, pilasters and entablatures
are mostly function as "support" the dome from below.

The dome might even be seen as the grandest
architectural feature of the whole tower.
All the other features nearly play supporting roles. Why?

Because it represents heaven.
The idea of a heavenly dome is not simply some 20th-century poetic metaphor.
It goes back much, much farther than that.
I'm talking about way back to the late Stone Age.

E. Baldwin Smith in his 1950 tome **The Dome** writes:

"**At the primitive level for most prevalent and usually the earliest type of constructed shelter, whether a tent, pit house, earth lodge, or thatched cabin, was more or less circular in plan and covered by necessity with a curved roof.**

Therefore, in many parts of the ancient world the domical shape became habitually associated in men's memories with a central type of structure which was a venerated as a tribal and ancestral shelter, a cosmic symbol, a house of appearances and ritualistic abode."

(E. B. Smith, p.6)

Long after men started building houses with flat or pitched roofs,
the domed shape was still used for sacred buildings.
Around 1500 BC, During the Bronze Age, royal burial tombs called tholoi (tholos tombs)
were constructed throughout the Mediterranean from Turkey to Spain (and even up into Ireland).

They are often called "beehive tombs" because of their corbelled shape.
They were built with successively smaller rings of flat stones that
gradually arched up into a dome. Most often the whole structures
were mounded over with dirt, making them underground tombs.

One type of tholos that was not buried underground is called a "nuraghe."
On the island of Sardinia (200 miles southwest of Rome)
there are still over 8000 nuraghi that still exist today (some are over 60 feet tall!)
It's estimated that there were once over 30,000 of them on the island.
They were built and were in continuous use from 1500 BC to around 200 BC.

a domed nuraghe

Nuraghi can also be seen in plentitude
on the southern mainland of Italy.
This nuraghe is on the outskirts of Bari,
which is near the "heel" of Italy.

It is not known if nuraghi were temples,
houses, forts, meeting houses or
perhaps a mix of many of these uses.

what appears to be an
illustration of a "nuraghe,"
a domed structure,
in the Element of Earth
on the Title page

It's apparent that Dee was aware of this tradition,
because, as we've seen, he drew a small domed
structure on the Title page of the *Monas*.

It can be seen if you closely inspect the circular
illustration representing the element Earth.
(Behind foreground foliage a nuraghi sits on a shoreline
against a backdrop of mountains and clouds.
There appears to be a person in front of it for scale.)

E.B. Smith asserts that the the sacred architecture
of the Indian, Islamic, Roman, and Christian worlds all
fused this "divine, royal, celestial," dome idea with building
methods using bricks or squared-off stones.

As he puts it, the ancient "*tentorium*," "*vihara*", or "*kubba*"
became a "divine helmet," "cosmic egg," "umbrella," or "***mundus***".

Our modern word “dome” comes from the Greek and Latin word *domus* meaning “house.”
This can be seen in our modern-day words like “domestic” and “domicile.”

In the Middle Ages and during the Renaissance, the word was used
“all over Europe to designate a revered house, a *Domus Dei* (or House of God).
This “house” idea survived in the Italian word *doumo* and in the
German, Icelandic and Danish word *dom* meaning “cathedral.”
(E.B. Smith, p.5)

When we use the word “mundane” it usually means a dull or
unexciting, but it comes from the Latin word “mundus” meaning “world.”
The Romans envisioned a dome building to be a “mundus,”a representation of the world.
A.L. Frothingham, in his 1914 article on the “Circular Templum and Mundus” in the *American Journal of Archaeology* writes to that:

“**there is the explicit testimony of Cato, quoted by Festus (154)
that the mundus itself was circular,
on the model of the heavenly hemisphere and that this was,
in fact, with the origin of the name**:

The “mundus”
or dome of the world

“**Mundo nomen impositum est ab eo
mundo qui supra nos est: forma enim eius est,
ut ex his qui intravere cognoscere potui adsimilis illi**.”

E. B. Smith translates this as:
“**the mundus gets its name
from the** “**sky**” **above our heads;
indeed it resembles the shape of the sky**”

Frothingham also cites Varro who asserts,
“**The form of the subterranean templum was the
same as that of the heavenly templum, that is, circular**.”
(A. L. Frothingham, p. 315)

Dee was certainly familiar with the writing of Marcus Terentius Varro (116 BC to 27 BC)
as he had a copy of his *Opera* (Works) in his library.
He also had in his collection 2 copies of a manuscript that included
the works of both Varro and Marcus Porcius Cato (234 BC to 159 BC)
(Roberts and Watson, book numbers 1067, 1112 and 1913)

The sky was a dome in the Biblical tradition as well.
The word “firmament” referred to so frequently in the Bible,
is a translation of the original Hebrew word “raqiya,”
meaning “a dome beaten out of metal sheets”.

J. Edward Wright in *The Early History of Heaven* suggests even the Sumerians
considered the sky to be a dome “**because they describe heaven as having a zenith**.”

[Wright, (New York, Oxford University Press, 2000), p. 29]

All this gives new meaning to Dee's words
"SIC FACTUS EST MUNDUS"
"Thus the World Was Created."

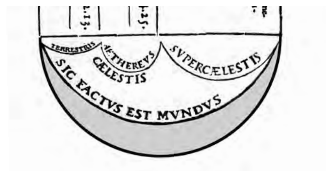
We might also read it as
"Thus the ***Dome*** Was Created."

In fact, these words type sit right on the circle segment that we "ballooned" into a full semi-circular "dome."

And, with that dome was superimposed on the Title Page, it's apex and the Title Page dome's apex are the exact same point.

Furthermore, this is the design plan for the John Dee Tower "dome."

Dee's Latin word "MUNDUS" means "WORLD", but it was also used to mean "DOME".

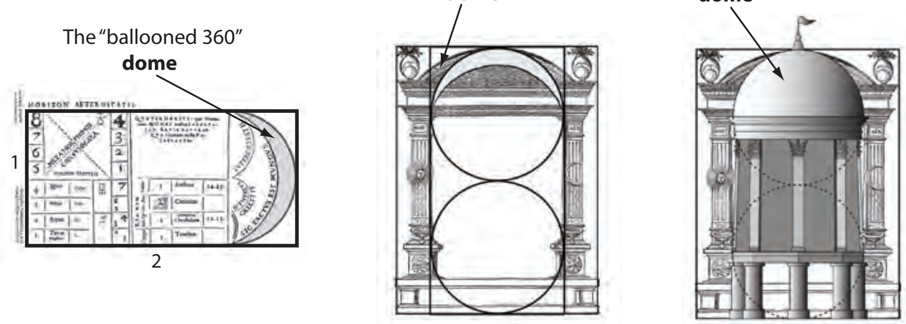


In this respect,
"Thus the World was Created"
might also be translated
"Thus the Dome was created"

The "ballooned 360" **dome**

The Title Page **dome**

The design plan of the Tower **dome**

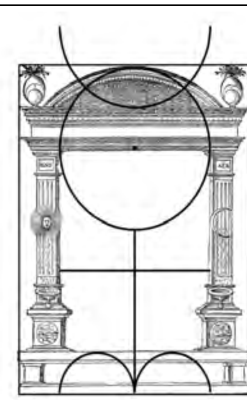


"Thus the Dome Was created"

Plus the "dome" also corresponds with the top of the Monas symbol's Sun circle!

Remember, the "ballooned 360 dome" is ***hidden*** until one discovers that the areas of the circle segments are in the proportion of 12 : 24 : 72 : 360.

I jokingly refer to the dome as the "Mundus Hieroglyphica"
(the sacred symbol of the world).



And remember,
the dome also corresponds
with the top of the
Monas symbol's Sun circle

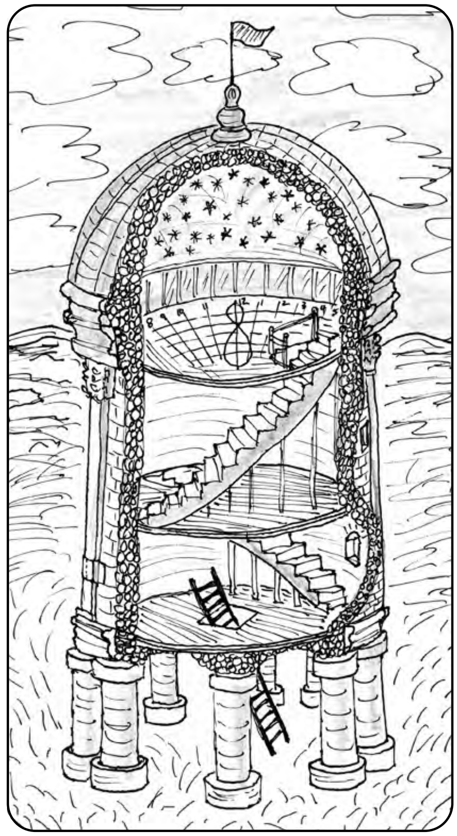
What color was the Tower?

As this cut-away view of the Tower shows,
the dome room is the "climax"
of the journey up through the Tower.

Starting from Earth, one ascends a ladder,
then a spiral stairway, then another,
and ends up in this 16-foot-tall
"Heavenly Dome."

Its beehive shape and solid walls
would probably make voices
reverberate in strange ways.

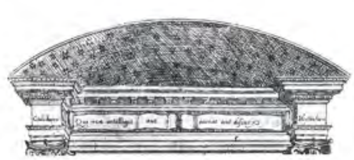
When it was not being used as a
camera-obscura solar-disc calendar
it might be illuminated by light
from as few small peepholes.
(Similar to those found in the lower rooms.)



The Title pages of Dee's "sister books" both include domes festooned
with sparkling stars against a dark background.

I envision that the interior of the dome room
of the Tower was decorated in a similar fashion,
perhaps with gold stars on a dark blue background.
(Perhaps the stars were arranged in as prominent constellations,
including part of the band of the zodiac.)

Dee's books were printed in black-and-white,
but extant paintings show that Elizabethans
had an exuberant sense of color.



Stars in the Dome
of Dee's 1558
Propaedeumata Aphoristica

Stars in the Dome
of Dee's 1564
Monas Hieroglyphica

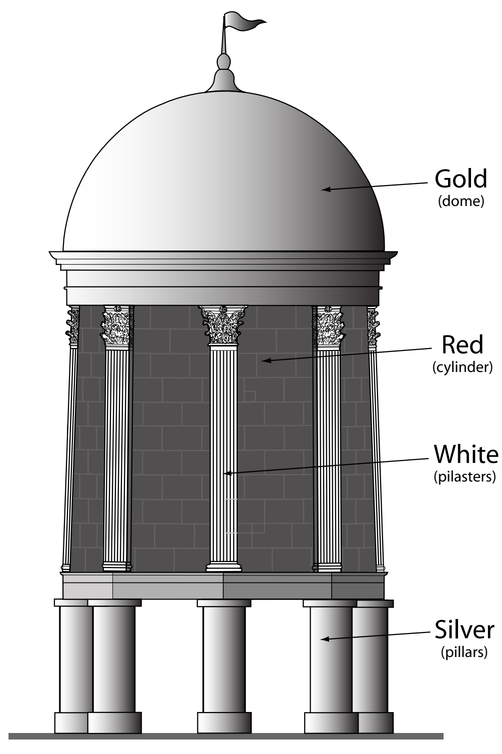
I envision that a whole exterior of the Tower was not only ornately decorated, but also painted in vivid colors. One possibility would be that the crowning dome was painted Gold and that the 8 pillars were painted Silver.

Dee was a big fan of Pantheus' *Voarchadeumia*,
a technical and mathematical manual on refining gold and silver.
These two metal are a classic pair of opposites,
as in the "golden sun" and the "silvery moon."

I also envision the 8 Corinthian pilasters were painted white to look like marble.

This light color would contrast nicely against a red cylinder
sgraffitoed to look it was built from bricks.

As Dee shows in his "Thus the World Was Created" chart,
Red (Anthrax) is a "Solar" color and
White ("Chrystallina, Serenitas" or clear crystal) is a "Lunar" color.



Possible Colors of the Tower

The colors, decorative elements, and engraved letters I've described are all conjectural.
No evidence of them can be seen today.
What do you envision the coloration and decor looked like?

The Best reason for why Dee tapered the Tower

The third reason why Dee built a tapering tower has to do was some of his favorite things: **numbers.**

Dee knew that the circumference of a 24-foot diameter Tower would be about 75’ 4 1/2”. It’s obvious that this is ***very close*** to the Metamorphosis number 72, which Dee undoubtedly would have included in the design plan of his Tower of number.

If he tapered the Tower to 72 feet in circumference, the dome would overhang the cylinder by only 8 inches on each side.

But he did not conclude his taper at 72.

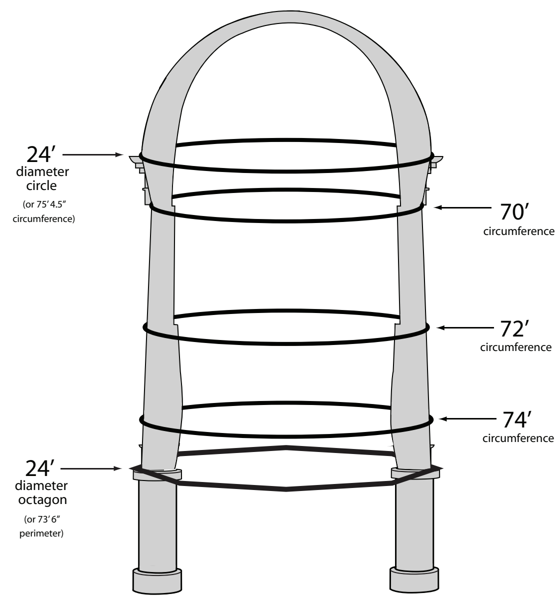
My extrapolation shows that he tapered it to around 69 feet 9 inches.

As this result is based on a mathematical extrapolation, and on measurements that don’t take into account the plaster “skin” of the Tower, let’s round this off to 70 feet.

Down below, the widest part of the cylinder fits within the octagon of the pillar entablature.

As the cylinder is less than 24 feet in diameter, its circumference is less than 75 feet 4 inches.

Indeed, at its widest section its only about 74 feet in circumference.

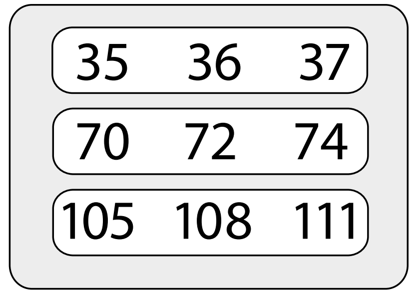


So the Tower seems to taper from 74 feet (just above the arches) to 72 feet (near of the middle) to 70 feet (at top, under the dome.)

These three numbers reminded me of something that Bob Marshall told me. In his investigations of number he found three “important triads” that kept coming up over and over again.

As you can see the members of the “70, 72 and 74” triad are ***double*** those of the “35, 36 and 37” triad.

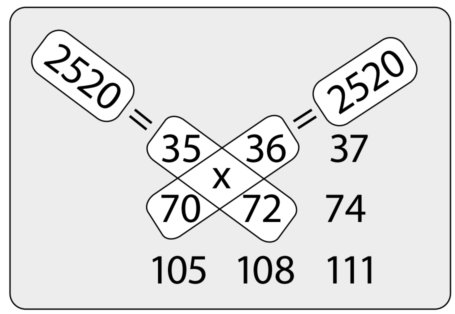
And the members of the “105, 108 and 111” triad are ***triple*** those of the “35, 36 and 37” triad.



Marshall explained that importance of 35, 36 and 37 can be seen in the fact that they are a composite, a square, and a prime in consecutive order.

But that’s another story.

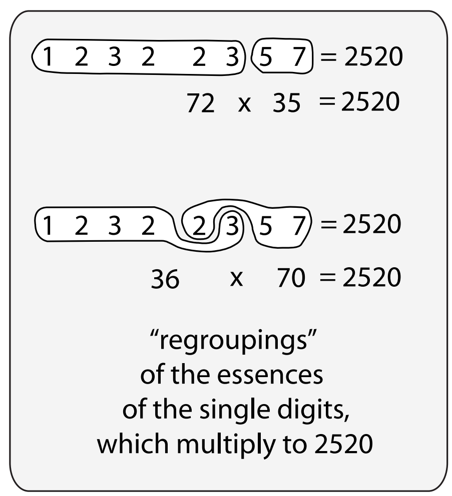
What’s most pertinent about these “important triads” in this story is the following amazing relationship. Both 35 x 72 and 36 x 70 equal same number, 2520, Dee’s “sabbatizat”!!!



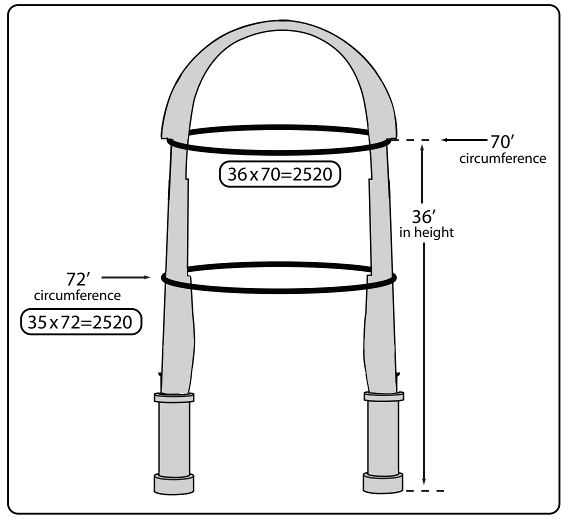
It’s clear that he would have known about this arrangement, as the two equations are simply “re-groupings” of the “essences” of the single digits, which multiply to 2520.

In this first example,
Dee’s Artificial Quaternary
1 x 2 x 3 x 2,
times another 2, and times another 3
(the essences of eight and nine respectively)
multiply to 72.
The remaining primes,
5 and 7, multiply to 35.

In the second example,
Dee’s “Artificial Quaternary”
times 3 makes 36.
And the remaining primes, 2, 5, and 7,
multiply to 70.



The connection between these two special equivalent equations and the Tower is that the 70-foot circumference is right under the dome, exactly 36 feet of the original ground level!



I've previously noted that there really is no 70-foot circumference, because the top of the cylinder splays outward to a 75' 4 1/2" circumference in order to fully support the dome.

But that entire splay is hidden behind a 4-foot-tall entablature. So, to anyone looking at the exterior of the Tower, the cylinder would appear to taper to 70 foot circumference.

Knowing Dee's penchant for "hiding things" which can only be found through mathematical reasoning, this appears to be what he had in mind

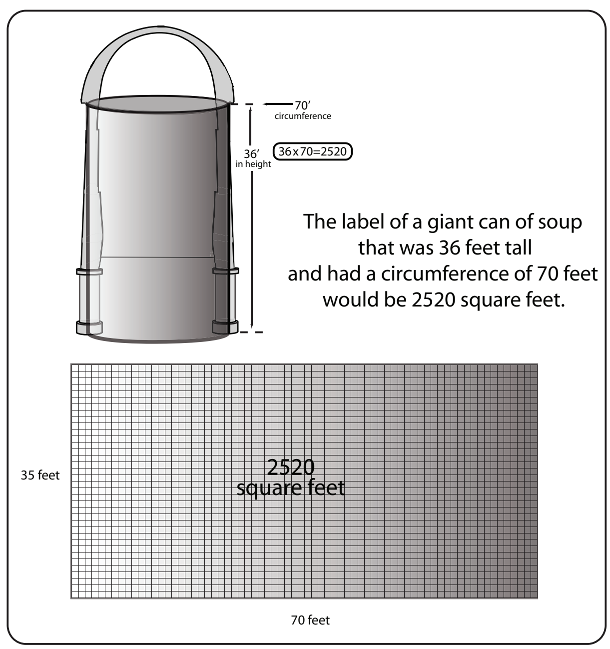
Any good geometer knows that to find the surface area of a cylinder one simply multiplyies its circumference times its height.

Think of the part of the Tower beneath the dome as a giant cylindrical can of soup.

If you were to cut the paper label off and flatten it out, its surface area would be 2520.
(36 feet x 70 feet = 2520 square feet)

That's a pretty clever way to conceal 2520 in the Tower. It's a measurement you can never physically measure.

Yet it can be intuited mathematically.



Indeed, Dee has involved the first four Metamorphosis numbers in the Tower in subtle ways.

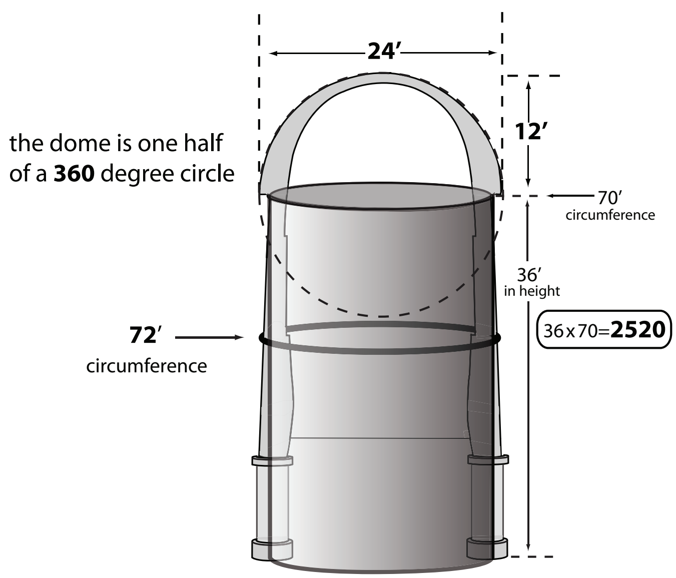
The number 12 can be seen in the 12-foot radius (height) of the dome
and the 12-foot radius of the octagonal pillar entablature,
whose top edge is 12 feet above the ground.

The number 24 can be seen in the 24-foot diameter of the dome,
the 24-foot diameter of the pillar entablature,
and a 24-foot high of the "Column Section"
(which includes the 20 foot pilasters
and the 4 foot pilaster entablature).

The 72 can be seen in the 72-foot circumference at about
the middle of the tapering part of the Tower.

The 360 can be seen in the 360° circles that make up the overall design plan, part of which corresponds to the hemispheric dome (which is that "ballooned 360").

And we've just located 2520 in the surface area of the cylinder.



Cryptic expression of
Metamorphosis numbers
12, 24, 72 ,360 and 2520
as dimensions of the Tower.

I'll admit that this type of architectural analysis might be silly if applied
to most other buildings, but we're talking about Dee's building here.

The John Dee Tower appears to be the one and only building
that he designed that ever actually got constructed
(aside from some additions to his house at Mortlake and perhaps some work on St. Mary's Church next door).

The Tower was an expression of the *Monas Hieroglyphica* and
the numerical cosmology that he had concealed within it.

Remember the way Dee hid 12 : 24 : 72 : 360 : 2520 in the
proportions of the “Thus the World Was Created” chart.

Remember the clever way he hid 2520 in the “Vessels of the Holy Art” illustration
using Roman numerals MMDXX (with the two X’s hidden in the word LVX”).

Remember the first two letters of each Theorem jumble together to spell
Mane Mane Thequel Phares, the expression of 2520 on Belshazzar’s palace wall.

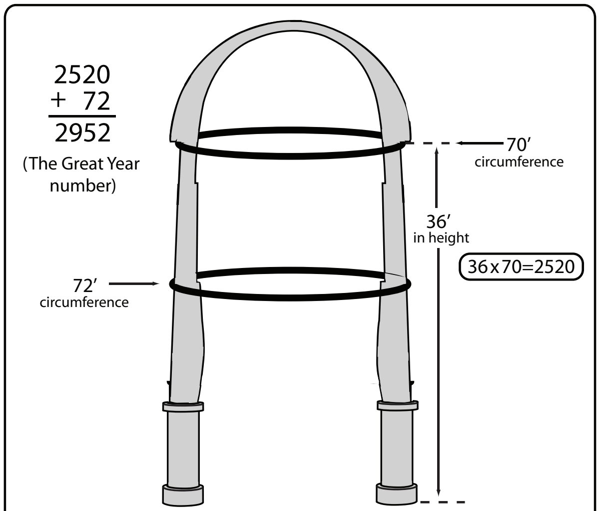
Remember the way he adds 20 + 200 + 21 + 1 to make 252.

The Tower was designed and built to reveal (yet conceal) these same numbers.

While were on the subject of numbers, there are two more numbers that pop up in this analysis.

If we take that 70’ circumference times 36’height = 2520 feet,
and add to it the 72 foot circumference that occurs about mid-Tower,
the total is 2592, the Great Year number.

This might seem like an unusual thing to do, but there
is a confirming clue that it’s not that odd after all.

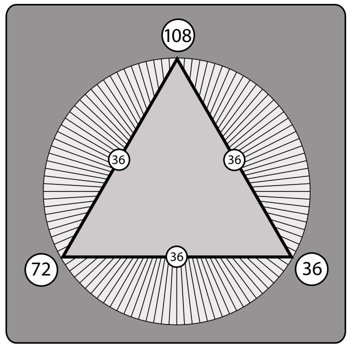


Cryptic expression of
the Great Year number
in the dimensions of the Tower.

70
+72
+74
216

If we add up those three circumferences
that the tower “tapers through” (70, 72 and 74),
they sum to 216, which is the Great Month number.

12 Great Months of 2160 years each = a Great Year (25,920 solar years)



Perhaps the most important relationship among the 9 members of the
3 “important triads” is the relationship between **36, 72 and 108**.

And as we’ve seen, the multiples of 108 include the Yuga numbers 432, 864, 1296, and 1728, the Great Month number 2160 (and also 216), and that Great Year number 2592.

But, what I didn’t show before are amazing ways in which the **Metamorphosis** numbers, 12, 24, 72 and 360 and 2520 (along with 108 and 252) interrelate with these key numbers.

For example, the first three Metamorphosis numbers 12+24 +72 sum to 108.

As we’ve seen, 108 plus Dee’s Magistral number 252 sums to the Metamorphosis number 360.

The two Metamorphosis numbers, 12 and 72 multiply to Treta Yuga number 864.

Two others, 72 and 360 multiply to the Great Year number 25,920.

Perhaps you can find even more “amazing relationships.”

Some of the amazing
interrelationships between
the Metamorphosis numbers ,12,24,72,360, and 2520,
and the multiples of 108
(and also 36 and 252).

	(36)	← 12+24=36
	(72)	← Metamorphosis number
(The “Yuga numbers” are all multiples of the number 108)	108 X 1 = **(108)**	← 12+24+72=108 108+252=360
	108 X 2 = **216**	
	108 X 3 = 324	← 72+252=324 108+252+360+2520=3240
Kali Yuga	108 X 4 = **432**	← 72+360=432 12x360=4320
	108 X 5 = 540	
	108 X 6 = 648	
	108 X 7 = 756	
Dvapara Yuga	108 X 8 = **864**	← 12x72=864 24x360=8640
	108 X 9 = 972	
	108 X 10 = 1080	
	108 X 11 = 1188	
Treta Yuga	108 X 12 = **1296**	← 12x108=1296
	108 X 13 = 1404	
	108 X 14 = 1512	
	108 X 15 = 1620	
Krita Yuga	108 X 16 = **1728**	← 24x72=1728
	108 X 17 = 1836	
(Precession of the Equinoxes)	108 X 18 = 1944	
	108 X 19 = 2052	
Great Month number	108 X 20 = **2160**	← 2160+360=2520
	108 X 21 = 2268	
	108 X 22 = 2376	
	108 X 23 = 2484	
Great Year number	108 X 24 = **2592**	← 24x108=2592 72+2520=2592
		72x360=25920

12+24+36+72+108=252

What this brings to mind are the "visual equations" we found by comparing the proportions of Dee's illustrations that multiply to 2592/864.

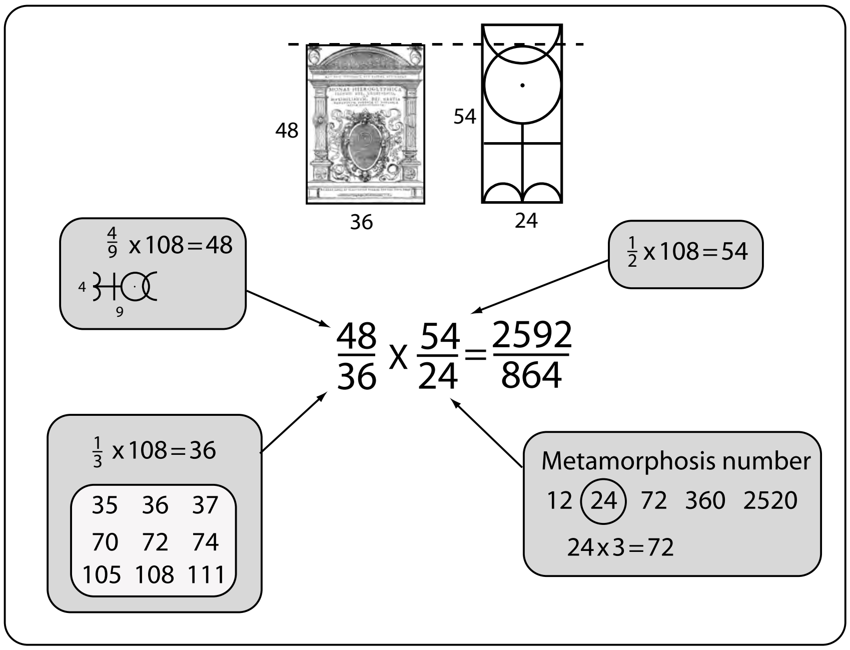
Recall that the 48 /36 Title Page times the 54 /24 Monas symbol equals 2592/864.

Of these numbers, only 24 is a Metamorphosis number.
But 36, 48 and 54 all interrelated with 108 which
is intrinsically involved with 252 and the Metamorphosis numbers.

For example, as we've seen in the analysis of Marshall's "important triads," 36 is one third of 108.

If we were to multiply 48 times the proportion of the upright Monas symbol, (9/4), the result would be 108.

And 54 is simply 108 chopped in half.



It's hard to put in words exactly what's going on here, but at the root of it are the numbers 12 and 9.

Not only do these two numbers multiply to 108, but they have a "Quaternary rests in the Ternary" relationship with each other. (9/12=3/4)

They each divide evenly into 72, and into 36 as well.

This 9 is the same "null nine" that does its magic in the "octave, null 9" of Consummata (36 and 72 are each members of the 9-wave).

$$9 \times 12 = 108$$

And 12 is the amazing "docena," first palindromable number, who, with its mate 21, multiplies to 252.

Another glimpse of what's going on here
can be seen by studying Marshall's "pretzel."

This is not precisely what Dee is expressing,
but it's quite related as you can see by the involvement
of the numbers 9, 12, 108 and 252.

$$\mathbf{12 \times 12 = 144}$$

(plus) $\underline{108} = 9 \times 12$

$$\mathbf{12 \times 21 = 252}$$

(minus) $\underline{189} = 9 \times 21$

$$\mathbf{21 \times 21 = 441}$$

What do the "multiplications"
of 9 x 12 and 12 x 21 have in common?
Well, obviously the number 12.

But beyond that, the numbers 9 and 12
can be seen as the fraction 9/12,
which is equivalent to 3/4.

$$\frac{9}{12} = \frac{3}{4}$$

An expression of "Quaternary rests in the Ternary"

The numbers 12 and 21
can be seen as the 12/21 fraction,
(which is equivalent to 4/7).

As 21 minus 12 = 9,
the "remaining"part is 9/21,
(which is equivalent to 3/7).

The fractions 4/7 and 3/7 have a
"Quaternary rests in the Ternary"
relationship with each other.

$$\frac{12}{21} = \frac{3}{7}$$

$$\frac{9}{21} = \frac{4}{7}$$

$$\frac{21}{21} = \frac{7}{7} = 1$$

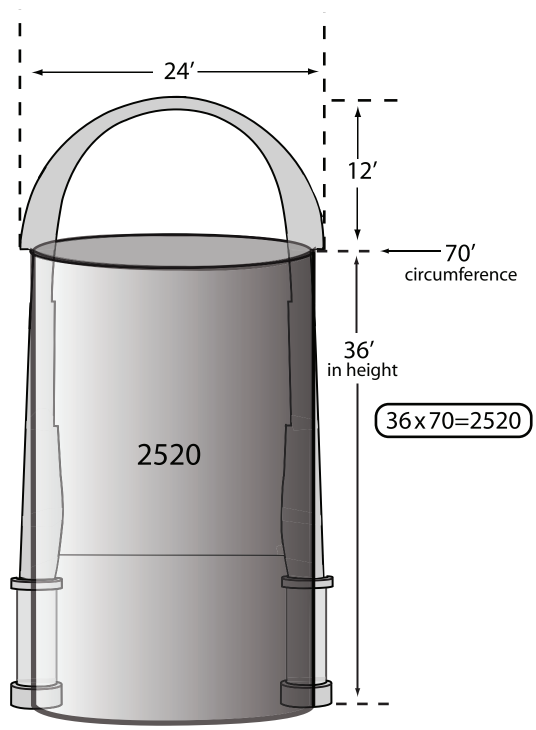
Another expression of "Quaternary rests in the Ternary"

As Dee has "hidden" 12, 24 and 2520 in the Tower,
he has also concealed his "***rare gift***" to Maximilian,
the Exemplar number, 12252240.

I'll admit its strange mathematics to ignore place values like this,
but both Dee and Marshall integrated 12, 24 and 2520 this way
to represent this wonderful number.

12252240 is the lowest number evenly divisible by
2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17 and 18.
In 12252240, all the primes and composites are arranged in perfect symmetry.

12252240



Cryptic expression of the
Exemplar Number
as dimensions of the Tower that
express various "parts" of
12252240

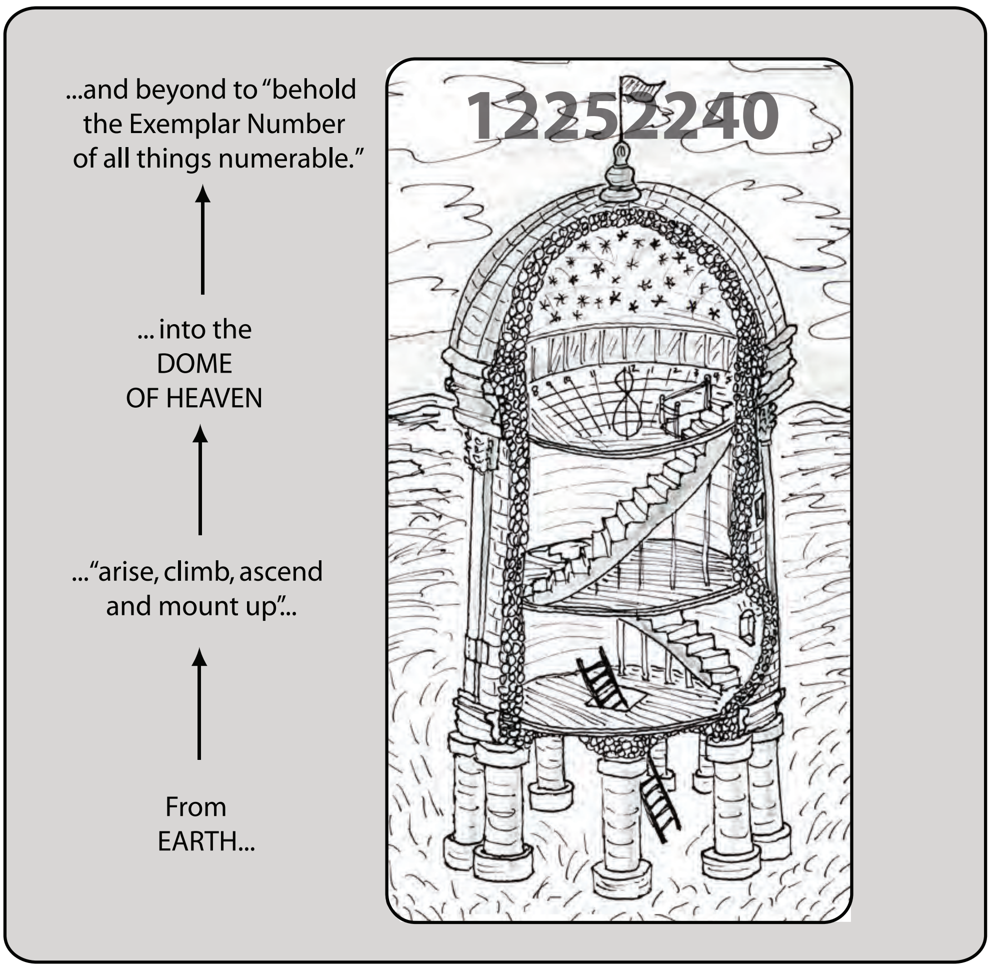
Not only is the Exemplar number implied by 12, 24 and 2520,
it is also implied by the finial of the Tower, that "ninth part" of the Tower,
just as 12252240 is the "ninth thing" which "encapsulates"
the first octave of Metamorphosis numbers.

To Dee, the pinnacle of his mathematical Tower was the Exemplary number.
The ladder, stairs and dome of the Tower and connote a "climbing upwards" or an "ascension."

It's worth repeating here what Dee wrote in his *Preface to Euclid* about the Exemplar number:

"**And also farther, arise, climb, ascend and mount up
(with Speculative wings) in spirit,
to behold in that Glass [Mirror] of Creation,
the Form of Forms,
the Exemplar Number of all things Numerable:
both visible and invisible,
mortal and immortal,
Corporal and Spiritual**."

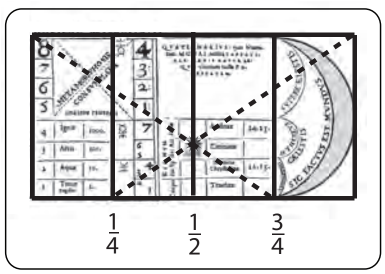
(Dee Preface, p.j)



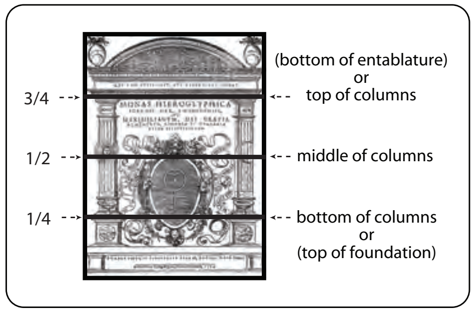
Dee's "4 Steps"
are
12, 24, 36, and 48
(and their mates)

Dee liked "quaternaries."
A number of them are listed in his "Thus the World Was Created" chart.
(1, 2, 3, 4), (earth, water, air, fire), (1, 10, 100, 1000), (1, 2, 3, 2), (black, white, yellow, red).

He also like "quartering" things.
By using the Engraved 2 as a hot spot" he effectively "quartered"
his "ballooned Thus the World Was Created" chart.



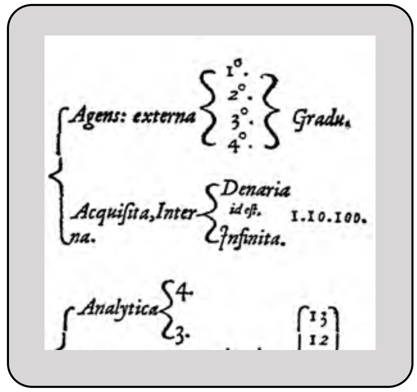
On his Title page architectural drawing,
he put the1/4 mark at the or base of the columns (or the top of the foundation),
the 1/2 mark at mid-column,
and the 3/4 mark for the top of the column (or bottom of entablature)



A fuller understanding of the importance of quartering in Dee's charts will help clarify how and why he used "quartering" in the design of the John Dee Tower.

But in its “sister” chart, the Artificial Quaternary chart,
the only clear reference to the idea of “quartering”
is in at the top of the chart where he writes the {1°, 2°, 3°, 4° } Gradu.

To my modern eye, I can’t help but see this as
“1 degree, 2 degrees, 3 degrees, 4 degrees.
At first , it seemed like a measure of temperature,
as in: “the front was quickly moving in
and the weatherman watched the thermometer
quickly rise by 1, 2, 3, then 4 degrees.”
However, in 1564, the thermometer
hadn’t been invented yet.



It also might be read as very skinny angles, as in: “1° is 1/360 of a circle.”

But, Florian Cajori, in his *History of Mathematical Notations*, claims the first printed use
of the small circle ° for angular degrees was in J. Pelletier is 1569 revision
of Gemma Frisius’ text called *Easy Method of the Practical Arithmetic*.
Peletier uses it as a substitute for the Latin word “Integra”
which means “whole, entire or complete.”

(Interestingly, *Arithmeticae practicae methodus facilis* was first published in 1540,
when Dee was studying under Gemma Frisuis, but Frisius did not use the symbol.)

It wasn’t until the 1570’s that degree symbol ° really caught
on as an expression angular degrees.

The brackets seem to indicate that the 1°, 2°, 3°, and 4° are labeled as “Gradu.”
which means “steps, stairs, stages, or the rungs of the ladder.”

If the degree symbol ° isn’t “temperature” or “angular
degrees,”
it seems like Dee is simply saying:
“step 1,
step 2,
step 3,
step 4”

What are the four steps?

They could be simply the digits 1, 2, 3, 4, which were certainly very important to Dee.
(the “4 great Wombs of the Larger World” from Aphorism 18).

However, Dee already lists these digits is in the middle category of the chart:
(1, 2, 3, 4, 5, 6, 7, 8, 12, 13, 24, 25).

The “4 steps” could simply refer to the “+4, -4, octave” rhythm of number, but again, the octave is more fully shown below in that longer number listing.
(and also cryptically in the sub-category “Particularia”)

The “4 steps” might be the 4 alchemical stages (black, white, yellow, red) or the 4 elements (earth, water, air, fire).

But the Artificial Quaternary chart is not about philosophy.
It’s about number!

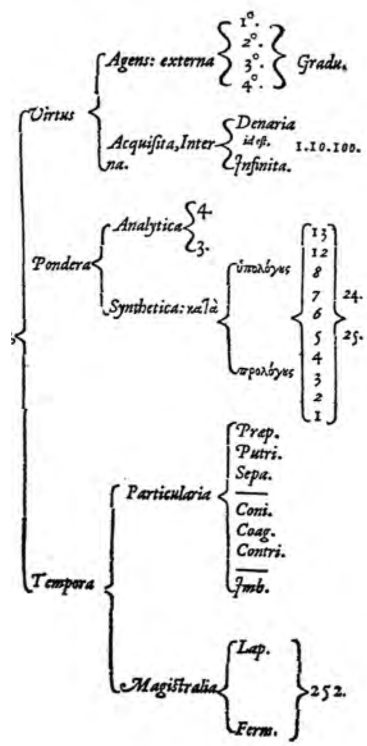
It’s a concise, cohesive synthesis about Dee’s view of how number works.
Nothing is superfluous.

It appears that Dee spent hours refining this chart so that it included only the bare essentials, which are somehow all integrated.

Concepts like “1, 2, 3, 2,” “4:3,” “252,” “the first 8 digits,” “12 and 13,” “24 and 25,” and “1, 10, 100, infinity” all work together in a system.

Thus, “step 1, step 2, step 3, step 4” must also be key players in this system.

What the heck are they?

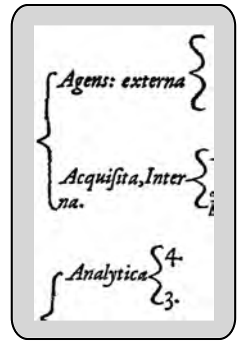


To help part in the clouds of obscurity about the “4 steps,”
Dee provides a few subtle clues to steer us towards the path of discovery.

Dee knew that an observant reader would see that four of the brackets in the chart were printed using a **hand-engraved** plate (they are the more curvy ones) and that all the rest were from **letterpress** type (the more linear ones).

This means that they were printed in separate passes through its printing press. Dee appears to have done this to cryptically highlight to the relatedness of the material enclosed in the 4 curvy brackets.

Seeing that one of the brackets contains the numerals “4 and 3,”
puts the expression “Quaternary rests in the Ternary”
into the reader’s mind.



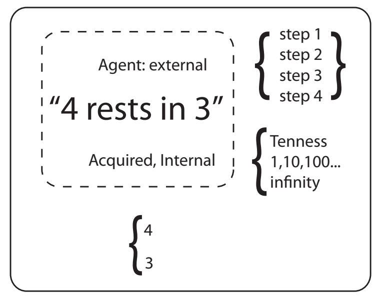
After scrutinizing the chart a bit more, the reader will suddenly see these words appearing in the letters which spell:

“Agens; externa” and “Acquisita, Interna.”
(“Agent: external” and “Acquired, Internal”)

As we’ve seen these words are an anagram for
“Quaternary rests in the Ternary.”

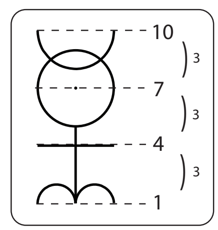
But what else might Dee mean by **external** and **internal**?)

Here, I have graphically simplified all this in English
to show more clearly what Dee is trying to express.

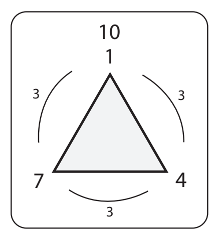


He seems to be saying that the idea
of “4 rest in 3” can be seen in two ways:

“**externally**” in “step 1, step 2, step 3, step 4”
and also “**internally**” in “1, 10, 100, to infinity.”



We’ve already seen how “4 rest in 3”
expresses the “symmetry of the Decad,”
in the Monas symbol.



And how the “symmetry of the Decad”
can be seen in a triangular arrangement.

This same proportioning applies to (what I refer to as)
the “symmetry of the centad” (10, 40, 70, 100),
and the “symmetry of the milliad” (100 400, 700, 1000).

If this “Denarian” stuff is “**internal**”.....

....then what is “**external**”?

One might suspect that Dee wants us to see results of:
“1, 2, 3, and 4” meets Tenness.
In other words, **10, 20, 30, and 40**.
But this particular sequence of multiples of 10 isn’t of any importance
anywhere else in the mathematical fabric of the *Monas*.

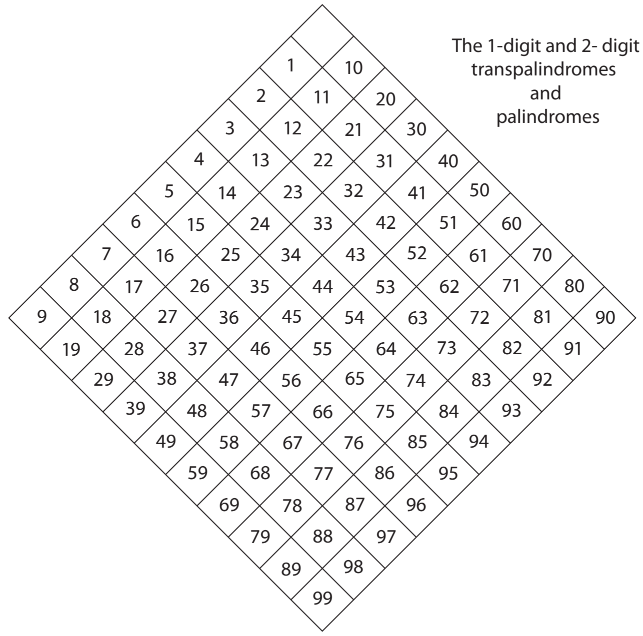
If these mysterious “4 steps” are so important,
they must be apparent in number itself.

So, I decided to return to the source.

I contemplated what else might be going on in the diamond-shaped chart of the 1-digit and 2-digit numbers.

The idea that
“1, 2, 3, 4 meets 10”
making “10, 20, 30, and 40”
is immediately apparent on the two top edges of the chart where the transpalindromic mates of 10, 20, 30, and 40 are 1, 2, 3 and 4.

But all this action takes place on the periphery of the chart, and it still doesn’t seem very important.



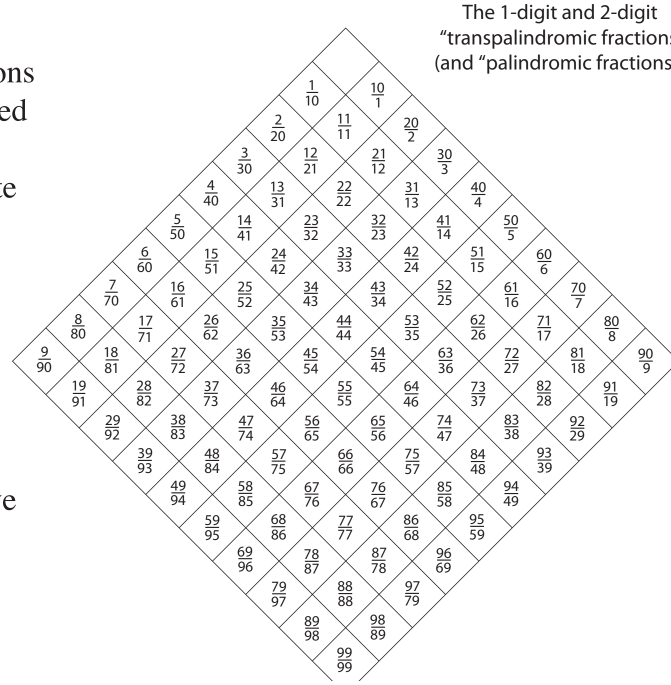
The 1-digit and 2- digit transpalindromes and palindromes

Being aware of Dee’s penchant for fractions and ratios, I decided to see what happened if each member of the chart was “compared” to its transpalindromic mate in the form of a “fraction.”

I call these
***“transpalindromic fractions*.”**

Note that with the palindomic 11 Wave (on the vertical spine), the fractions are all equivalent to 1.

The 1-digit and 2-digit “transpalindromic fractions” (and “palindromic fractions”)

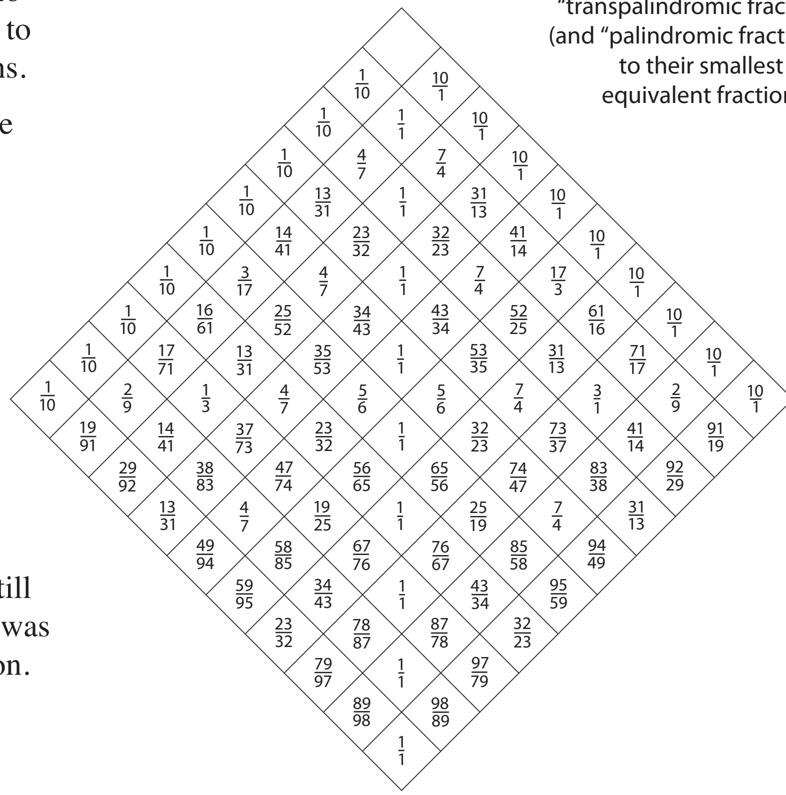


Suddenly, however, an already busy chart just got busier.

To simplify, I decided to reduce all the fractions to their lowest expressions.

(Many already include prime numbers and are thus not able to be reduced.)

Reducing the 1-digit and 2-digit "transpalindromic fractions" (and "palindromic fractions") to their smallest equivalent fractions

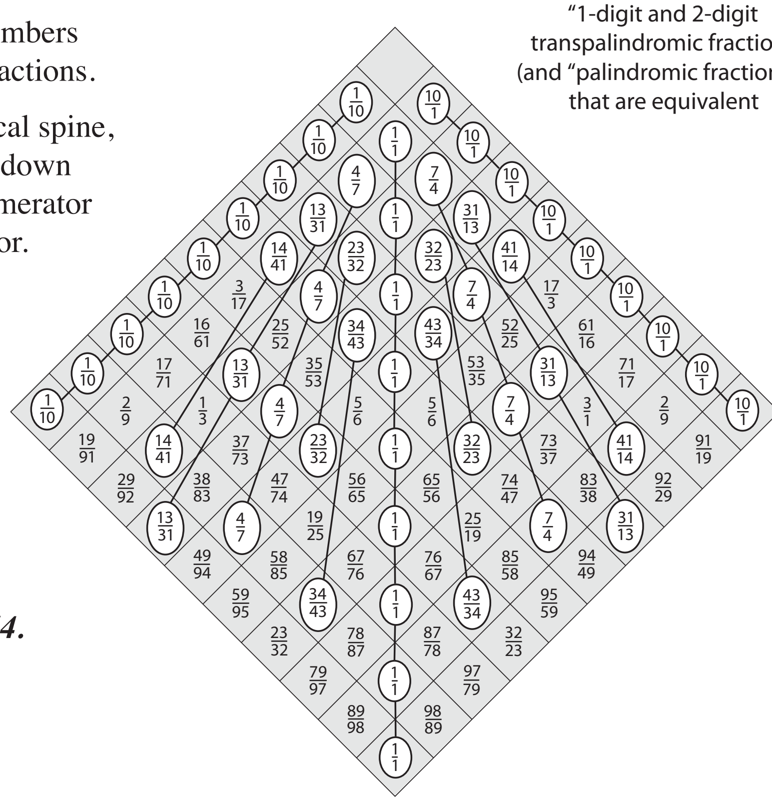


The resulting chart was still quite confusing, but there was a definite pattern going on.

To make things clearer, I used lines to connect all the members which reduce down to the same fractions.

Aside from all the 1/1's on the vertical spine, that there are ***only 8*** that reduce down to fractions with a single-digit numerator and a single-digit denominator.

Groupings of the "1-digit and 2-digit transpalindromic fractions" (and "palindromic fractions") that are equivalent



Interestingly, ***they are either 4/7 or 7/4.***

Let's look again at the original chart to see what numbers are responsible for this well-spun web.

At the top, there are lines that start at 1, 11, and 10.

To the left of the vertical spine, the lines start at 12, 13, 14, 23, and 34.

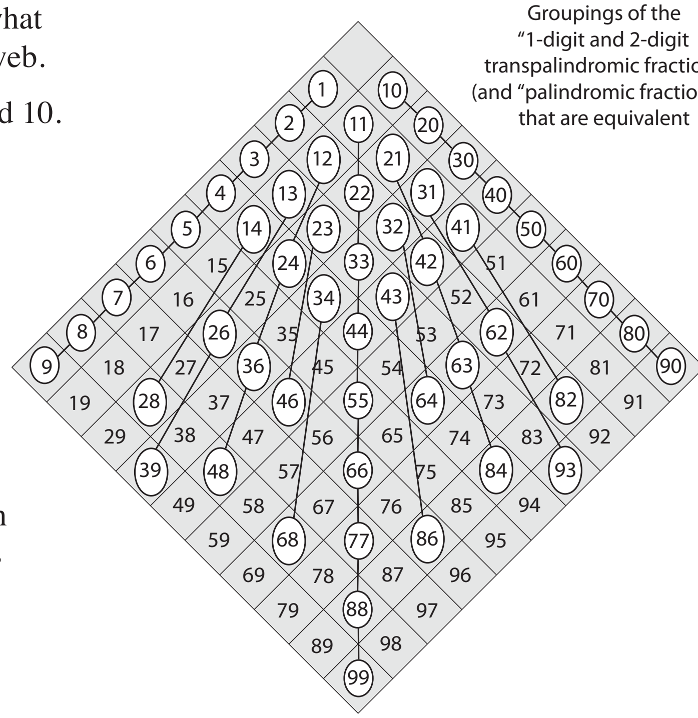
To the right of the spine, they start at 21, 31, 41, 32, and 43

(Notice that all these numbers are made from the digits 1, 2, 3, and 4).

But those members of the web which reduce down to those fractions with single-digit numerators and denominators "4/7 or 7/4" are very special!

They are 12, 24, 36, and 48 and their counterparts 21, 42, 63, and 84.

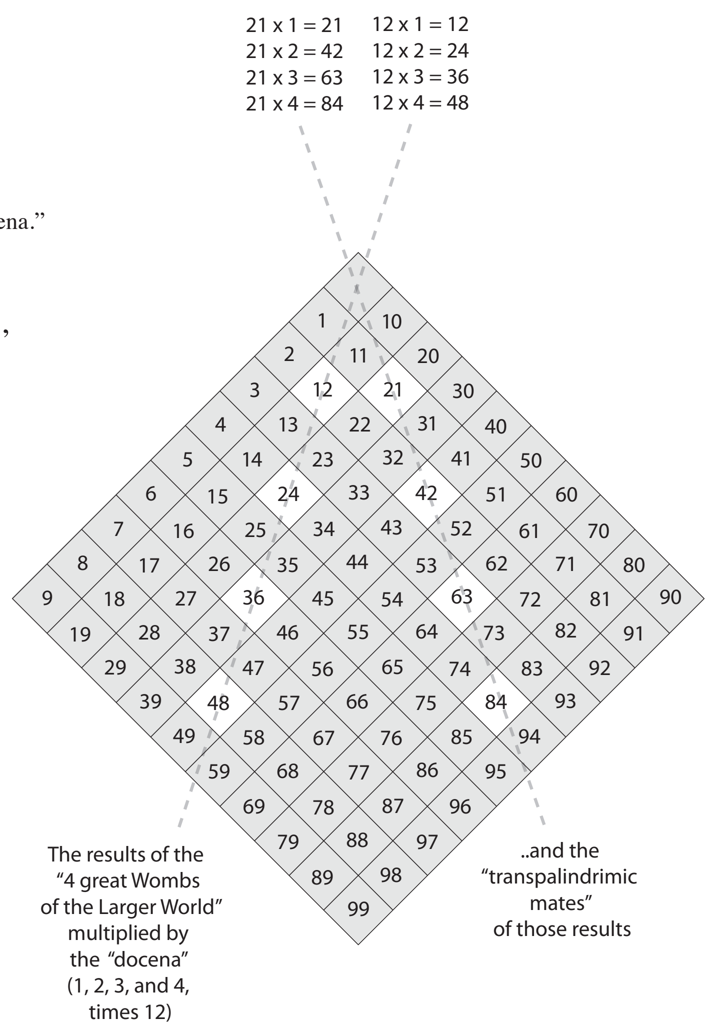
Only the numerators of the:
Groupings of the
"1-digit and 2-digit transpalindromic fractions"
(and "palindromic fractions")
that are equivalent



Suddenly Dee's "4 steps" have appeared before our very eyes!

It's not "1, 2, 3, 4 meets 10."
It's "1, 2, 3, 4 meets 12."

It's the "4 great Wombs of the Larger World meets the docena."



Notice how proudly symmetrical they are, cascading downwards from the "first possible transpalindromic pair" (12 and 21).

And also upward, to where they cross in the empty diamond at the top of the chart.

(which is one of the two diamonds I view as "zero-retrocity-one")

$$\frac{4}{7} = \frac{12}{21} = \frac{24}{42} = \frac{36}{63} = \frac{48}{84}$$

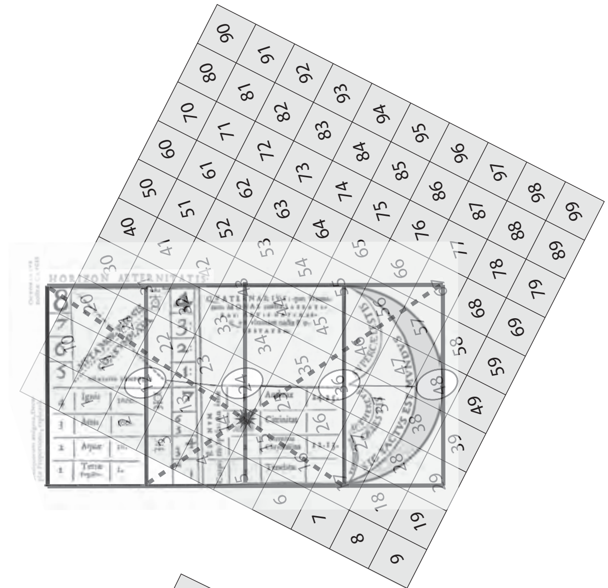
$$\frac{7}{4} = \frac{21}{12} = \frac{42}{24} = \frac{63}{36} = \frac{84}{48}$$

Amazingly, these four special pairs of transpalindromic mates are equal to the same thing, 4/7 (or 7/4).

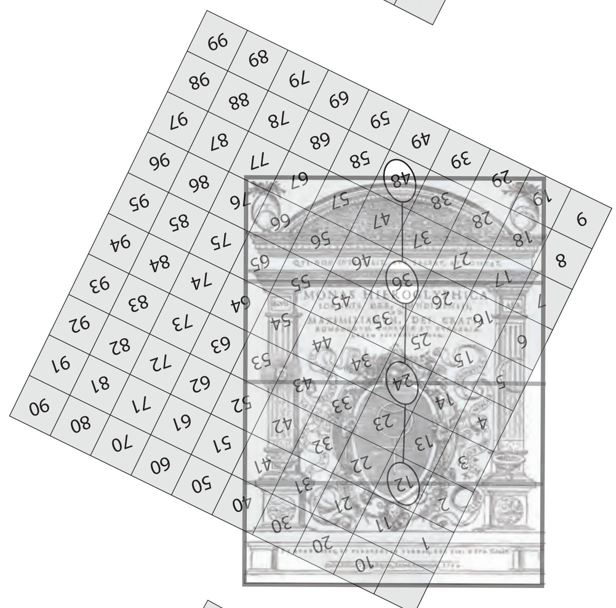
To show these correspondences visually,
let's superimpose the "quartering marks"
found when in 3 of **Dee's "graphic creations"**
are superimposed over the "4 steps,"
on the diamond-shaped chart.

Look very closely to see what
12, 24, 36, and 48 correspond to.

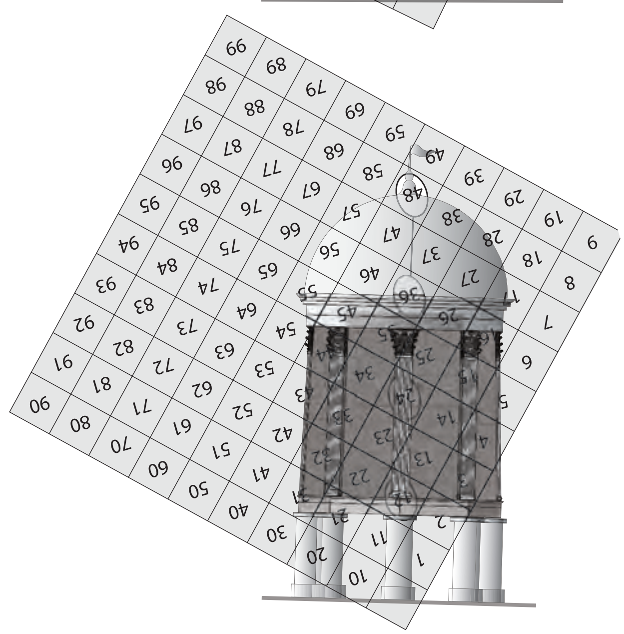
Here's the "ballooned 360
Thus the World
Was Created" chart
(which was made
with a 24 by 48 grid).

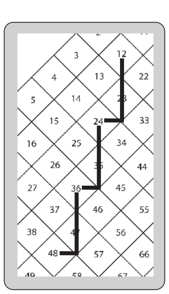


Here's the Title Page,
(which has a 48 by 36 grid).



And of the design plan
of the John Dee Tower,
which, excluding the finial was
48 feet tall and 24 feet wide.)





These overlaid charts look a little awkward,
being tilted and not symmetrical.
Because the chart is
10 boxes by 10 boxes square,
"12 and its multiples" are "off by two"
on each consecutive row.
Thus, to get from 12 to 24,
you have to move
in an L-shaped route,
like a knight on a chessboard.

But, 12, 24, 36, 48 are much more “organic and symmetrical” than “chart of single- and double-digits numbers” shows.

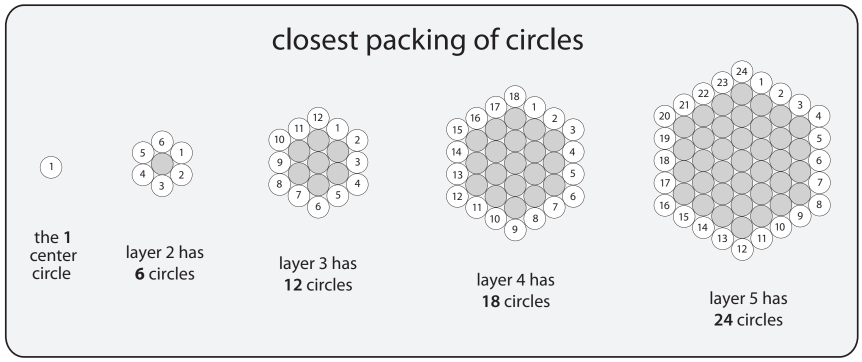
To see them really shine, let’s look at the number of “circles per layer” in that oh-so-natural “closest packing of circles” arrangement.

Layer 1 has 6 circles (around 1)

Layer 2 has 12 circles.

Layer 3 as 18 circles.

Layer 4 has 24 circles.

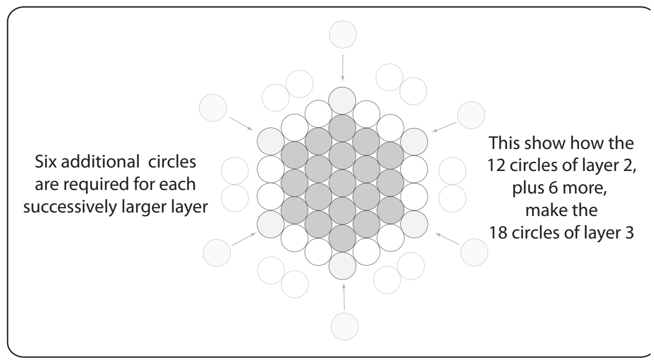


You can sense the pattern.

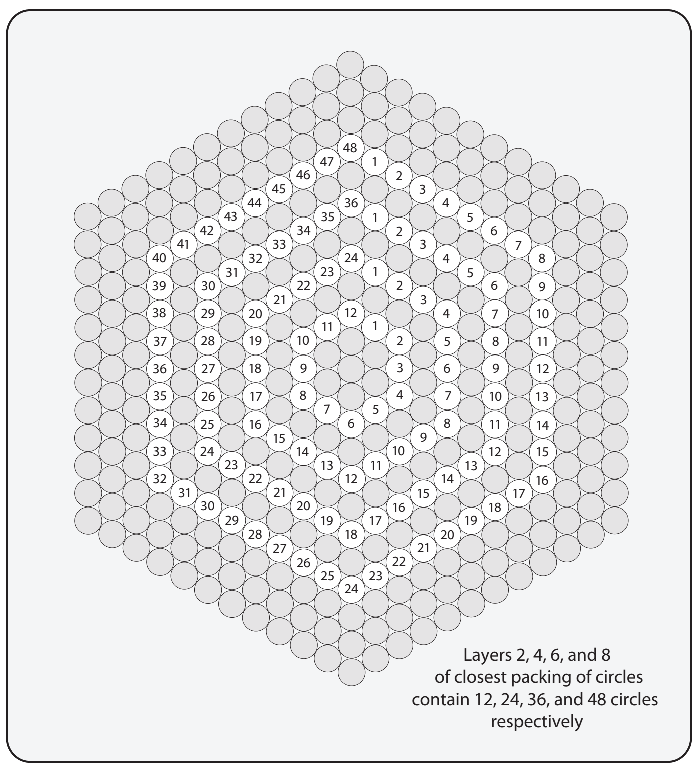
Every consecutive layer adds 6 circles.

It’s easiest to see them as being added to the 6 corners of the hexagonal snowflake which is expanding concentrically.

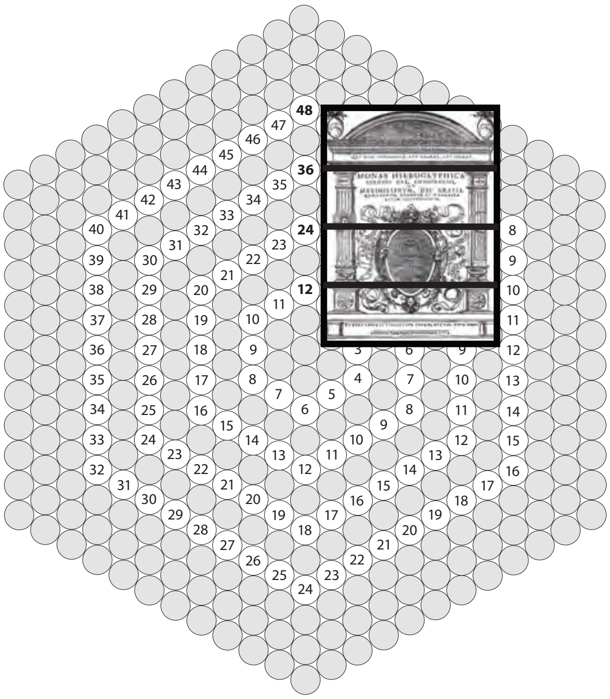
This example shows the 12 circles of layer 2, plus 6 more, making the 18 circles of layer 3.

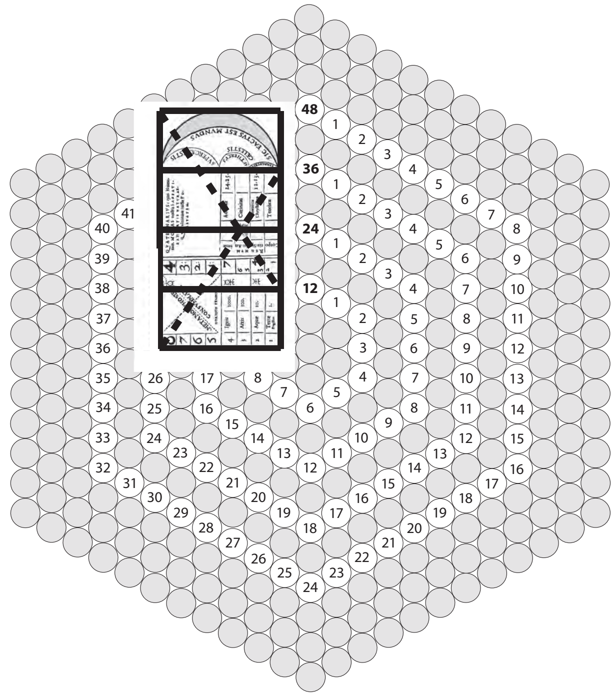


This larger snowflake shows how the “four 4 steps” (12, 24, 36, and 48) grow “organically and symmetrically.”

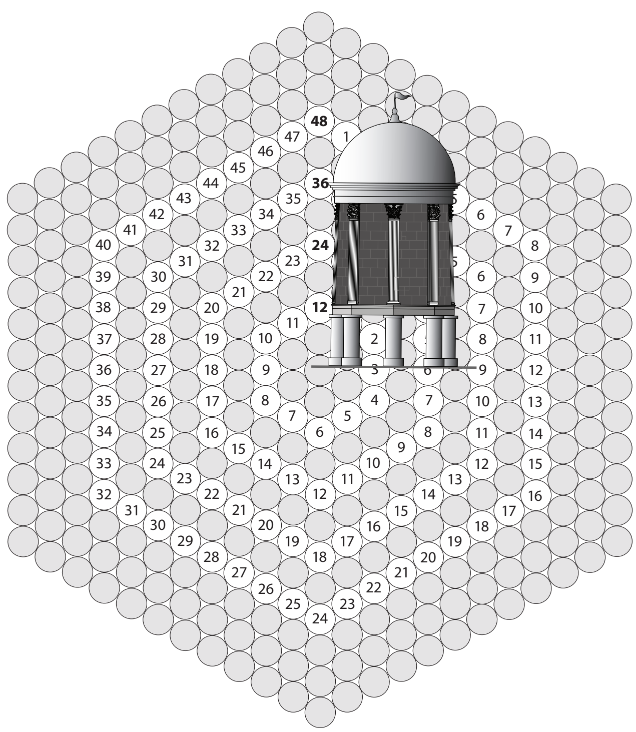


To emphasize this relationship, I've superimposed
two of Dee's drawings on this
"closest packing of circles"
chart





And here is the
John Dee Tower
on the same chart.



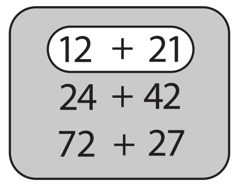
Dee cut these designs
"from the same cloth,"
mathematically and
philosophically.

An amazing connection between the "4 steps" and the Metamorphosis numbers

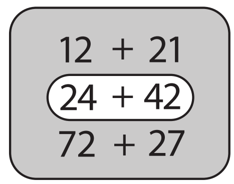
The first 2 Metamorphosis numbers,
12 and 24, ***are*** the first two "steps."
But what about 36 and 48?
How are they related to the Metamorphosis numbers?

Recall this powerful relationship.
The first three Metamorphosis numbers,
12, 24 and 72,
when added to their reflective mates,
are synchronous with
that key number of Consummata, 99.

$$12 + 21 = 33$$
$$24 + 42 = +66$$
$$72 + 27 = 99$$



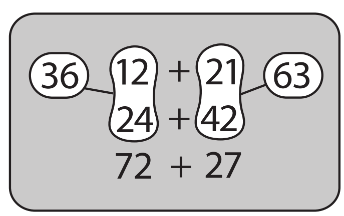
In this demonstation,
the first two "steps"
are easy to locate:



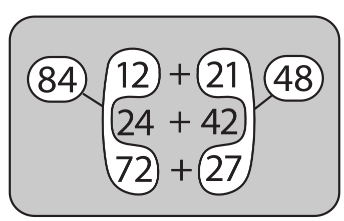
But what about 36 or 48?
Neither of them are
Metamorphohis numbers.

The Metamorphosis numbers	The first two "steps"
12	12 + 21
24	24 + 42
72	
360	
2520	
27720	

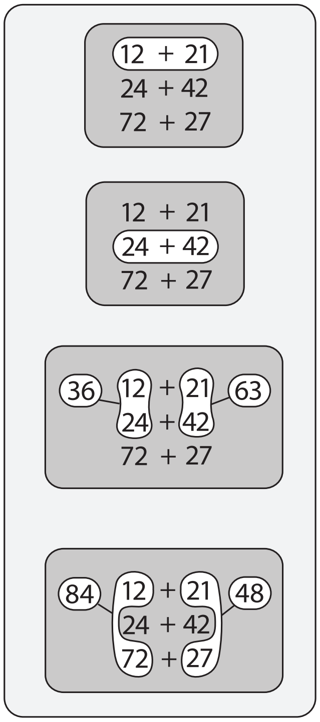
The third "step,"
36 and 63,
can be found quite simply:
12+24=36 and 21+42= 63

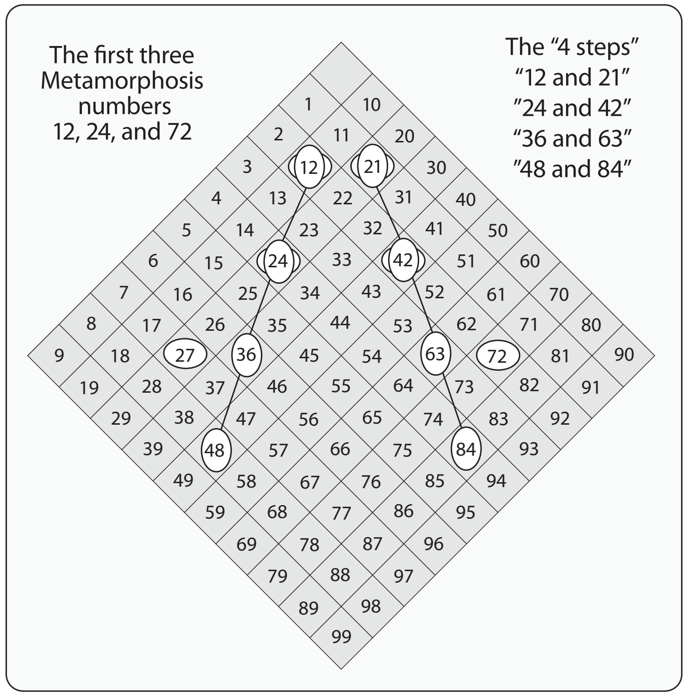


If that isn't cool enough,
check this out.
The fourth "step"is hidden
in here as well!
21+27=48 and 12+72=84.

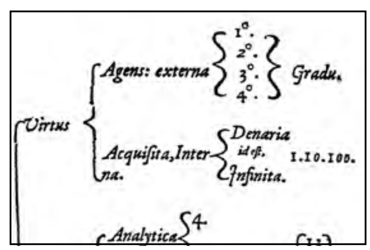


This is so thrilling, I must show it again,
in summary form,
and in the diamond-shaped chart
of 1-digit and 2-digit numbers.

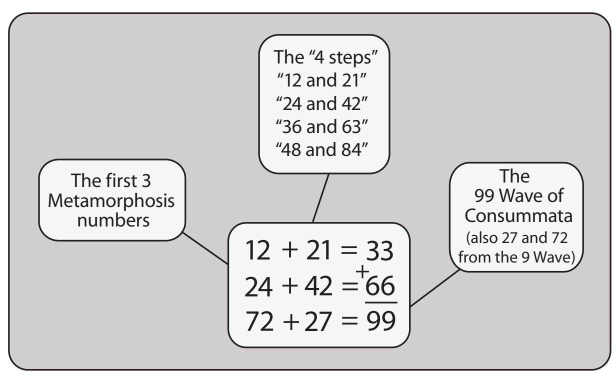




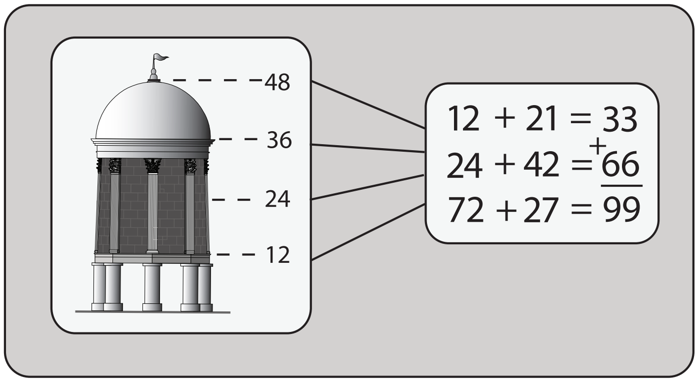
The "4 steps" integrate so nicely
with the first three Metamorphosis numbers,
its no wonder Dee included them in his
grand-summary Artificial Quaternary chart.



Metamorphosis, Consummata, and the "4 steps"
are all incorporated in this
concise mathematical depiction
of retrocity at work.

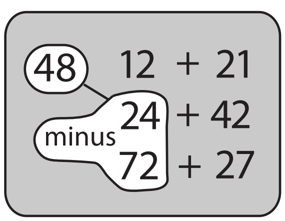


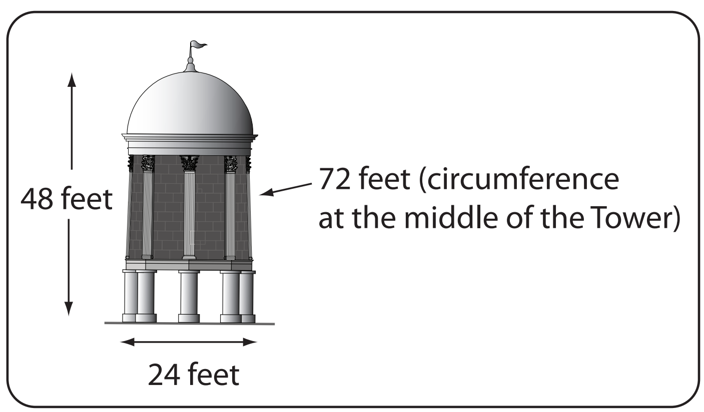
This also means is that the
powerful demonstration of synchrony between
Metamorphosis, Consummata, and the "4 steps"
is related to the heights of key architectural features
on the **John Dee Tower**!



As if this isn't enough, there is yet another connection between this "powerful display of synchronicty"and the Tower measurements. 72 – 24 =48.

48 feet is the height of the stone and mortar part of the Tower.
24 is the Tower's mean width as it is in the 2:1 proportion.
72 is the circumference of the tower's cylinder, halfway up.

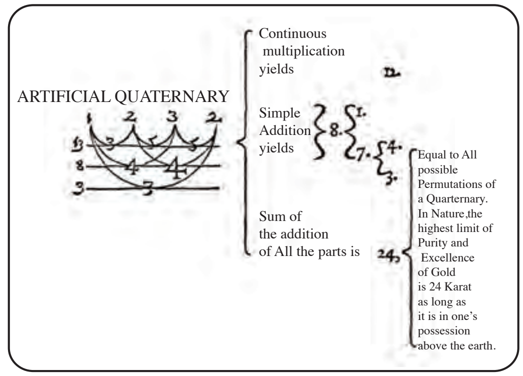




How can I be certain that Dee actually thought this way?
He tells us.
As he writes regarding the "Mathematical Arte" of Architecture in his 1570 *Preface to Euclid*:

"**By Arithmetike, the charges of Buildings are summed together.
The measures are expressed, and the hard questions of Symmetries,
are by Geometrical Meanes and Methods discoursed on, & etc**."
(Dee, *Preface*, p. diij,verso)

Dee "hidden gold" in the Monas Hieroglyphica

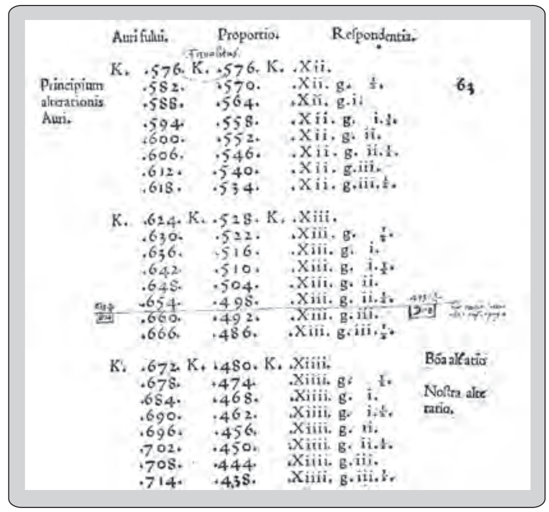


In the text following the Artificial Quaternary result of 24, Dee reminds us that the "**highest limit of Purity and Excellence of Gold is 24 Karat**."

As Dee's *Monas Hieroglyphica* has exactly 24 Theorems, the implication is that his book is pure gold.

But, even if gold has fewer than 24 carats, it's still considered gold.(You won't find a jeweler advertising 14 Karat "almost gold" necklaces.)

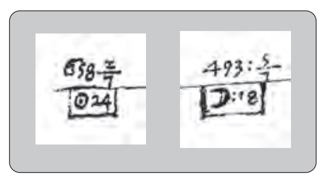
As we've just seen, the essence of the "4 steps" is the fraction 4/7.



	Auri fului.	Proportio.	Respondentia.	
	K. .576.	K. .576.	K. .Xii.	
Principium alterationis Auri.	.582.	.570.	.Xii. g. ½.	63
	.588.	.564.	.Xii. g.i.	
	.594.	.558.	.Xii. g. i.½.	
	.600.	.552.	.Xii. g. ii.	
	.606.	.546.	.Xii. g. ii.½.	
	.612.	.540.	.Xii. g.iii.	
	.618.	.534.	.Xii. g.iii.½.	
	K. .624.	K. .528.	K. .Xiii.	
	.630.	.522.	.Xiii. g. ½.	
	.636.	.516.	.Xiii. g. i.	
	.642.	.510.	.Xiii. g. i.½.	
	.648.	.504.	.Xiii. g. ii.	
	.654.	.498.	.Xiii. g. ii.½.	
	.660.	.492.	.Xiii. g.iii.	
	.666.	.486.	.Xiii. g.iii.½.	
	K. .672.	K. .480.	K. .Xiiii.	Bōa alťatio
	.678.	.474.	.Xiiii. g. ½.	
	.684.	.468.	.Xiiii. g. i.	Noſtra alteratio.
	.690.	.462.	.Xiiii. g. i.½.	
	.696.	.456.	.Xiiii. g. ii.	
	.702.	.450.	.Xiiii. g. ii.½.	
	.708.	.444.	.Xiiii. g.iii.	
	.714.	.438.	.Xiiii. g.iii.½.	

And you'll recall that Dee's annotation in his copy of Pantheus' *Voarchadumia* ("on Gold Refining") denotes a purity of 24 parts gold to 18 part silver. (or 658 2/7 carob beans of gold to 493 5/7 carob beans of silver) (which is 4/7 to 3/7) (which is 57 1/7% to 42 6/7%)

Four sevenths of 24 is approximately 13 7/10. You can see on Pantheus' chart that Dee's hand-drawn line marks 13 7/10 karat gold. (The line is just below "xiii. g. ii. ½." which is "13 karats" plus "2 1/2 out of 4" karats)



So, if Dee's twenty-four-Theorem-*Monas* is pure gold, we might expect to find a treasure 4/7 of the way into his book.

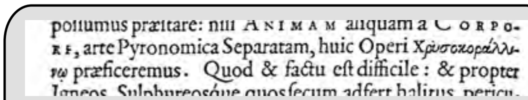
Well, this measurement is hard to determine because the 24 Theorems are all different lengths. (For example, Theorem 1 is only one sentence long and Theorem 23 is over 8 pages long)

I conjectured Dee might have considered his Theorems to be like a "whole numbers," and decided to investigate Theorem 13. Sure enough, about 7/10 of the way into the Theorem, I struck gold!

Dee uses the Greek word Χρυσοχοράλλινω (*Xrysoxorallino*) in the expression "Operi *Xrysoxorallino*" or "Golden Work."

Certainly this may all be coincidental. However, *Xrysoxorallino* is the only Greek word on the entire page (page 14 verso) and it stands out because it's written in italics.

Knowing Dee's love for the number 24, for gold, for the fraction 4/7, and his penchant for hiding things, it certainly looks like Dee's hidden cache of gold to me.



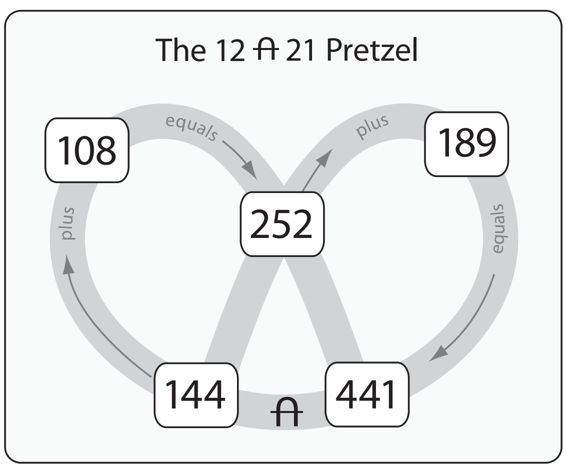
poſſumus præſtare: niſi ANIMAM aliquam a CORPO-
RE, arte Pyronomica Separatam, huic Operi Χρυσοχοράλλι-
νω præficeremus. Quod & factu eſt difficile: & propter
Igneos Sulphureosque quos ſecum adfert halitus, pericu-

the phrase "Operi *Xrysoxorallino*"
(Golden Work), in Theorem 13

The "Magistral Number" meets the "Quaternary Rests in the Ternary" in the 12 and 21 Pretzel

Two major themes of the *Monas* are the
"Quaternary rests in the Ternary,"
(or the 3:4 "part to part" ratio)
and Dee's Magistral number "252."

They seem unrelated, but they are
actually quite entwined in this pretzel.



The good way to start exploring the pretzel
is by first painting a clear picture of the
multiples of the 3: 4 "part to part" ratio.

This chart starts off with the 3:4 “part to part” ratio
and then shows its multiples. (All the way up to the 3:4 ratio times 66.)

The 3:4 “part to part” ratio... ...and its multiples

	3	4	7	(x 2)	6	8	14	(x 3)	9	12	21
(x 4)	12	16	28	(x 5)	15	20	35	(x 6)	18	24	42
(x 7)	21	28	49	(x 8)	24	32	56	(x 9)	27	36	63
(x 10)	30	40	70	(x 11)	33	44	77	(x 12)	36	48	84
(x 13)	39	52	91	(x 14)	42	56	98	(x 15)	45	60	105
(x 16)	48	64	112	(x 17)	51	68	119	(x 18)	54	72	126
(x 19)	57	76	133	(x 20)	60	80	140	(x 21)	63	84	147
(x 22)	66	88	154	(x 23)	69	92	161	(x 24)	72	96	168
(x 25)	75	100	175	(x 26)	78	104	182	(x 27)	81	108	189
(x 28)	84	112	196	(x 29)	87	116	203	(x 30)	90	120	210
(x 31)	93	124	217	(x 32)	96	128	224	(x 33)	99	132	231
(x 34)	102	136	238	(x 35)	105	140	245	(x 36)	108	144	252
(x 37)	111	148	259	(x 38)	114	152	266	(x 39)	117	156	273
(x 40)	120	160	280	(x 41)	123	164	287	(x 42)	126	168	294
(x 43)	129	172	301	(x 44)	132	176	308	(x 45)	135	180	315
(x 46)	138	184	322	(x 47)	141	188	329	(x 48)	144	192	336
(x 49)	147	196	343	(x 50)	150	200	350	(x 51)	153	204	357
(x 52)	156	208	364	(x 53)	159	212	371	(x 54)	162	216	378
(x 55)	165	220	385	(x 56)	168	224	392	(x 57)	171	288	399
(x 58)	174	232	406	(x 59)	177	236	415	(x 60)	180	240	420
(x 61)	183	244	427	(x 62)	186	248	434	(x 63)	189	252	441
(x 64)	192	256	448	(x 65)	195	260	455	(x 66)	198	264	462

The top row of the chart

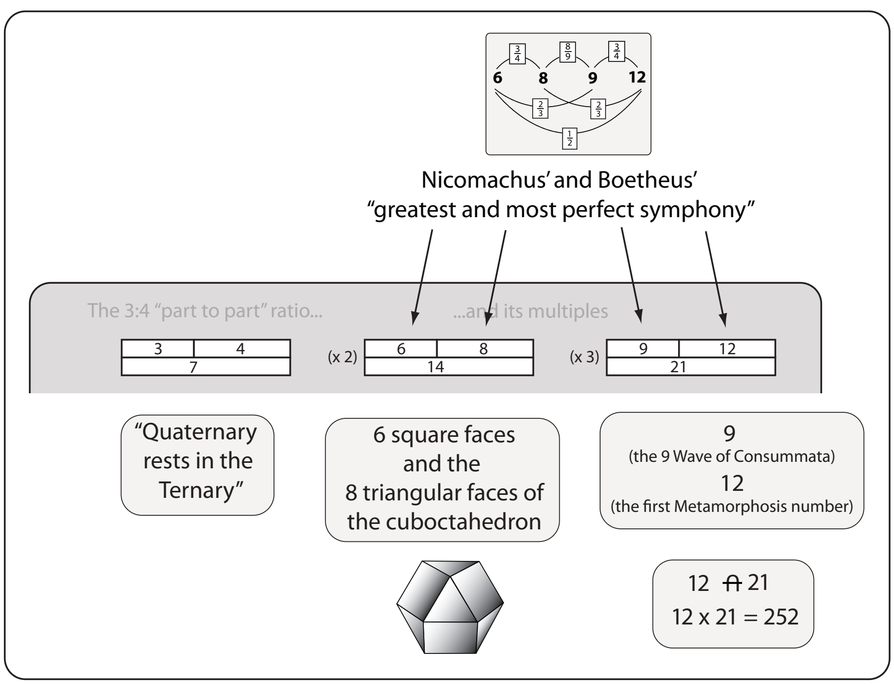
Even across the top row we can find concepts which Dee has integrated into the *Monas Hieroglyphica*.

In the second set of boxes, the "6:8 ratio" expresses the 6 square faces and the 8 triangular faces of the 14 -sided cuboctahedron.

In the third set of boxes, we can find the "first transpalindromable pair," numbers "12 and 21," which multiply to 252.
We also find 9 (of the 9 Wave of Consummata)
being "compared" to 12 (the first member of the Metamorphosis sequence).

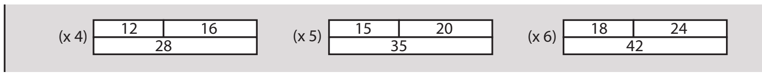
On the tops of these two sets of boxes we can also find the numbers 6, 8, 9, and 12, which Nicomachus and Boethius referred to as the "greatest and most perfect harmony."

These 4 harmonious numbers express not only diatesseron (3:4), but also diapente, diapason and epogdous (2:3, 1:2, and 8:9)

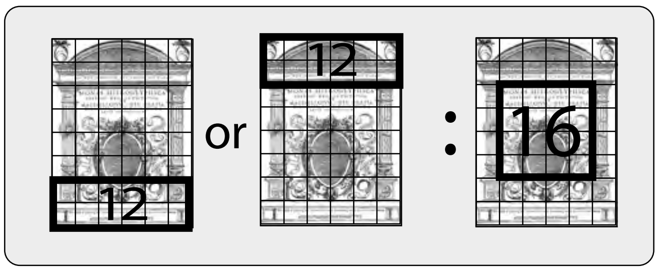


The second row of the chart

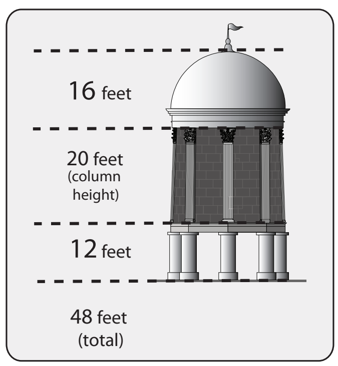
At the beginning of the next row, "3:4 times 4" results in 12:16.



On the Title Page, this is the ratio between the area contained
in either the "Foundation area" or the "Dome area," (12 units)
compared to area of the "Theater" between the columns (16 units).

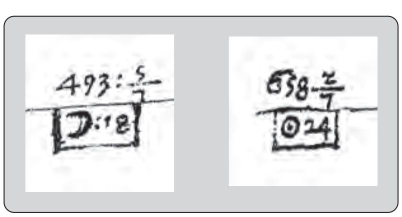


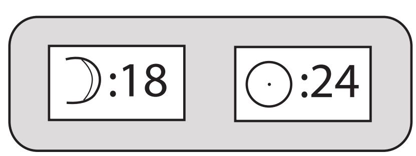
In my reconstruction, the Tower, the ratio of the "pillars plus the pillar entablature" (12 feet) to the "dome plus the column entablature" (16 feet) is 12/16.
(When the height of the 20-foot-tall columns is added, the total is 48 feet.)



Next, the is "3:4 ratio times 5" makes 15:20 ratio is not very noteworthy.

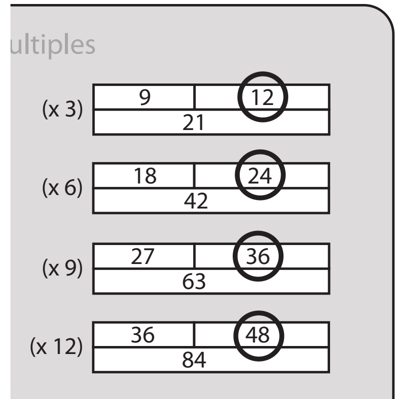
But, following that, the "3: 4 ratio times 6" makes 18:24 is significant.
This is the ratio Dee wrote in the margin of his copy of Pantheus' *Voarchadeumia*.It representing one of his favorite ratios of "silver to gold" (which he illustrated with as Sun to Moon).





Next, let's focus on just the upper-right corner of the chart.
In the upper right part of each of these boxes is are
Dee's "4 steps," **12, 24, 36, 48**,
as well as their reflective mates **21,42,63, and 84**!

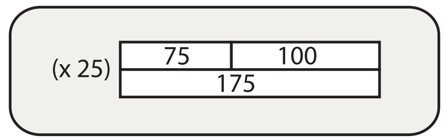
(Note that they result from the 3:4 ratio
times 3, 6, 9, and 12, respectively.)



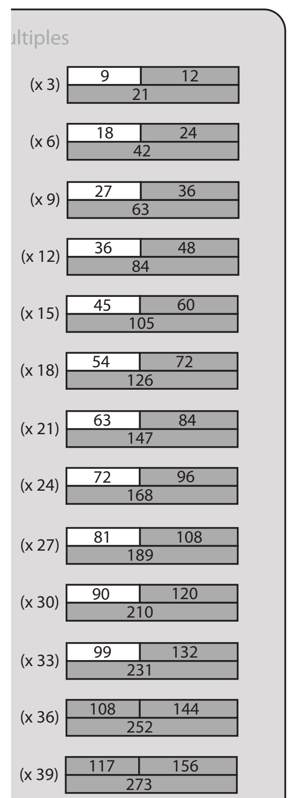
Another noteworthy member of this chart
is the "3:4 ratio multiplied by 25."

Dee use this example in the *Preface to Euclid*,
when explaining the Lex Falcidia,
the Roman inheritance laws:

"**For, what proportion, 100 hath to 75:
the same hath 17 1/6 to 12 6/7:
which is Sesquitertia:
that is, as 4, to 3. which make 7**"
(Dee, *Preface to Euclid*, p. a.j. verso)



Also notice that the members of the
9 Wave can be found among the boxes:



Here I have encircled only the 4/7 part of these 3:4 “part to part” ratios.
I’ve also provided the “sum of the their numerator and denominator” in bold type.

Sum of the numerator and the denominator in the 4/7 fraction...(and its multiples)

The 3:4 “part to part” ratio... ...and its multiples

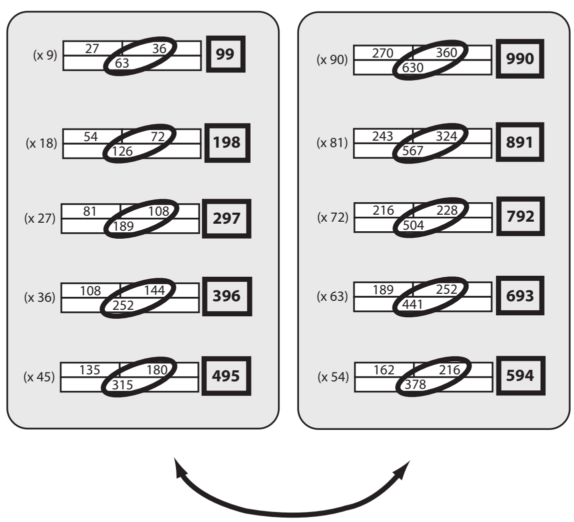
Multiple	3 part	4 part	7 (whole)	Sum
	3	4	7	**11**
(x 2)	6	8	14	**22**
(x 3)	9	12	21	**33**
(x 4)	12	16	28	**44**
(x 5)	15	20	35	**55**
(x 6)	18	24	42	**66**
(x 7)	21	28	49	**77**
(x 8)	24	32	56	**88**
(x 9)	27	36	63	**99**
(x 10)	30	40	70	**110**
(x 11)	33	44	77	**121**
(x 12)	36	48	84	**132**
(x 13)	39	52	91	**143**
(x 14)	42	56	98	**154**
(x 15)	45	60	105	**165**
(x 16)	48	64	112	**176**
(x 17)	51	68	119	**187**
(x 18)	54	72	126	**198**
(x 19)	57	76	133	**209**
(x 20)	60	80	140	**220**
(x 21)	63	84	147	**231**
(x 22)	66	88	154	**242**
(x 23)	69	92	161	**253**
(x 24)	72	96	168	**264**
(x 25)	75	100	175	**275**
(x 26)	78	104	182	**286**
(x 27)	81	108	189	**297**
(x 28)	84	112	196	**308**
(x 29)	87	116	203	**319**
(x 30)	90	120	210	**330**
(x 31)	93	124	217	**341**
(x 32)	96	128	224	**352**
(x 33)	99	132	231	**363**
(x 34)	102	136	238	**374**
(x 35)	105	140	245	**385**
(x 36)	108	144	252	**396**
(x 37)	111	148	259	**407**
(x 38)	114	152	266	**418**
(x 39)	117	156	273	**429**
(x 40)	120	160	280	**440**
(x 41)	123	164	287	**451**
(x 42)	126	168	294	**462**
(x 43)	129	172	301	**473**
(x 44)	132	176	308	**484**
(x 45)	135	180	315	**495**
(x 46)	138	184	322	**506**
(x 47)	141	188	329	**517**
(x 48)	144	192	336	**528**
(x 49)	147	196	343	**538**
(x 50)	150	200	350	**550**
(x 51)	153	204	357	**561**
(x 52)	156	208	364	**572**
(x 53)	159	212	371	**583**
(x 54)	162	216	378	**594**
(x 55)	165	220	385	**605**
(x 56)	168	224	392	**616**
(x 57)	171	288	399	**627**
(x 58)	174	232	406	**638**
(x 59)	177	236	415	**649**
(x 60)	180	240	420	**660**
(x 61)	183	244	427	**671**
(x 62)	186	248	434	**682**
(x 63)	189	252	441	**693**
(x 64)	192	256	448	**704**
(x 65)	195	260	455	**715**
(x 66)	198	264	462	**726**

Following the lead of the first box (in which 7+4 = 11),
all of the subsequent boxes are "multiples of 11."
(You'll recognize the first grouping of them ,11, 22, 33… 99, as the vertical spine
of the "diamond-shaped chart of single and double digit numbers."

Along the right edge, I've highlighted another important set,
the 99 Wave (99, 198, 297…).
If continued, this chart would also include 792, 891, 990, 1089 (which is 11 x 99).
(As we shall see, it also includes 1089 Wave, the 10890 Wave and beyond).

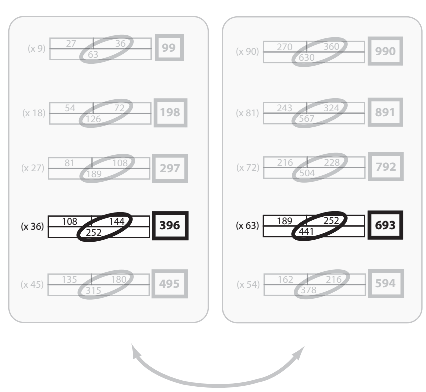
Starting from the strange fraction 4/7,
suddenly we have an expression of **Consummata**!

Sums of the numerator and the denominator of multiples of the 4/7 fraction which are members of the "99 wave"



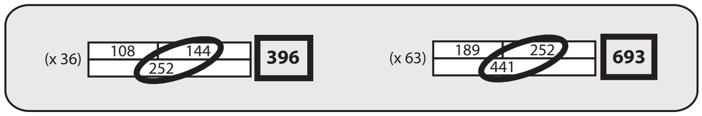
Studying this summary
of the 99 wave,
(arranged as
transpalindromic pairs),
we find an even more
exciting result.
Up pops all the
key numberts
of Marshall's
"12 and 21 pretzel."

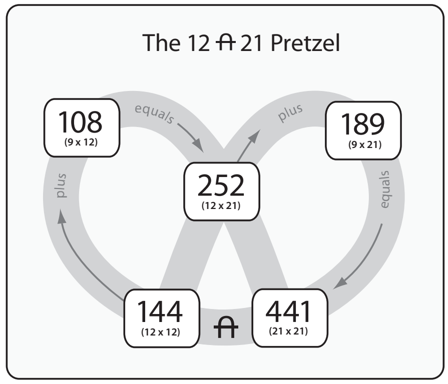
Sums of the numerator and the denominator of multiples of the 4/7 fraction which are members of the "99 wave"



There is so much
"retrocity"
going on here,
you can
practically
smell its
sweet fragrance
in the air.

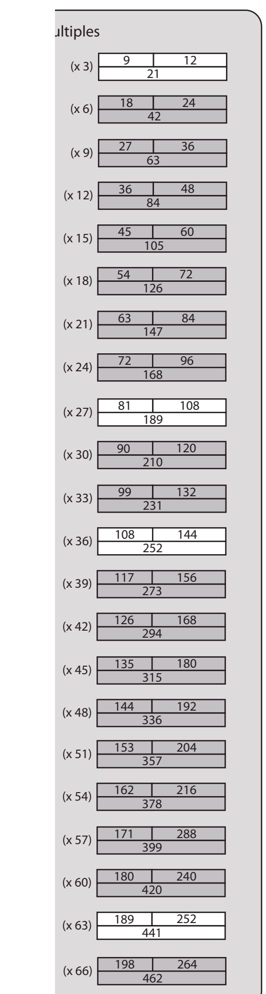
All the numbers of Marshall's " Syndex Pretzel"





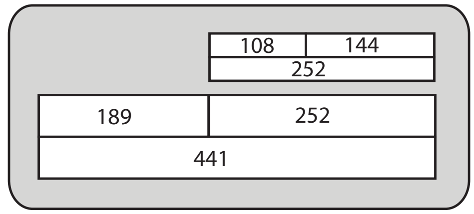
A simple way to describe the wonder of this pretzel is:

The squares of the first transpalindromable numbers (12 and 21) are also transpalindromes(144 and 441).

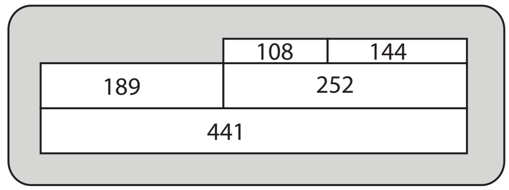


All the key numbers of the pretzel are in the chart,
9,12,21,
144, 441
108,189,
and of course 252

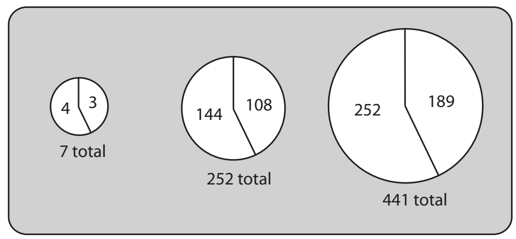
As the 252 in each these boxes
is really the "same number"...



...we might rescale the boxes
and combine them like this:



Another way to look
at the this interrelationship
is using a circular pie chart
(with seven pieces of pie in each chart).



The whole pretzel revolves around 252!

In Theorem 17 of the *Monas*,
Dee combines various numerical descriptions
of the Cross to sum up to the Magistral number
(20 + 200 + 10 + 21 + 1 = 252).
He adds:

"There are two other logical ways that we can draw forth this Number from our premises.

For the sake of brevity, we recommend these reasons be rooted out by Beginning Kabbalists.

The various artificial productions of this Magistral Number are also worthy of the Consideration of Philosophers."

Those "two logical ways" are most likely the two
loops of the pretzel, both of which involve 252.

The fabulous 252 dances with 144 and 441
in two ways, each of which expresses 4/7.

$$\frac{4}{7} = \frac{144}{252} \qquad \frac{4}{7} = \frac{252}{441}$$

Let's explore this a bit further by summing the
numerator and denominator of these fractions:

$$\begin{array}{r} 144 \\ \underline{252} \\ 396 \end{array} \qquad \begin{array}{r} 252 \\ \underline{441} \\ 693 \end{array}$$

Let's rephrase these results
in terms of "252."

$$\frac{7}{11} \times 396 = 252$$

$$\frac{4}{11} \times 693 = 252$$

Type 7/11 or 4/11 into your hand calculator
and you'll get very interesting pair
of repeating decimals:
.636363…
and .363636…

(Again, Dee might not have used decimal numbers
like this, but if he simply divided "11 into 700,000"
or "11 into 400,000" he could certainly
see what was going on.)

$$.636363\ldots \times 396 = 252$$

$$.363636\ldots \times 693 = 252$$

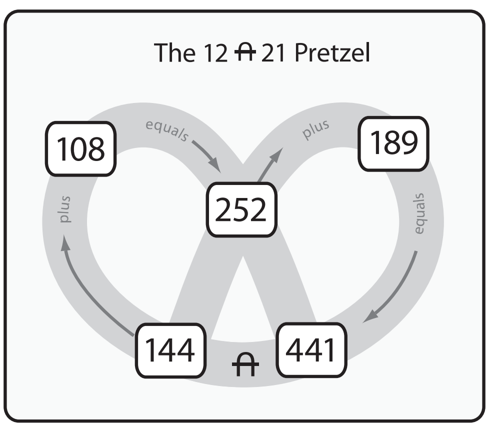
Seeing 252 made in two ways, with the numbers involving only the digits 3, 6, and 9 is pretty remarkable.

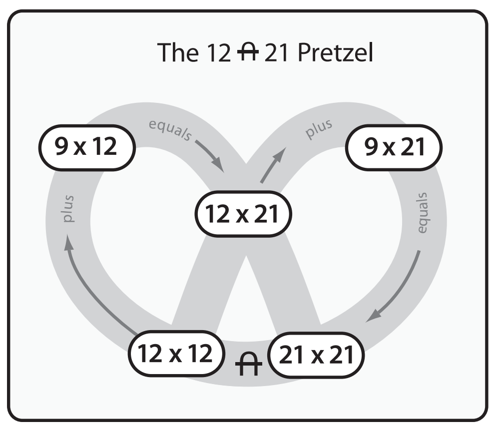
Dee was aware of the "camaraderie" of 3, 6, and 9, as in
Theorem 21 he illustrates 3 orientations of the Aries symbol,
which he claims might be seen as 6 things (6 half-circles).

But he adds 3 and 6 "summed together make 3 x 3."

[Dee's way of expressing nine without actually saying it]

To summarize, look at the pretzel as a roadmap.
From 144, take Route 108 to get to 252.This route its involves numbers 9 and 12.
Then from 252 take Route 189 around to 441. This route its involves with 9 and 21.





This is a taste of Metamorphosis (the first metamorphosis number being 12)
meeting Consummata (the 9 of the 9 Wave).

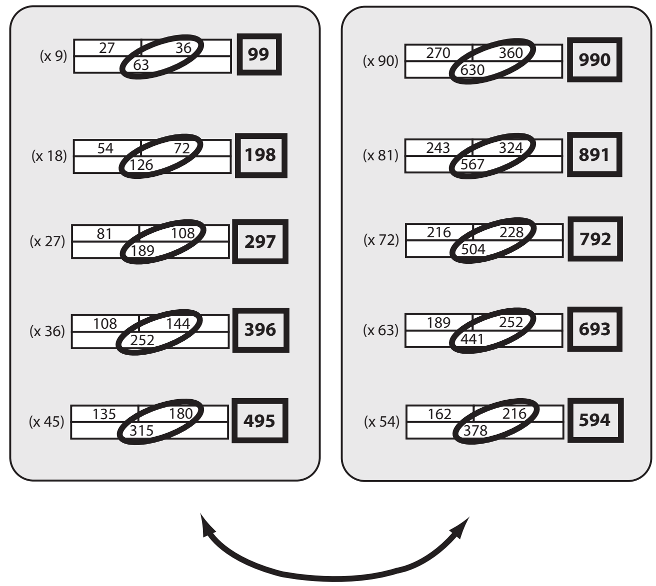
But in the chart we can also find Consummata's
99 Wave, 1089 Wave, 10890 Wave,... and beyond...

The 99 Wave

We've seen that if the chart was extended
we would find following multiples,
which comprise the 99 Wave,
or "Consummata of the
3-digit number range":
(99, 198, 297, 396… 990).

Next, let's find the results of the
99 Wave **times** the 3:4 ratio.

Sums of the numerator and the denominator of multiples of the 4/7 fraction which are members of the "99 wave"



The 1089 Wave

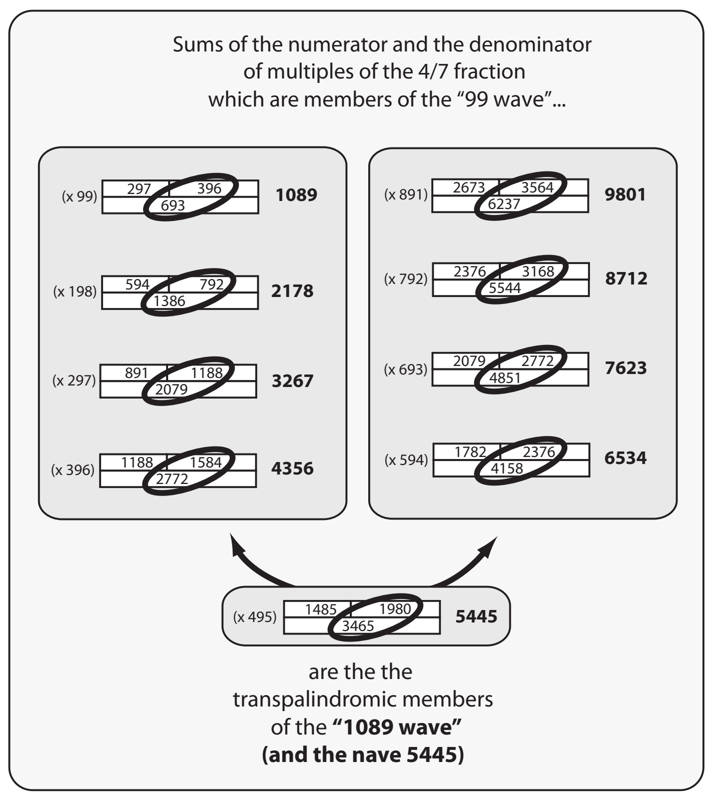
Adding the "4" (part)
and the "7" (whole)
sections of these boxes
generates the 1089 Wave,
"Consummata of the
4-digit number range":
(1089, 2178, 3267, 4356,
5445 (a nave is born, more on this later),
6534, 7623, 8712, 9801)

In the position where 252
was located in the chart above,
here we find the number 2772.

That's because 252 x 11 = 2772.

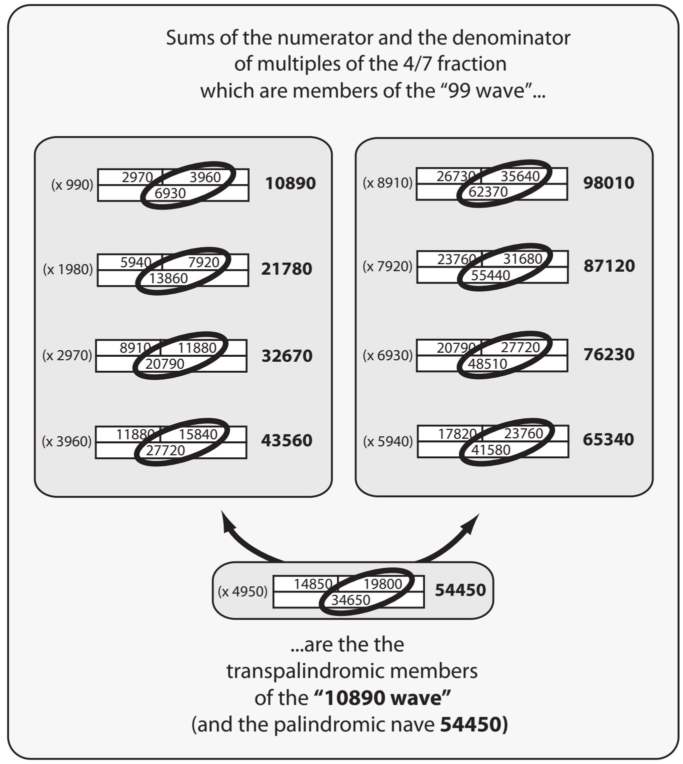
Notice the similarity between this and
with how Metamorphosis number 27720
is derived from its predecessor 2520.

12 x 2 = 24
24 x 3 = 72
72 x 5 = 360
360 x 7 = 2520
2520 x 11 = 27720



The 10890 Wave

Next, starting with the
"3: 4 ratio times 990,"
we enter into the 10890 Wave,
or "Consummata of the
5-digit numbers":
(10890, 21780, 32670…98010)



and beyond...

And the pattern continues indefinitely.

As far as you want to go, Consummata continues to be involved with the 4/7 ratio.

The 3:7 ratio in the chart

This time, let's sum the numerator and the denominator of the "3/7 fraction and its multiples." As 3 +7 = 10, all the results (shown in **bold** type) are multiples of 10.

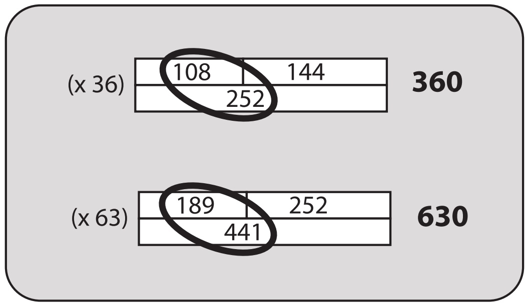
Sum of the numerator and the denominator in the 3/7 fraction...(and its multiples)

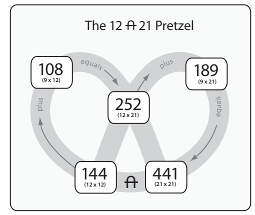
The 3:4 "part to part" ratio... ...and its multiples

	3	4	7	**10**	(x 2)	6	8	14	**20**	(x 3)	9	12	21	**30**
(x 4)	12	16	28	**40**	(x 5)	15	20	35	**50**	(x 6)	18	24	42	**60**
(x 7)	21	28	49	**70**	(x 8)	24	32	56	**80**	(x 9)	27	36	63	**90**
(x 10)	30	40	70	**100**	(x 11)	33	44	77	**110**	(x 12)	36	48	84	**120**
(x 13)	39	52	91	**130**	(x 14)	42	56	98	**140**	(x 15)	45	60	105	**150**
(x 16)	48	64	112	**160**	(x 17)	51	68	119	**170**	(x 18)	54	72	126	**160**
(x 19)	57	76	133	**190**	(x 20)	60	80	140	**200**	(x 21)	63	84	147	**210**
(x 22)	66	88	154	**220**	(x 23)	69	92	161	**230**	(x 24)	72	96	168	**240**
(x 25)	75	100	175	**250**	(x 26)	78	104	182	**260**	(x 27)	81	108	189	**270**
(x 28)	84	112	196	**280**	(x 29)	87	116	203	**290**	(x 30)	90	120	210	**300**
(x 31)	93	124	217	**310**	(x 32)	96	128	224	**320**	(x 33)	99	132	231	**330**
(x 34)	102	136	238	**340**	(x 35)	105	140	245	**350**	(x 36)	108	144	252	**360**
(x 37)	111	148	259	**370**	(x 38)	114	152	266	**380**	(x 39)	117	156	273	**390**
(x 40)	120	160	280	**400**	(x 41)	123	164	287	**410**	(x 42)	126	168	294	**420**
(x 43)	129	172	301	**430**	(x 44)	132	176	308	**440**	(x 45)	135	180	315	**450**
(x 46)	138	184	322	**460**	(x 47)	141	188	329	**470**	(x 48)	144	192	336	**480**
(x 49)	147	196	343	**490**	(x 50)	150	200	350	**500**	(x 51)	153	204	357	**510**
(x 52)	156	208	364	**520**	(x 53)	159	212	371	**530**	(x 54)	162	216	378	**540**
(x 55)	165	220	385	**550**	(x 56)	168	224	392	**560**	(x 57)	171	288	399	**570**
(x 58)	174	232	406	**580**	(x 59)	177	236	415	**590**	(x 60)	180	240	420	**600**
(x 61)	183	244	427	**610**	(x 62)	186	248	434	**620**	(x 63)	189	252	441	**630**
(x 64)	192	256	448	**640**	(x 65)	195	260	455	**650**	(x 66)	198	264	462	**660**

(As 3+7 =10, all the results are multiples of 10)

Let's take a closer look at what's going on
in the boxes involved in making the pretzel.
Here we see that the ancient Hindu number 108
added to Dee's Magistral number 252 makes 360,
the fourth number of Metamorphosis.





In the lower boxes, we find 189 and 441 summing to 630.
Ignoring those zeros, 360 and 630
are transpalindrimic mates 36 and 63,
(also famous for being one of the "4 steps").

To summarize,
this 3:4 chart brings together many of Dee's mathematical concepts.

It's an expression of "Quaternary rests in the Ternary."

Nestled inside in the chart are the cuboctahedron...
(6 square faces and 8 triangular faces),

and the" greatest and most perfect symphony" (6, 8, 9, and 12)
and thus the "3 main harmonies" (1/2, 2/3, 3/4).

Also within it are the "four steps" (12, 24, 36, and 48).

Hidden a little deeper is the "12 and 21 pretzel,"

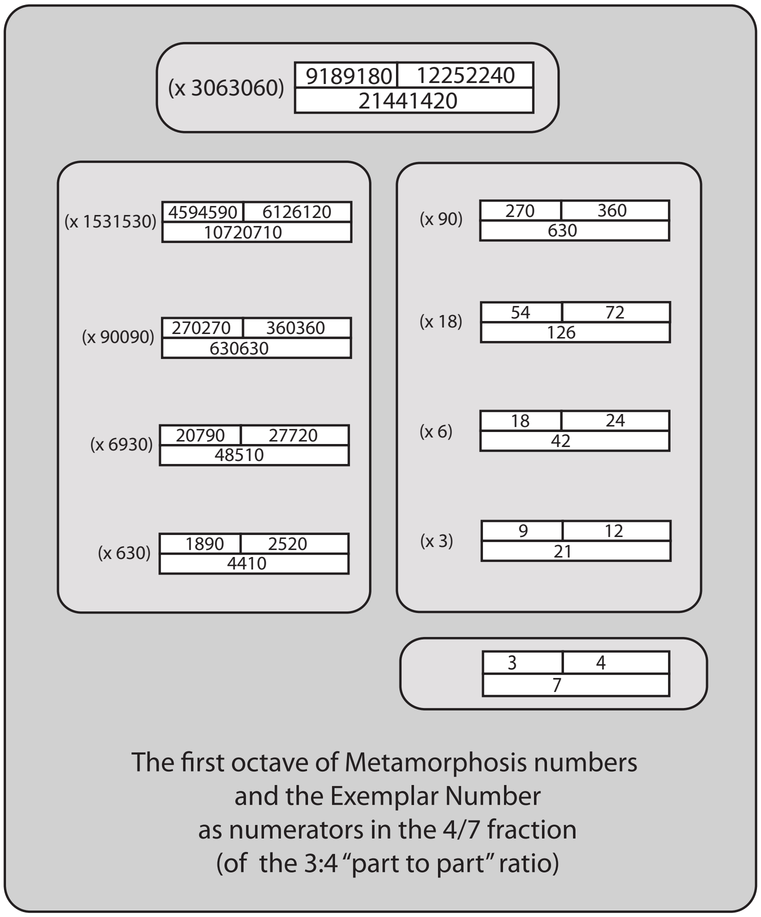
as well as the 9 Wave, 11 Wave,
99 Wave,
1089 Wave and 10890 Wave… of Consummata.

That's pretty power-packed!

But it has even more stories to tell.
Let's take a closer look at how the Metamorphosis sequence fits into this picture.

The 3:4 ratio and the Metamprphosis numbers

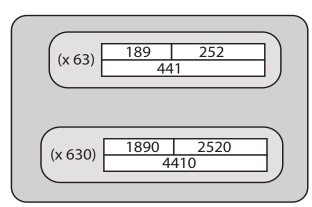
Here, I’ve put the first 8 Metamorphosis numbers
and the Exemplar Number in the upper right section of each box.



The first octave of Metamorphosis numbers
and the Exemplar Number
as numerators in the 4/7 fraction
(of the 3:4 “part to part” ratio)

First, let’s look at the “4 and 7 boxes” for the first 4 Metamorphosis numbers:
“**12** and 21” are a transpalidromic pair.
“**24** and 42” are another transpalidromic pair.
“**72** and 126.” Note that126 is half of 252.
“**360** and 630.” Their reflective mates are transpalindromes

In the boxes for Metamorphosis number 2520,
you can see that its contents 1890, 2520, and 4410
are simply “10 times 189, 252, and 441” (all of pretzel fame).
(And remember, 252 and 2520 are reflective mates)



We’ve already come across the box involving for Metamorphosis number 27720
while exploring the 10890 Wave (3:4 ratio multiplied by 6930).
Keep an eye out for the “nineness” in this key number
(693 = 99 x 7) and (6930 = 1089 x .6363636....).

Next, look at the boxes for Metamorphosis number 360360:
there's a whole lot of "reverberating nineness" going on here
in several "stuttering" numbers 270270, 360360 and 630630.

Metamorphosis number 6126120 is found by multiplying
"sort of stuttering" number, 1531530 times 4.

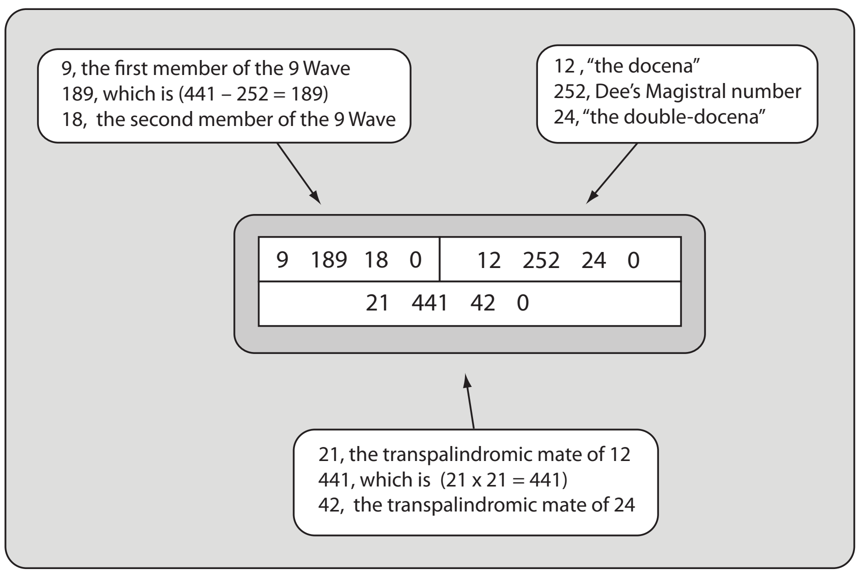
Last, but certainly not least, centered at the top of the chart,are the boxes which have
the Exemplar Number, 12252240 in their upper right-hand corner.

Note that these boxes are the "3:4 ratio" times a number
conaining *even more* 3's and 6's: "3063060"

Once again I'm going to unconventionally "ignore place values"
and "ignore the zeroes" to dissect these numbers.

(It's uncanny how the same key numbers
keep popping up in this opera of retrocity.)

All these numbers are old friends:

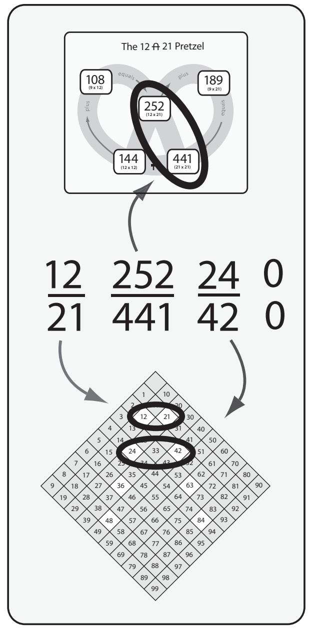


Furthermore, comparing parts of the Exemplar Number, 12252240,
with the number below it, 21441420,
yields some familiar fractions, which are all equivalent to 4/7.
This is not that startling,
as can be seen when we **do** look at the place values.

$$\frac{4}{7} = \frac{12252240}{21441420}$$

$$\frac{4}{7} = \frac{12}{21} = \frac{252}{441} = \frac{24}{42} \quad \begin{matrix} 0 \\ 0 \end{matrix}$$

$$\frac{4}{7} = \frac{12{,}000{,}000}{21{,}000{,}000} = \frac{252{,}000}{441{,}000} = \frac{240}{420}$$



What is startling
is that these are the relationships
which we found in the "pretzel"
and in the "diamond shaped chart
of 1-digit and 2-digit numbers"!

Let's push this a step further and look at the
transpalindromic mates all of **both** of these numbers.

12 252 24 ₳ 42 252 21

21 441 42 ₳ 24 144 12

Where we started with a fraction equivalent to 4/7,
these reflective mates make a fraction which is equivalent to 7/4.

12252240 ₳ 04225221

21441420 ₳ 02414412

$$\frac{4}{7} = \frac{12252240}{21441420} \qquad \frac{04225221}{02414412} = \frac{7}{4}$$

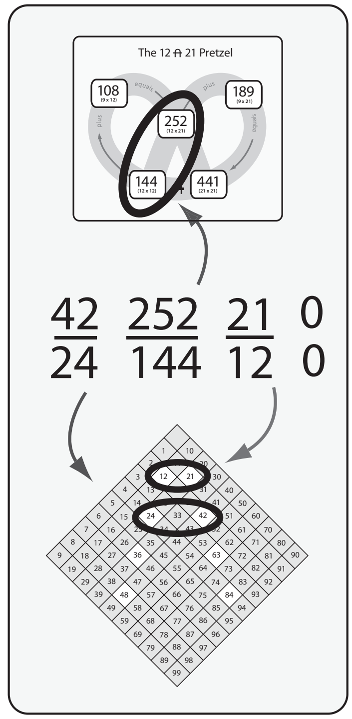
This might not seem unusual, but it doesn't seem to happen very often.
Test this for yourself using **any** transpalindromable numbers as numerators and denominators.
You won't often find the results to be reciprocals like this.

This phenomenon only seems to happen with fractions equivalent to 4/7 (or 7/4).

Recall that on the chart of "1-digit and 2-digit transpalindromic **fractions**,"
the only results that reduced down to fractions having single-digit numerators
and single digit denominators were those that were equivalent to 4/7 (or 7/4).
(And those were the "4 steps")

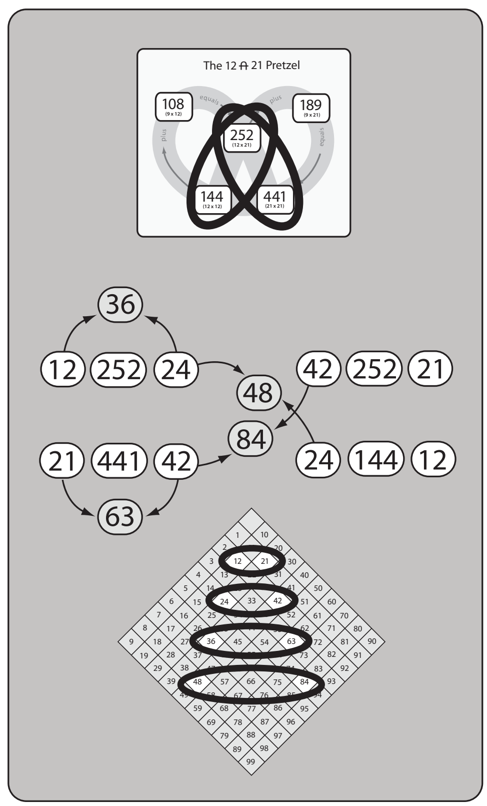
Next, let’s look at the “component parts”
of these newly derived numbers.

We still have expressions
of two of Dee’s “steps”
and now, the “other side”
of the “pretzel”(12 x12 =144).



Furthermore, by creatively combining
some of these numbers, we can
find all of the “4 steps”
(12, 24 36, 48)
as well as their
transpalindromic mates
(21, 42, 63, 84).

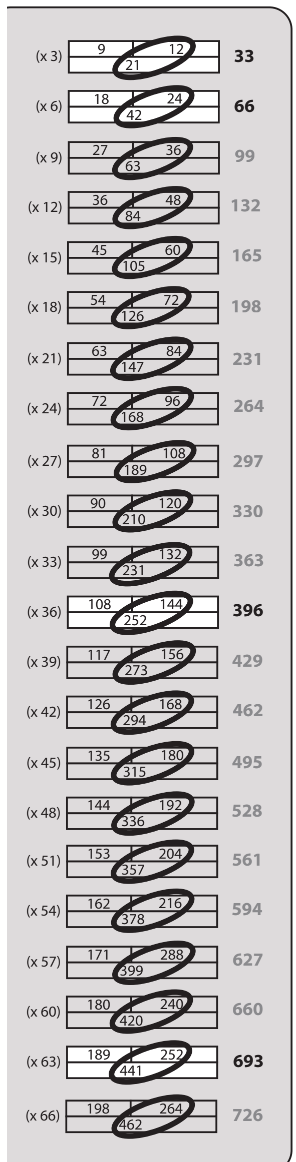
Besides the wondrous fact that
the Exemplar Number is
divisible by all the digits from 1–18,
and that it
perfectly distributes the primes,
all this
interconnectedness
is another stellar reason
why Dee felt this number was
worthy of being a “rare gift” for the
King of the Holy Roman Empire.



Let's push this analysis
of the Exempar Number a step further
by **adding** these 2 pairs of numbers.

The results are all made from
3's, 6's, and 9's:
(33, 66, 396, and 693).

12	252	24			42	252	21
+ 21	441	42		+	24	144	12
33	693	66			66	396	33



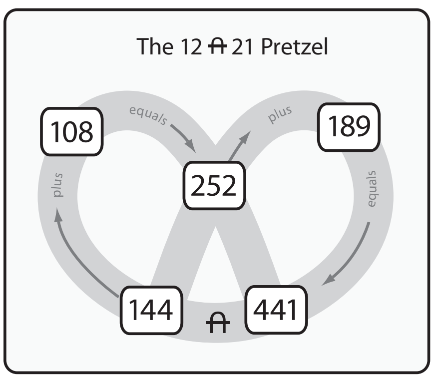
As we found earlier
in the analysis ofthe
chart, these **are** all
important numbers.

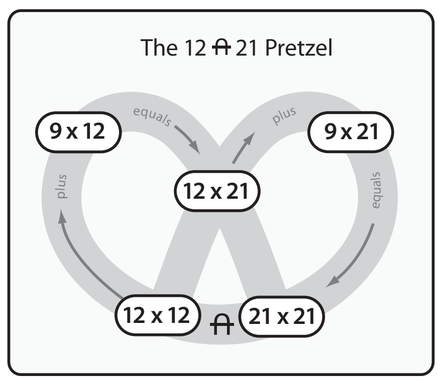
In summary,
the pretzel can be boiled down to
the interactions of 9 and 12 (and its reflective mate, 21).
This suggests a synchrony between Consummata (9 Wave)
and Metamorphosis (first Metamorphosis number being 12).

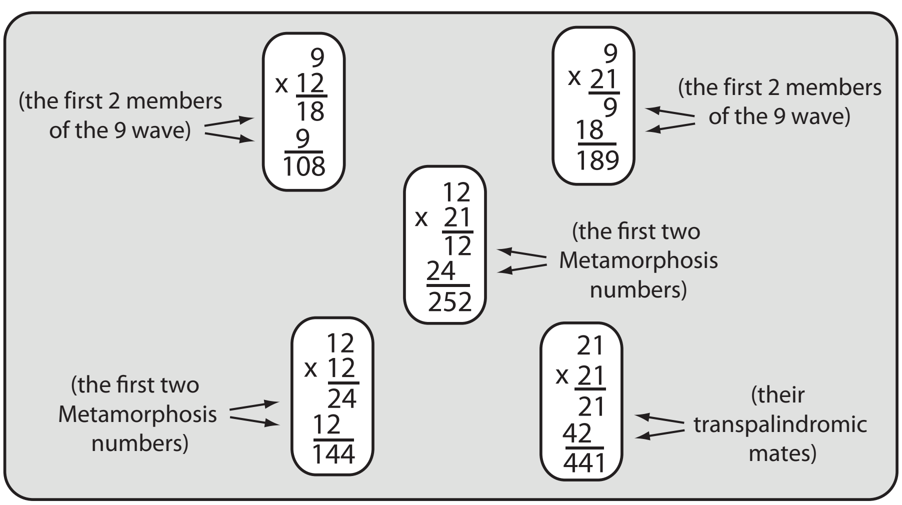
This synchrony can be seen more fully by putting the calculator aside
and using “long multiplication x-ray vision.”
The “internal numbers” here are “9 and its double, 18,”
“12 and its double, 24,” (and their mates 21 and 42).

108	144
252	

189	252
441	

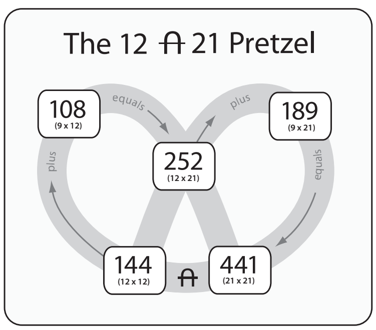


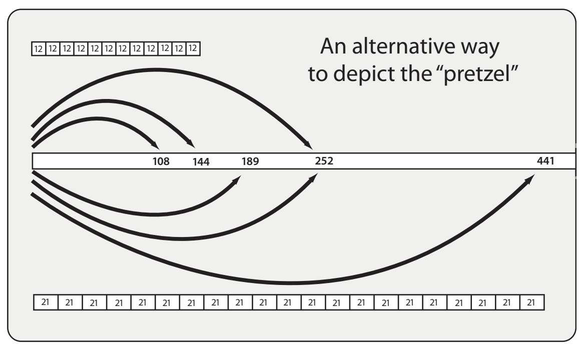




The "252 pretzel" is hidden in Title page of the Monas

Given Dee's excitement about his Magistral number, 252, I had a hunch that somehow concealed it in the grid of the Title page. The splendor of 252 is how it integrates with 108, 144, 189, and 144, in what Marshall calls the "Syndex Pretzel." I suspected that he might have hidden this whole "Pretzel" interrelationship there as well.

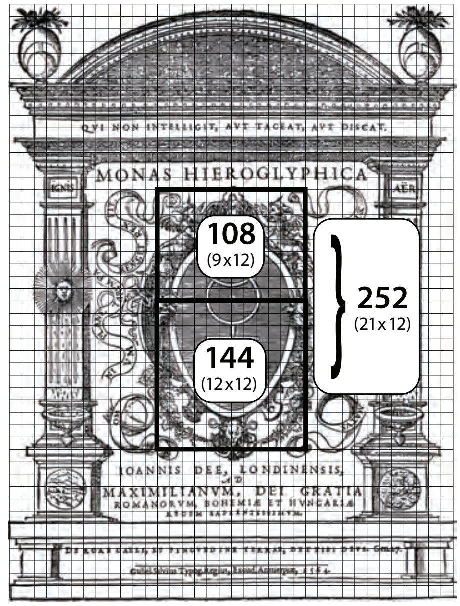


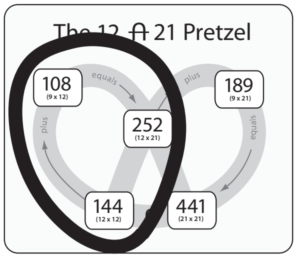


As the most important factors of 252 are 12 and 21, (the first transpalindromic pair), I first looked for a block of 12 x 21 grid squares.

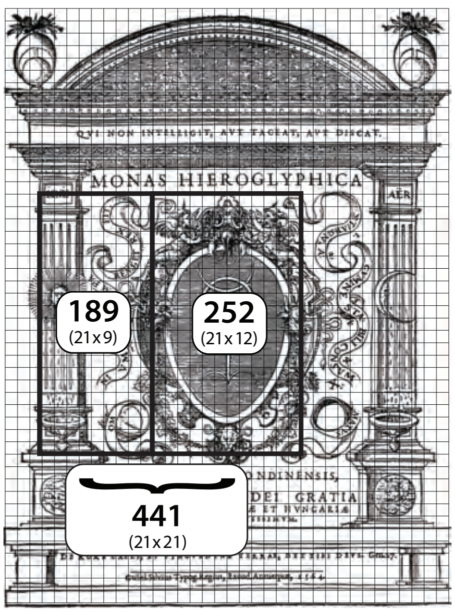
Such a block nicely frames the central egg and the Monas symbol. The top edge is at the top of the emblem, the bottom edge touches the Lion's chin (King Maximilian's chin). And the two side edges come very close to the eye of the ram and the eye of the bull.

When a horizontal line is made through the center of the Sun Circle, two blocks are created, one with 108 grid squares and the other with 144 grid squares.





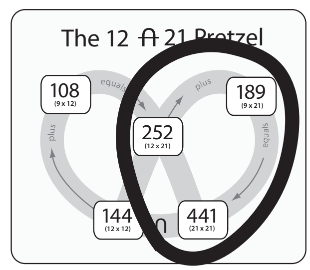
This represents the equation on the left side of the Pretzel.



Next, I search for the other half of the Pretzel, a block of 189 grid squares (which is 9 X 21).

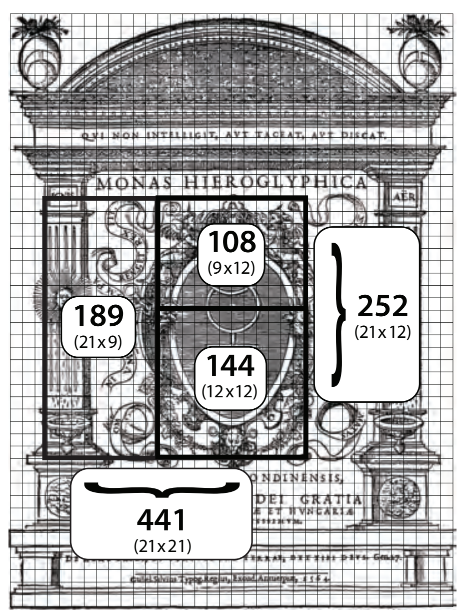
As the 21 dimension had already been established, I measured 9 grid squares to the left of the 252 block. Bingo. It exactly reached the outer edge of the left.

Adding the 189 block and 252 block together makes 441 grid squares! This is the equation on the right side of the Pretzel.



Here is a full summary of the "252 Pretzel" on the Title page.

If this cofiguration is what Dee had in mind, he would undoubtedly have left a confirming clue. And indeed he did!



Let’s zoom in for a close-up of the top left column. Along the right edge of the capital, hiding in the shadows, is what he appears to be a “missing chunk” of the architecture. This could be an attempt to make us the structure look a bit more antique,or could be shoddy engraving technique. However there are no other chunks missing anywhere else on the architecture, and we know that Dee was fanatic about details.

As you can see, the top of Dee’s “geometric pretzel” cuts right across this “missing chunk.”

Unless one knows about Dee’s grid and his blocks of “Pretzel numbers,” the “missing chunk” is meaningless. Once understood, it confirms puzzle has been solved correctly. It also confirms that the “restoration” of the whole central emblem has been done correctly as well. If the emblem had not been moved upwards to “fit into the theater,” this horizontal line wouldn’t cross the column at the “missing chunk.”

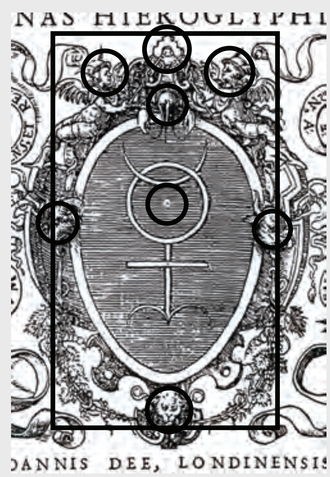
Within the 252 block another curiosity. There are ten “eyes.” Eight of them are somewhat “paired, as in the “+4,–4 octave.”

The lion has a pair of eyes. (King Maximilian’s). The crustacean has a pair of eyes (Dee’s), the 2 Mercuries are in profile like a pair of “one-eyed Jacks”, and finally the single eyes of the ram and the bull symmetrically grace the two sides of the egg. This makes an octave of “eyes.”

The central point of the Sun Circle apperars to be the “null ninth” eye. Remember, Dee labeled this Cyclops eye of with the letter “I”, in his geometric construction of the Monas symbol in Theorem 22. And “I” is the ninth letter in the Latin alphabet.

The tenth “eye” is the hole where the two Mercuries’ spear tips meet. It represents the aperture of a camera obscura–a giant model of how an eye works.

A denary of “eyes”



252 grid squares

This might all sound highly imaginative, but 252, the Denary, and the camera obscura are key themes of the *Monas Hieroglyphica*. Knowing Dee, front and center on the Title page is right where he would put them (albeit discreetly veiled).

How the tower expresses the Exemplar Number, 12252240

Dee liked to hide things in fireplaces.

When Dee was beginning to converse with the angels through his scryers Barnabus Saul and Edward Kelley, he took copious notes of his questions and the angels' responses.

To ensure no one would find his notes, he hid them in a "cap case" which he somehow hid in his chimney at Mortlake.

At the end of the April 18, 1583 "action,",the spirit "IL," (speaking through Kelley) says:

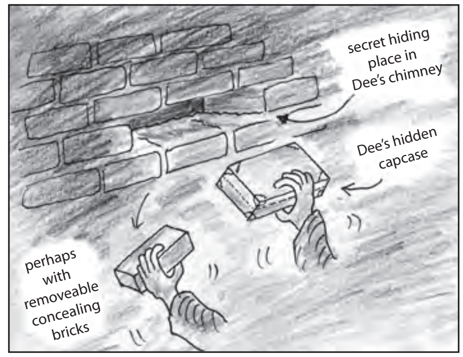
Conjectured view of Dee hiding his "cap case" inside his chimney at Mortlake

"**Your Chimney here**
will speak against you soon:
yet I am no brick layer.
I must be gone."

In the margin Dee notes,
"**True it is, I had hidden there,**
in a cap case the recordes
of my doings with Saul & other & c."
(Peterson, *Dee's 5 Books*, p. 359.)

A “cap case” is a small box that would hold a few books. “Organists” kept their music in a small cap case. Ladies would keep “lace, pins, and needles” in a cap case. And “unthrifty heirs” could keep their wealth in a cap case. (OED, cap case, p. 90.)

Elizabethan fireplaces were often large enough to stand inside. Perhaps Dee had a hidden shelf or place behind a loose brick where he could put his “cap case” of notes. It needed been be protected from the smoky fumes and hidden well enough that anyone doing house-search wouldn’t notice it it.



Regardless of how well concealed it was, Kelley could have picked up clues as to where Dee kept it. Perhaps he noticed soot marks on the capcase or of that Dee’s nots smelled smoky. Perhaps he was spying on Dee, as Kelley had taken up residence in Dee’s house. Regardless, it appears that he was aware of where Dee hid it and he was metaphorically conveying the idea that he didn’t like being recorded. As Peterson puts it, Kelley seemed “upset about the diaries hidden in Dee’s chimney.” (Peterson, *Dee’s 5 Books*, p. xi.)

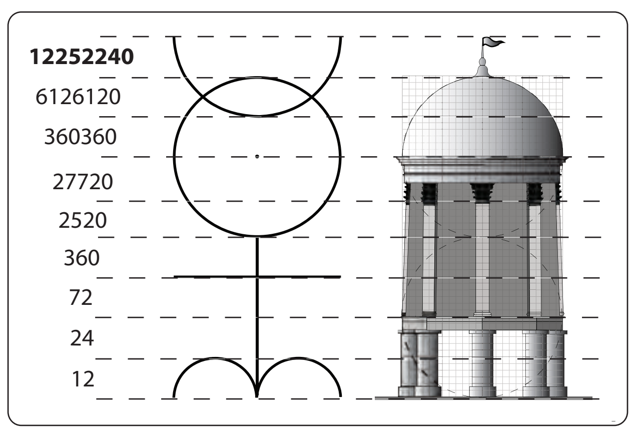
(A cop, sometimes spelled cap, is a unit of measure of ¼ of a Scotch peck. The volume of a peck “varied greatly according to locality and commodity measured” but on average it seems to be the size of our modern peck, which is 2 gallons. A “cap case” would thus the volume of a half gallon of milk (4’ x 4’ x 10’ = 160 square inches), but flatter, perhaps (8’ x 10’ x 2’ = 160 square inches). In other words, a “cap case” could hold about a dozen copies of the 5’ x 7’ *Monas Hieroglyphica*.)

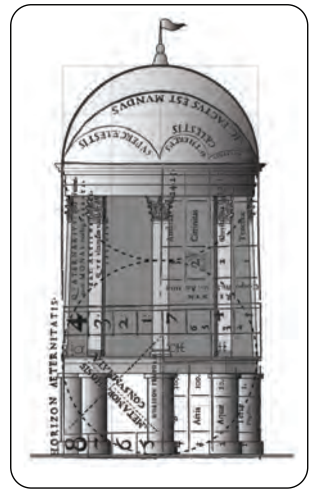
The point here is that Dee hid something very important and special to him in his fireplace. It seems as though he hid something very special in the fireplace of the John Dee tower as well. (Someting small, but very “rare.”)

How the fireplace of the Tower expresses 12252240

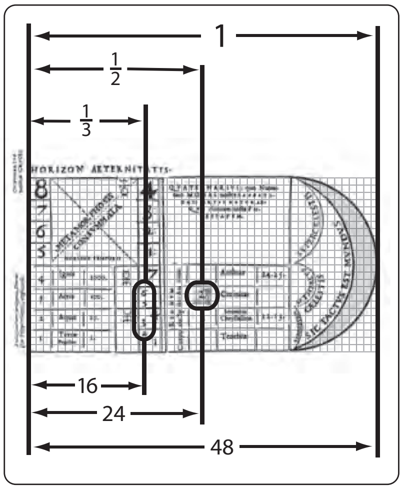
As we’ve seen, the 9 parts of the Monas symbol can be seen as the ocave of Metamorphosis numbers and the Exemplar number.

As the Monas symbol is the overall plan for the tower, it also expresses these special numbers. (The Exemplar number corresponds with the “finial” at the top of the Tower.)



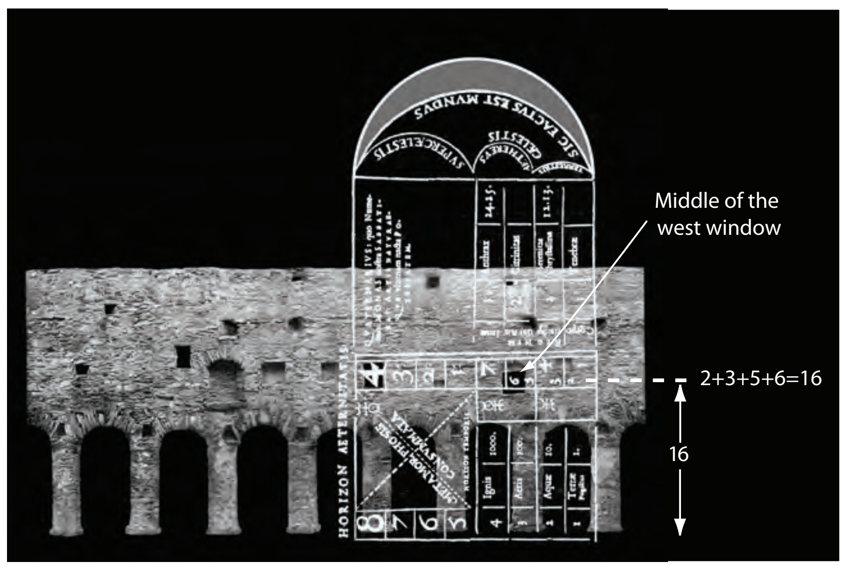


Also, we've seen that the "balooned 360" Thus the World Was Created chart is based on two circles, just like the stone-and-mortar part of the Tower.



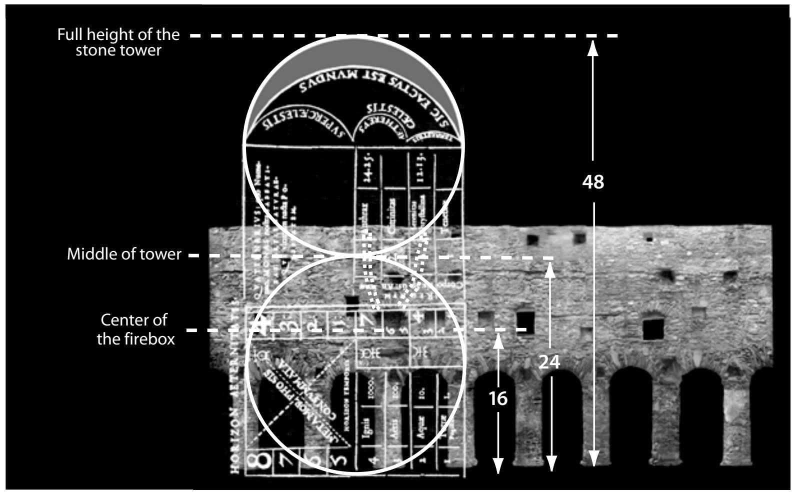
Applying a 24 by 48 grid, the numbers (2, 3, 5,6) ,which also happen **add to 16**, are at the "one-third mark." The Artificial Quaternary, with it's large,"Engraved 2" is at the "one half mark."

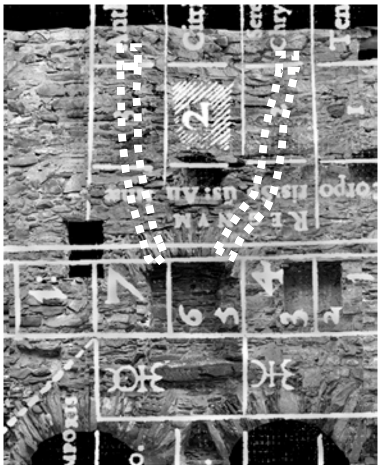
Overlaying this chart on the panoramic view of the interior of the Tower, the (2, 3, 5, 6) is the same height as the center of the West window, or **16 feet** above original ground level.



This means that the center of the fireplace is also **16 fee**t above original ground level.

With this overlay arrangement, the "Engraved **2**" falls between the **two** flues of the fireplace.

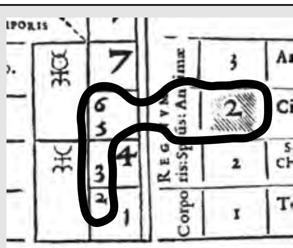




Here is a close-up view of that detail. The second floor is 22 feet above ground level (the pillars and the entablatue are 12 feet, plus the first floor room is 10 feet). And as you can see, the "Engraved 2" is about 2 feet above the dark beam socket for the second floor, bringing the total to 24 feet, exactly the mid-height of the 48 foot tall Tower.

Indeed, it certainly seems strange to express a mathematical concept with architectural features, but Dee has done this with many other features of the Tower. It's odd, but its something Dee would do.

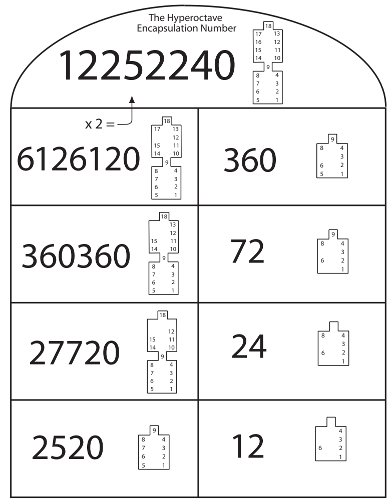
Remember, the fireplace is not just any old feature. It's the heart of the Tower– not just because it is the sole source of warmth, but because it is carefully aligned with the West window so the the camera obscura projection on the equinox creates a blaze of "fiery water "in its center.



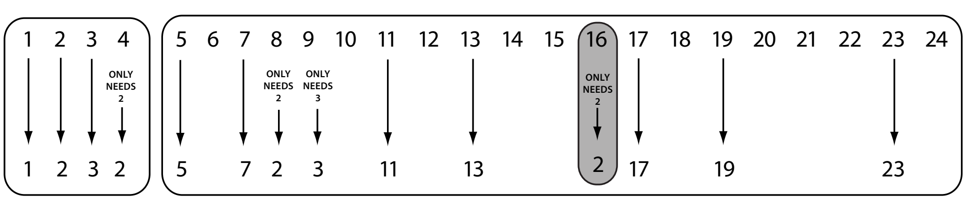
"(The sum of 2+3+5+6) or **16**, **needs the Engraved 2** " expresses the final step to **12252240**

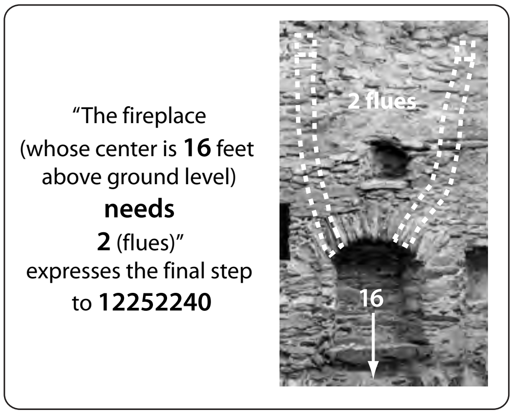
In Dee's chart, the (2, 3, 5, 6) represent the "**16 which needs that pesky, Engraved 2**" (confirmed by the word Anus or Annulis, the prized gold ring).

Recall that the number 6126120 is divisible by all the numbers from 2 to 18, except for 16. The **"16 needs a 2,"** resulting in 12252240.



The "Hyperoctave" or eight Holotomes

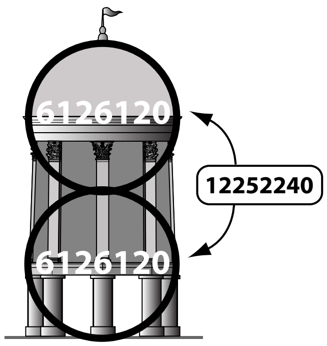




Well, Dee's fireplace expresses the same thing!
"**The fireplace, (whose center is 16 feet above ground level, needs 2 (flues)**"
In this strange way, the fireplace represents 12252240.

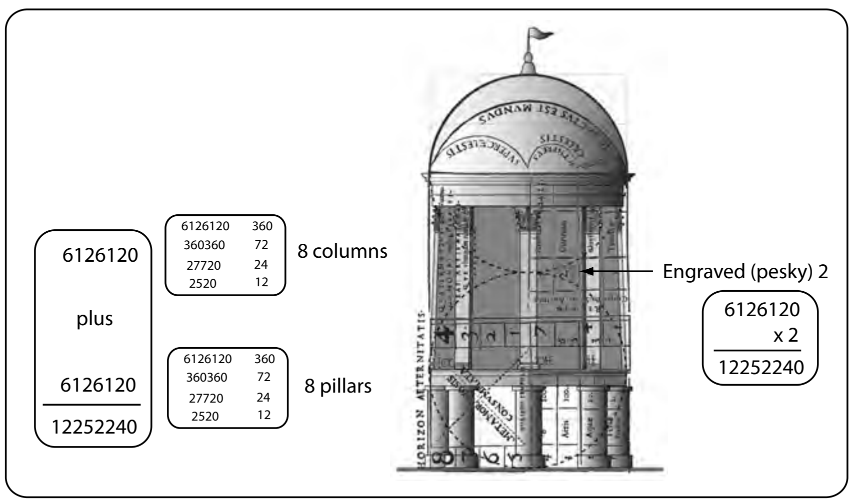
I realize that equating a math concept
with an architectural feature is quite odd,
but 12252240 was ultra, ultra special to Dee.
It was his "rare gift" to the King of the Holy Roman Empire.
This unique number organizes over 12 million
primes and composites in perfect symmetry,
and it was how Dee envisioned that "...the World Was Made."

Other ways to see "16 needs 2"in the Tower.



The exterior of the Tower might also be seen as an expression of this "16 needs 2" theme. The Engraved 2 marks the exact middle height (24 feet above ground level) of the 48 foot stone-and-mortar part of the Tower. If each of the two grand circles that are tangent at that height represents 6126120, their sum is 12252240.

As visual corollary to this idea, if the 8 pillars and the 8 columns are each seen as representations of the octave of the first eight Metamorphosis numbers (which culminate at 6126120), the pillars and columns combined can be seen as expressing 12252240.



The "16 needs 2" theme might also be seen as
creatively expressed by the height of the dome room:

The 16-foot-tall dome room needs 2 rooms below it.
The 16-foot-tall dome room needs two stairways to access it.
The 16-foot-tall dome room needs the 2 circles of the
figure-8 solar disc analemma projected on its floor to describe a full year.

The Sun is the source of Fire

Becauce of its wooden floor, the fireplace was the only location in the Tower where a firecould be made. In Dee's symbolic mind, "fire" meant more than a "a heater " or "a place to cook." Fire was one of the 4 Elements, and an important one to Dee, who signed his name with its symbol (Δ).

The main source of fire is the sun. Sure wood and coal are sources, but as the sun is our sole photosynthesizer. Wood and coal are simply forms of sun-energy that has been in storage for a while. Dee saw the sun as "Fire," the ultimate source of heat and light

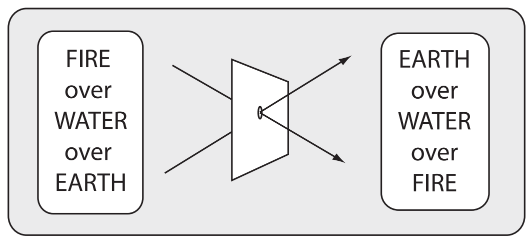
In his *Letter to Maximillian*, Dee describes how the "**Theorems of our MONAD**" are useful to a person skilled in the "Art of Hydraulics" (like an aqueduct builder)

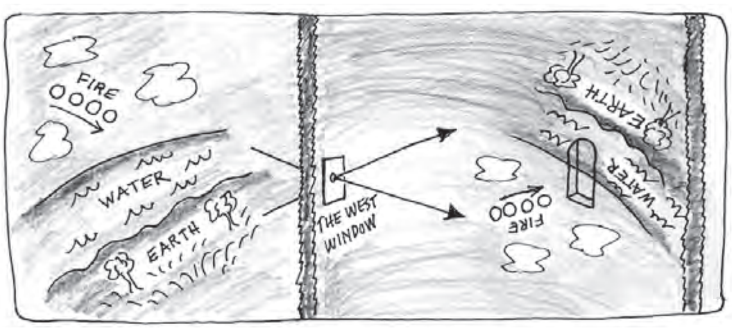
„ **But no one of that Profession can claim to have made a Machine,**
„ **which could raise the Element of Earth Upwards,**
„ **through Water,**
„ **and into Fire.**
However, the Theorems in our MONAD demonstrate that this is possible.

(Dee, *Letter*, p. 6 verso)

(This was an important point for Dee, as he highlighted it with four quotation marks in the margin. This is the only passage so marked in his whole *Letter to Maximillian*.)

The "Theorems of our MONAD" contain many clues about the "oppositeness" that takes place in a camera obscura. The first floor rooom of the Tower was a "machine" that could do what Dee claimed.





View of Newport Harbor

Cutaway view of the first floor room of the Tower.

In the view out the West window,
Fire (of the sun) was "above" Water,
which was visually "above" Earth.

But inside the Tower,
Dee had moved the
"Element of Earth Upwards"
and it was now on top!

In Theorem 10 Dee infers that the Aries symbol represents Fire. On The first day of spring, the "heat and light"starts to return to the northern hemisphere bringing songbirds and daffodils.. One woud think that Dee would have positioned the fireplace so that the solar disc hit is center on March 21. (It actually hits there about March 6 [and October 6]). Instead, Dee positioned it so it would tell a story involving Fire **and** Water.

He did so because he concealed a whole different story about Earth and Air.
Can you figure out how?

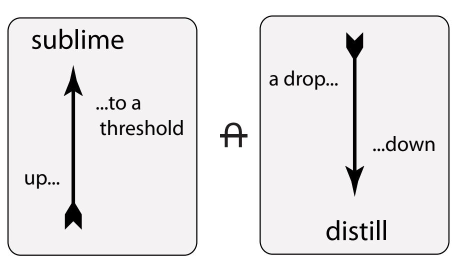
Sublimation and Distillation in the Monas and in the Tower

Dee's *Monas Hieroglyphica* incorporates another important concept in alchemy that we have not touched on yet: the idea of sublimation and distillation. These two "opposing" processes are a specific kind of "separatio and conjunctio."

Contrary to how it sounds, to sublimate means "to raise to a higher status." ("Sub" means "up to" and "limen" is perhaps related to "threshold.")

In chemistry, sublime means "to change a substance into a vapor by heating."

(OED, sublimate, p. 1694.)



There's a real sense of "up-down" going on here.

Distillation is just the opposite. It cools the vapor so it forms condensation, which is then collected.

The word distill is derives from the Latin word *destillare* ("de" meaning "down or away" and stilla meaning "a drop.")

(OED, distill, p. 523)

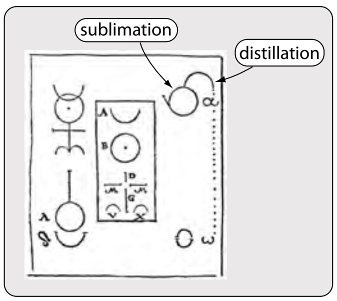
The Neoplatonic philosopher Syneseus (370-413 AD) wrote:

"Thus when our stone is in the vessel,
and that it mounts up on high in fume,
this is called Sublimation
and when it falls down from on high,
Distillation, and Descension."

(Syneseus in Abraham, p. 55.)

Dee was very familiar with this famous philosopher. He had 6 copies of Syneseus' works in his library. (Roberts and Watson p. 227.)

Sublimation and distillation are frequently referred to as the nigredo (black) and albedo (white) respectively. In Dee's "Thus The World Was Created" chart, they are the first two alchemical stages: Tenebrae (darkness) and Chrystallina (crystalline, clear, white).

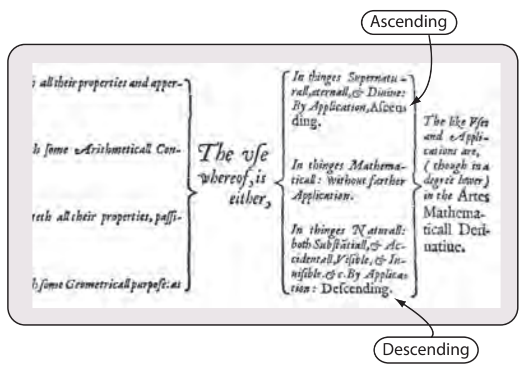


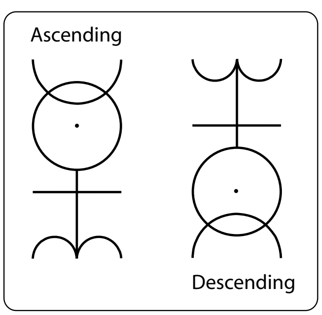
He illustrates them in his Vessels of the Holy Art diagram of Theorem 22.

The sublimation part take place when liquid in the round retort boils and steam goes upward.

The distillation part takes place in the neck of that vessel, where steam condenses and drips downward.

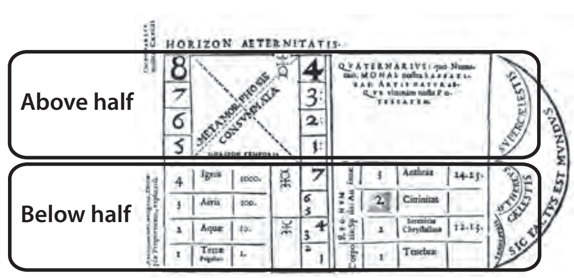
Dee refers to Ascending and Descending in the "Groundplat" of the two "Principal Mathematical Arts," Arithmetic and Geometry. Each of these subjects has a Supernatural aspect (ascending) and a Natural aspect (descending).





Even the upright Monas symbol and the inverted Monas symbol seem to express Ascending and Descending.

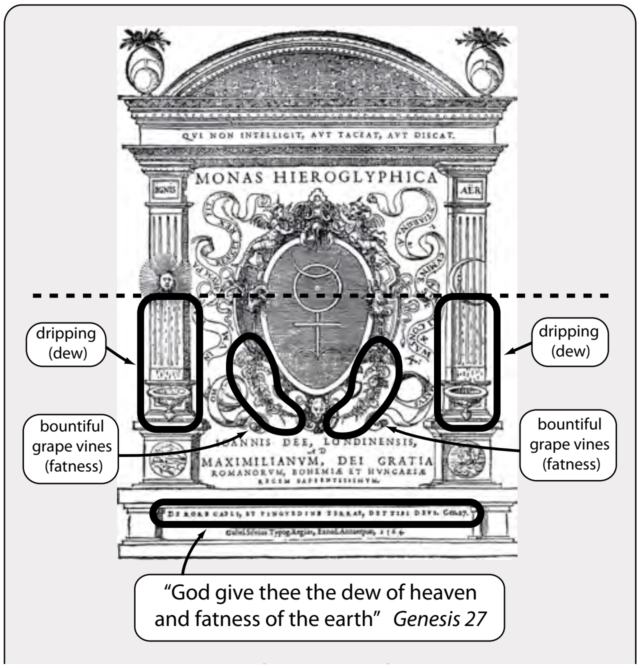
And of course the "Thus the World Was Created" chart has an "Above and a "Below."



Distillation is below the midline.

The drops dripping from the retort in the Vessels of the Holy Art diagram can also be seen on the Title page. Both the sun and the moon are raining drops of water. This confused me at first because the moon is generally associated with water, but the sun is more associated with fire.

This dripping is related to the Biblical quote that Dee wrote in the foundation of the Title page (Genesis 27:28): "**God give thee the dew of heaven, and of the fatness of the earth**." The "drippings" are the "dew of heaven" and earth's bounty is represented by the grapevines that drape around the lower part of the central egg.



Notice that all these distillation or condensation related things are in the bottom half of the Title page.

Dee left a subtle confirming clue. Look very closely at the drips coming from the moon. They don't start at the lower edge of the crescent, but instead start partway up between the moon's horns. This starting point is precisely the midline of the Title page.

Not only that, but because the 24-grid-square tall columns are centered vertically, the moon's dew starts at the exact midline of thewhole Title page illustration. (The sun and moon on the columns are not exactly on that midline; their height is related to a different theme so let's ignore them for a moment.)

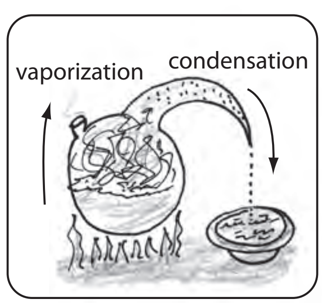
Sublimation is above the midline

During sublimation, or "blackening," heat creates a volatile vapor often called a "cloud." These smoky fumes swirl around and accumulate at the top of the vessel. (Abraham, p. 42.)

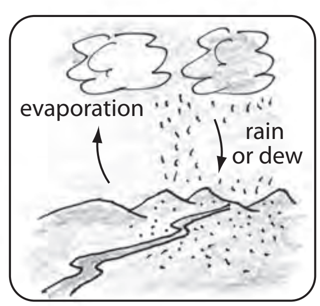
This smoke or cloud is sometimes called Green Lion; green because it's in the early stages of the alchemical process (as unripe fruit is green); and Lion because it has the power to "reduce bodies into their first matter." (Abraham, Green Lion, p. 92.)

When the "cloud" is cooled, in the "long, gradually bent neck of the retort" it condenses and drips. The alchemists saw this whole process as a metaphor for the earth's evaporation/condensation cycle of rain. The alchemist Jodocus Greverius (ca.1667) wrote: "**Turne thy clouds into raine to water thy Earth, and make it Fruitful.**" (Abraham, Cloud, p. 42.)

This "up and down" can be seen, in a larger sense, as the cycle of evaporation of water into clouds and condensation as rain or dew.



Microcosm



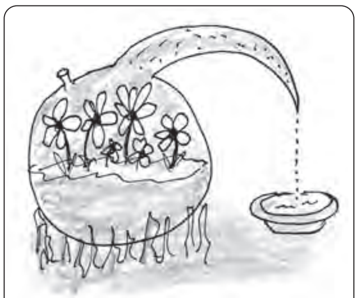
Macrocosm

The elements of Fire and Air are located in the top half of the title-page, but Dee would have made a more specific refence to sublimation than that.

The solution struck me as I was searching through the best modern day reference on alchemy, Lyndy Abraham's *Dictionary of Alchemical Imagery*. There on the front cover was a bearded, alchemist boiling a pot of water on an outdoor fire. In his hand he held a ribbon that rose from his hand like smoke. His other upraised hand gestures that the ribbon represents the fumes wafting upwards towards the sun.

Dee uses similar ribbons that also seem to curl upwards filling the upper corners of the "theater" between the columns (on the restored Title page). These ribbons, as we've seen, have other meanings as well, but they certainly are a graceful, airy, visual complement to all the right-angled architecture.

But Abraham's book alerted me to another feature in the top half of the Title page that says "sublimation." And in alchemical jargon, "flowers" are another name for the cloudy smoky substance obtained by sublimation.



sublimation seen as "flowering"

The English alchemical poet, Basset Jones (b. 1616), defined "sublimation" in his 1650 "*Lithochymicus, or a Discourse of a Chymic Store...*":

"that whereby the flower or subtile partes of a body are Elevated unto the topp of the Vessell and there, by vertue of the Ayer congeal'd."

(Jones in Abraham, p. 55; also in Robert Schuler Alchemical Poetry 1575-1700, p. 227-358.)

The alchemical author Artepheus wrote that:
"by reason of too much heat you will burn the flores auri, [the golden flowers]."

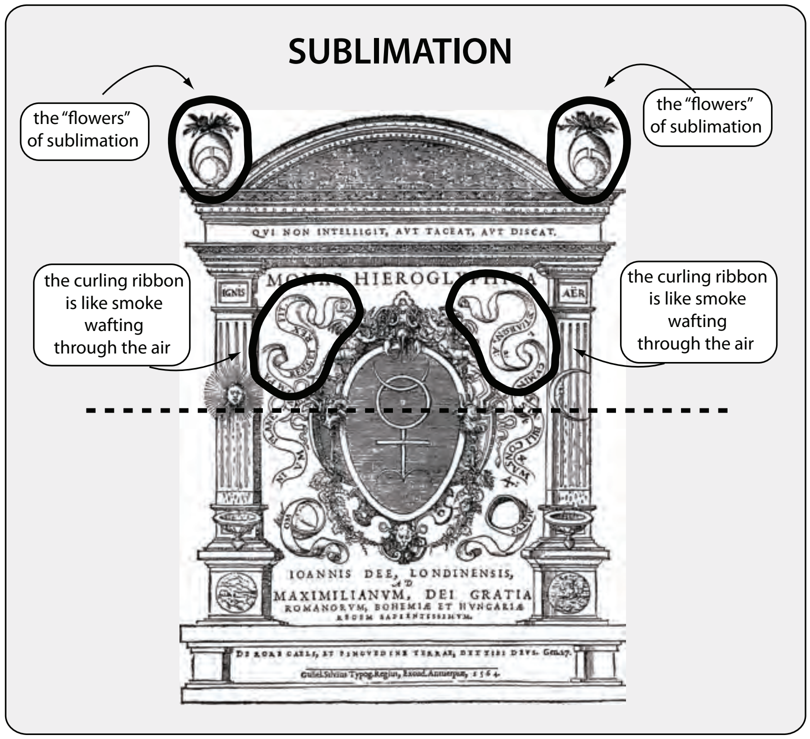
Dee mentions Artephius in his *Preface to Euclid*, (p. A.iijv.) citing him as the author of *Ars Sintrilla,* a work on Archemastrie of the "Science Alnirangiat." Avicenna used this word, which comes from "nirang" meaning a magical charm or spell. (Note that its prefix and suffix echo Dee's word "althalmasat," from thalamus (room), a camera obscura).

Dee owned the Artepheus manuscript before 1556. Nicholas Clulee researched Artephius and found his identity was "obscure," but he was mentioned in a manuscript from the 1100's.
(Nicholas Clulee, *Crossroads of Magic and Science*, p. 61, *John Dee's Archemastrie* in Vickers, B., p. 21.)

And indeed on the Title page are flowers, bursting forth from the tw0 urns, way up at the top of the illustration.

Just as the bottom half of the Title page represents the dripping water of distillation, the upper half represents the vaporous fumes of sublimation.

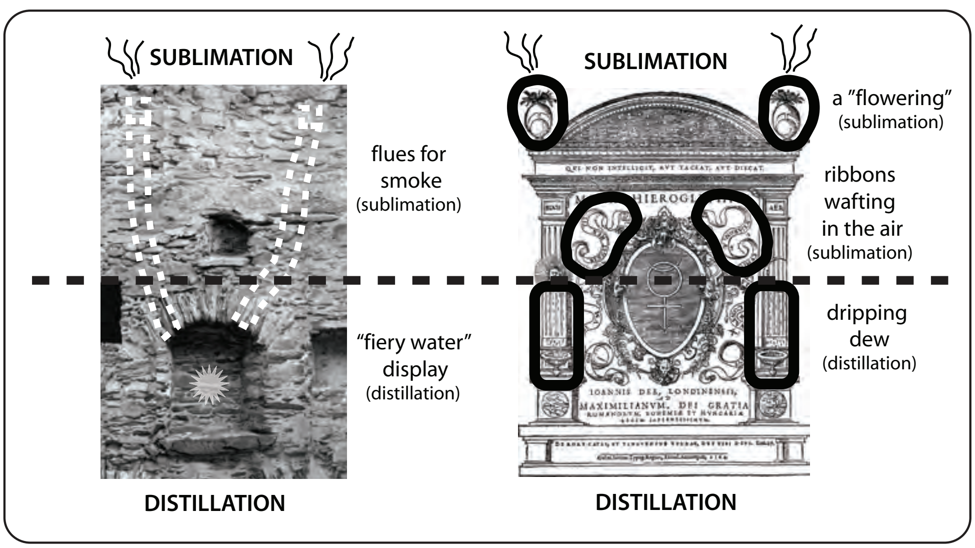
It's almost as if the "theater" is a firebox, and the smoke somehow rises through the walls and "flowers forth" through the openings in the two urns at verythe top of the illustration.



In these respects, the Title page is like a representation of what happens in the fireplace of the John Dee Tower.

The fiery water display is like the watery dew of distillation.

The two flues are like the wafting ribbons and flowering forth from the urns.

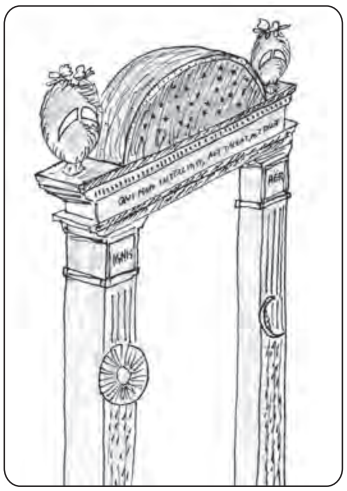


I realize this interpretation sounds highly imaginative, but Dee left another small clue that this is what he had in mind. The clue is quite small (only about one grid square high), and is found each of the urns.

I had long puzzled over what the crescent shapes on the side of the urn were. They resemble the blade of a scimitar, a short Arabian sword, but they have a gap in the middle. They also seemed like representations of the moon, but again that space in the middle was perplexing. They could be animal horns, but why would they be pointing downwards?

The "reflections" in the urns each have a "gap"

By studying the engraver's shading technique, I determined that the urns were meant to be seen as a shiny mirror-like metallic surface. There is a definite sense that the source of light is from the left, outside the frame of the Title page. This light direction matches the light and shadow patterns on the two columns and the rest of the architecture below.



My conjectured side view of the Title page architecture

The crescents therefore are something reflected in the mirror surface. They could be the "viewer" but that's not likely. They seem to be a reflection of the top of the architecture, the dome and the entablature. This illustration shows that what I call a dome is really flat like a slice of watermelon." And the urns are precariously placed on the outer shoulders at the cornice.

Realistically, in such an convex mirror, a flat shape would reflect as an upturned crescent, not a downturned one. But, allowing for a little artistic license, the downturned crescent suggests the top of the architecture more convincingly.

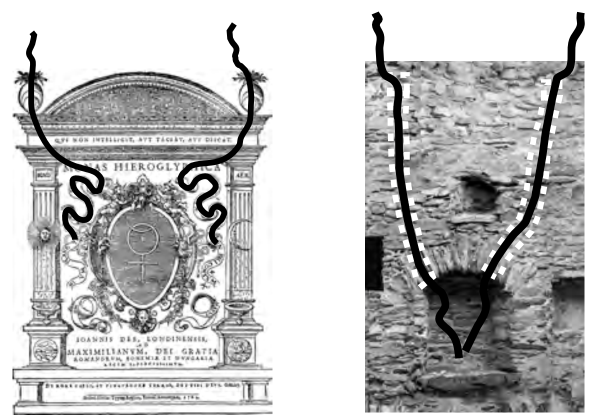
Then what is the gap in the crescent?

I think Dee put it there to hint about a "gap in the wall," in other words a "flue."

That's right. I mean to suggest that he small gaps in the "reflection" in the urns of Dee's illustration are the flues that still can be seen inside the walls of the Tower in Newport.

The gaps in the reflections in the urns suggest "flues"...

...like the two flues that run through the wall of the Tower



A brief history of fireplaces

Fireplaces with chimneys originated in chilly northern Europe in the Middle Ages. Before that, there was simply a fire pit were in the center of a room, with an opening in the roof above.

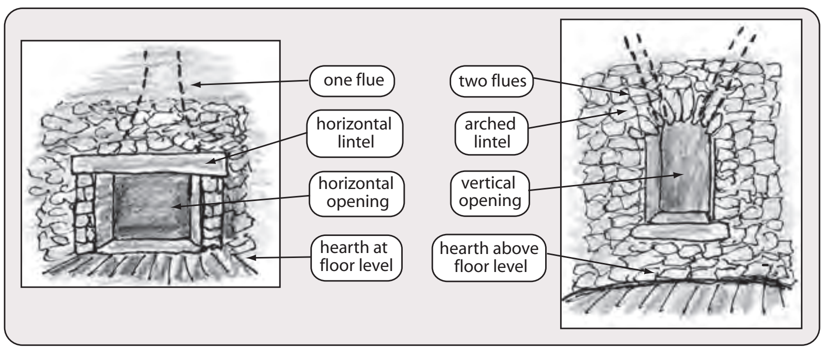
Moving the fireplace to the side wall not only facilitated the exhausting of smoke, but allowed buildings to have two floors.

Nowadays, most fireplaces are more for decoration than as a source of heat. People who do heat with wood usually use a wood stove rather than a more hazardous open fire. But in days of old, hearths were the central feature of households. Aside from providing warmth and a place to cook, there's something mesmerizing about watching a flickering fire. Even up until the early 1900's, families gathered around the fire for conversation. In the Great Depression, Franklin Roosevelt gave his weekly speeches, or "fireside chats," over the radio in the early evening when Americans were huddled by their fireplaces.

A clue in the fireplace that is no longer there

But the fireplace in the Tower is not a typical fireplace. It's not even a typical Tudor fireplace – for several reasons.

Fireplaces are generally at floor level, with a horizontal opening, a flat lintel, and have one flue.



The fireplace in The John Dee Tower is in the wall above floor level, has a vertical opening, has an arch for a lintel and has 2 flues.

It's more than a fireplace. It's a concept, an artistic expression, a sculptural metaphor.

It occured to me that the firebox looked to be about the same "4 to 3" proportion as the Title page. The firebox is splayed slightly, so I based my measurements on the width of the back wall, which is 30 inches. Measuring up 40 inches brought me to the height of where the arch starts.

For kicks, I superimposed the Title page on the fireplace in Photoshop. Now the two urns **really** seemed to correlate with the 2 flues.

This got me pondering how much the *Monas Hieroglyphica* and the John Dee Tower were similar. Sure, one was words and drawings on paper and the other was stones and wood that you could walk inside, but they both were expressing the same concepts. They were singing the same song.

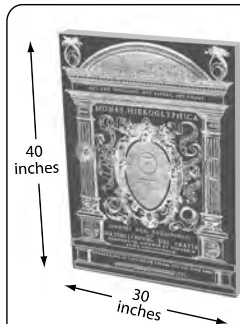
They existed independently, but there was a strong synergy that improved each off them. The John Dee Tower was the Monas Hieroglyphica made real.

In the midst of my musings, suddenly it struck me what I had just depicted in the back wall of the fireplace: **a fireback!**

What is a fireback?

It's a thick iron plate which stands upright and covers the back wall of a fireplace. It not only protects the back wall of the fireplace so that the rocks don't crack or burst, but it holds heat that would otherwise be lost. Long after a blazing fire has died down, the fireback continues to radiate heat into the room. It can increase the efficiency of a fire by as much as 50%. The thicker the fireback, the more heat it can store. (In a sense, it's like a one-sided wood stove.)

Firebacks are not as common as they used to be in Elizabethan and Colonial days. Old, restored firebacks, as well as new ones, are still available, though many people have never even heard of such a thing.



What a "Title page fireback" might look like

Firebacks were first used in the palaces of the French aristocracy, starting around 1460. They were luxury items that only the wealthy could afford and were usually adorned with family crests. When the general populace started using firebacks, they were decorated with designs about nature or classical stories.

The design is generally not carved or etched on the surface of the fireback. The design is first carved in wood, which is pressed onto a large tray of wet sand. Then molten iron is poured into the impression. After it has cooled, it is cleaned up and polished. The finished piece might be several feet wide by several tall and about one-half inch thick. The areas where there were raised decorations could be about an inch thick.

Dee's fireback for the Tower would most likely have been made at one of the great iron foundries in the "Weald." This great forest, that once was 40 miles wide, is in the county of Sussex, 25 miles south of London. In Old English, Weald was spelled "wilde," from which we get the words "wild" and "wilderness." (OED p. 1928, wild) and (OED p. 220, Weald)

Iron foundries not only required a source of iron, but also great quantities of wood to run their furnaces. The Weald had both, in addition to numerous small streams to provide water power for the bellows of the furnaces and forges.

Iron production in the Weald goes back to the Roman times, and even further, into the Iron age. In the 1500's, the Weald was the main iron producing region in all of England. By 1550 there were 50 furnaces and forges. By 1575, there were over 100.

At first, the furnaces simply made lengths of iron called "sows," but after 1540 they started making products like cannons, firebacks and iron memorials.

Many old homes in Sussex still have firebacks in their hearths. The church in Burwash has an iron memorial, which dates from the 1530's. These commemorative gravestones are approximately 2 feet wide by 4 feet long by an inch thick and are embedded between the flat stones of the church floor.

By 1583, firebacks were very much "in style," not only functionally but decoratively.

Sometimes fireback designs were made by pressing patterns of rope in the sand. For a thicker look, like a cable, a rope would be wound around a wire, and then pressed in the sand. Simple decorations used handprints or footprints (from animals or humans) or simple decorations made with farm tools.

Soon craftsmen got more creative, designing ornate scenes from Greek mythology, parables, or Biblical tales. The ultimate customization was to have your family coat-of-arms proudly displaced at the back of your hearth.

A large, hand-crafted fireback like this would be an ideal point of focus for the "first-floor room" of the "first Elizabethan building" in the "first American colony." Even if a viewer didn't fully understand the underlying signifigance of how it artfully integrated cosmologically with the rest of the building, it would have given the whole room a classical feel.

Dee would have had this fireback made in England prior to Brigham's departure. He had connections in the mining and metals trade. Pouring a custom fireback would be a minor detail in the scope of the whole Tower construction project. (Dee probably got the Catholic financiers, Peckham and Gerard, to pick up the bill anyway.) It would be heavy to transport, but might serve as ballast.

It might not have been as intricate as the Title page with all its minute engravings. But all the major features, architectural, the ribbons, the Monas symbol in the egg, etc., would be "raised surfaces," and (it goes without saying) in the correct proportions.

illustration of what a "Title page fireback" might look like in the fireplace

There seemed to be a few inconsistencies with my theory. First, with a fire blazing, it might look as though Dee's book was at a bookburning. Second, with so much intricate engraving on the Title page, the details would soon be blackened with soot and visually lost. A simpler, more graphic design seemed more plausible–like just the Monas symbol in an egg shape with a few decorative elements.

Then it occured to me that perhaps the whole fireplace might have been designed to look like the "theater"of the Title page!

The hearthstone would be like the foundation at the bottom of the Title page. A pair of classical pilasters would frame the firebox. And resting on top of them would be a small entablature. (These features might be made of either wood or metal.)

The whole thing would be in a 4 to 3 proportion, framing the smaller (but still 4-to-3-proportioned) fireback.

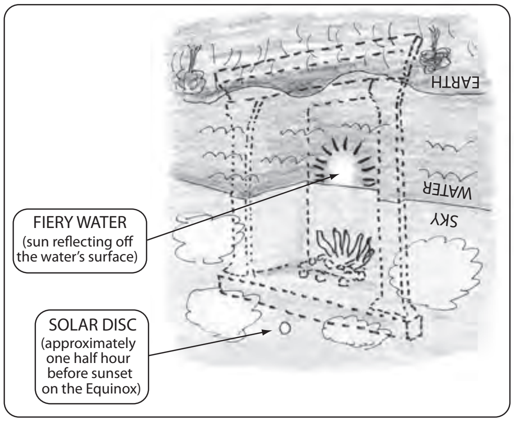
If the exterior of the Tower had a classy classic look (though faux), it makes sense that the the interior was similarly embellished.

Conjectured design of fireplace with Monas symbol on egg fireback

The entablature idea made a lot of sense. Not only would it help define the proportions of the opening, but functionally it would increase the size of the "smokebox."

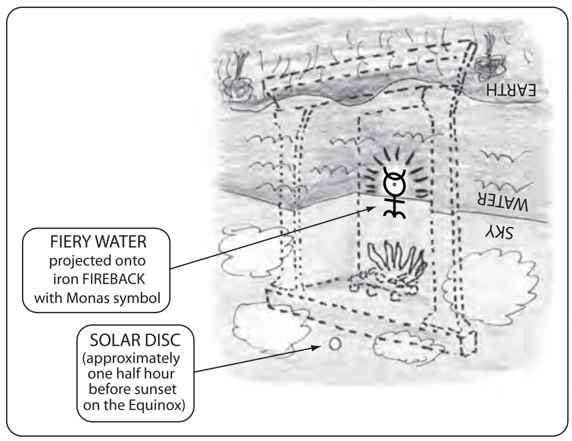
The "smokebox" is the upper part of the fireplace where smoke collects before finding its way up the 7-by-7 inch flues. The entablature would hold in smoke that might otherwise pour out the top of the arch and into the room.

The arch above the fireplace is nice, but it's hardly finished masonry. An arch can carry more weight than a stone lintel. Like the 8 arces spanning the pillars, I think it was there more for function than for form. And just as those 8 arches were hidden behind a "faux" entablature, I suspect this arch was hidden behind a "faux lintel."



Regardless of the design of the fireback, it most likely would have featured the Monas symbol in its center.

Thus, at sunset on the equinox, the "fiery water" display on the center of the fireplace ...



...would bathe the Monas symbol in shimmering light!

The "fiery water" display does its "light show" to the **right** of the firebox in the spring and summer. The Monas symbol on the fireback would be reignited on Sepember 21, the fall equinox. Then it progresses to the **left** of the fireplace during fall and winter. As the whole east wall celebrates this end-of-day phenomenon, I call the first floor room the "Sunset Room."

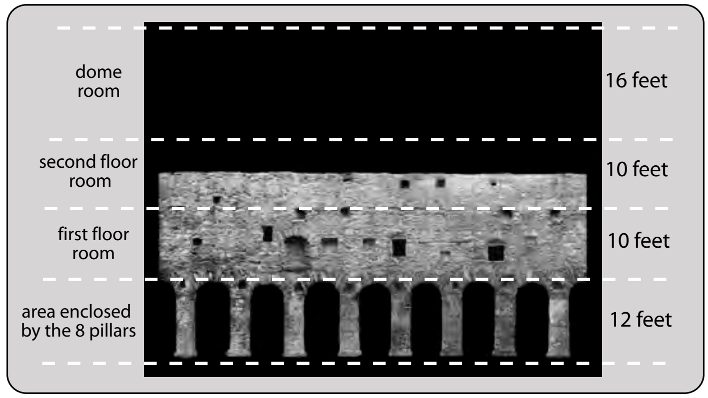
With an aperture in the roof of the building, a solar disc would march across the dome room floor from about 2-3 hours after sunrise to about 2-3 hours before sunset. A main feature of this display would be the meridiana line, where the solar disc crosses at noon. So I call the dome room the "Meridiana Room."

As per the "36 Boxes" chart of Theorem 22, we know that Dee saw "time periods" in terms of "Beginning, Middle, and End."

The Sunset room celebrates the End of the day.
The Meridiana Room celebrates the Middle of the day.
By simple deduction, the second floor room must celebrate the Beginning of the day.
It's the "**Sunrise Room**"!

Envisioning the Sunrise Room

The lower half of the second floor "Sunrise Room" (5 feet) still exists today.The upper 5 feet is missing. The top foot of what exists seems to have been have been reworked in the past . So it is possible that there once was a "sunrise" window on the eastern part of the wall.



Today, the eastern view of the horizon is obscured by the Newport Art Museum and surrounding buildings. But it's possible that from about 28 feet up in the Tower that a sliver of the Sakonnet River (where it meets the ocean) would be visible.

When the minister of the nearby Channing Church gave me permission to climb up into the belfry to photograph the Tower from above, I was able to see small areas of ocean to the east. But in general the Tower's view to the east is mostly EARTH, compared the sweeping vista over WATER to the west.

But there was an problem with my theory. Such an east-facing window would be about 3 to 4 feet above the 2 exhaust holes of the fireplace flues. An open window above billowing smoke just didn't make any sense.

Then it occurred to me that the second floor room was probably a camera obscura room just like the first floor room. The window could have been shuttered except for a small, one-inch-diameter hole in its center. (Actually the hole might was probably in metal insert that could be adjusted to make various size apertures.)

I tried envisioning what the image of the camera obscura projection would look like and realized Dee intentionally put the flue vents below the window!

The image of the smoke rising in front of the window would make a wonderfully mystical camera obscura image. It would fill the whole second floor with an image of puffs and swirls and wisps and billows of diaphanous smoke. (Outside the building, smoke is issuing forth from two vents distributes the smoke over a wider area than one vent would.)

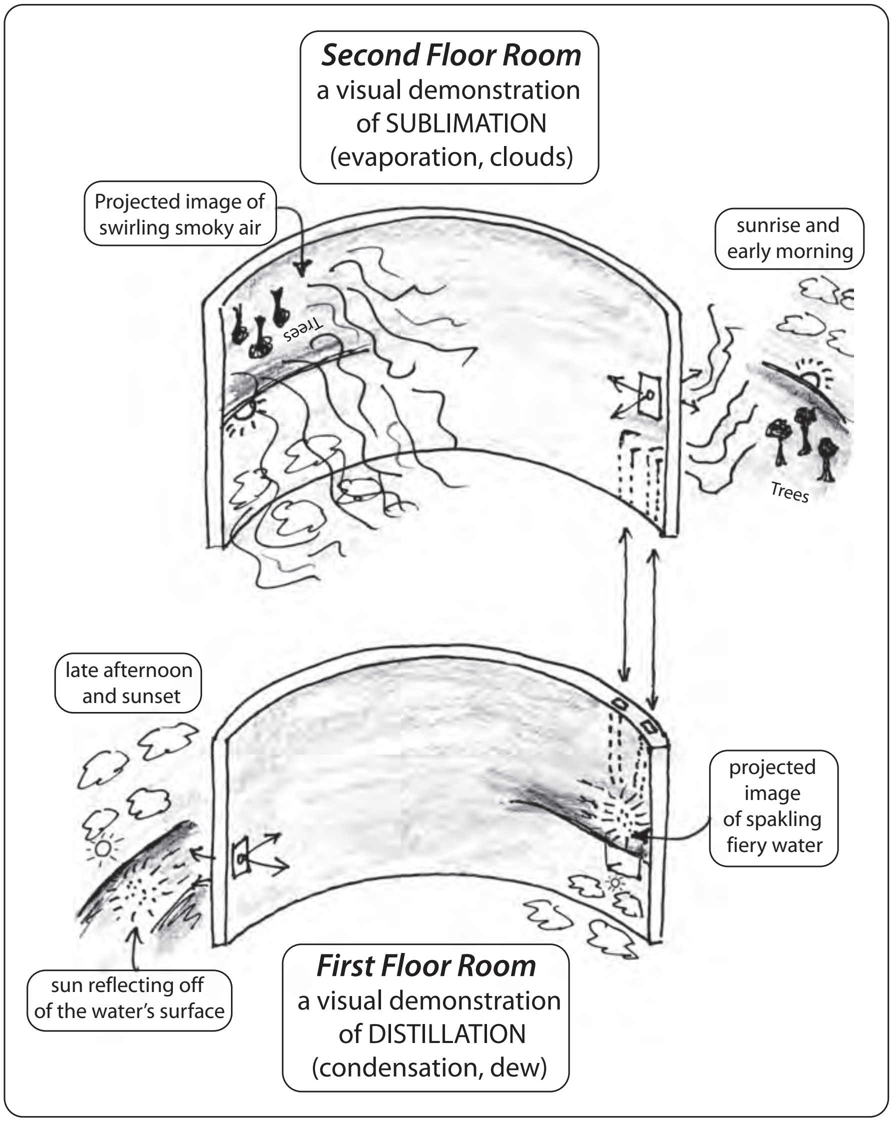
Having tried to photograph steam from a cup of coffee, I have learned that smoke needs to be viewed against a dark background and "back-lit" (meaning the light source comes from behind the subject). And this is exactly the lighting scenario Dee devised.

As the sun rises, looking out the east window, one would see the dark, shadowed side of any opaque objects like trees or hills. But the translucent smoke would be"back-lit"and light-colored, in dramatic contrast to those shadows.

The room would be smoke-free, but it would feel like you were "in a cloud." The smoke issuing from the vents would only partially obscure the scene outside. The effect would be a moving, translucent haze through which the bright, rising sun and some details of objects outside would still be visible.

Being in the room would feel like being inside a steaming retort during "sublimation."

In short, the second floor room featured a display of Sublimation. And in the first floor room featured its opposite. The water sparkling on the wall was a visual display of Distillation.



John Dee and Nature's dramatic lighting effects.

Would Dee have really thought about all this? **Yes**. Dee was a visual guy. He was interested in how the elements (fire, air, water, and earth) manifest and display themselves. And he was quite experienced with the wonders of various kinds of camera obscura images.

Dee seems to be describing such visual effects in Aphorisms 48, 49, and 52 of his *Propadeumata Aphoristica*.

In Aphorism 48, Dee reasons that just as we see the effects of the sun when it is below the horizon (the glow of dawn and dusk), we must also receive the light from planets and stars that are just below the horizon.

"When the sun is below our true horizon,
it furnishes rays of its accidental light to us through the air,
as it shown by the brightness of twilight.

Accordingly, the three superior planets and many of the fixed stars,
when they lie further below the horizon than the sun does
at the beginning of dawn or the end of twilight, will communicate the virtue
of their accidental light to us – though less sensible than the sun's light –
as if they had their own twilights.

I urge that the inferior planets should also be considered in this way.
As I have said, this is done not through any principal ray –
I mean direct, refracted, or reflected –
but through the species of a species, as philosophers
skilled in "optics and catoptrics" commonly say.
Observe why the sun's dusks are unequal, and study in the same way the dusks,
as I now call them, of the other planets."

(Dee, in Shumaker, p. 143.)

In Aphorism 49 he discusses the light from stars and planets (to Dee, the sun as one of the 7 planets) reflects on water (he actually describes "fiery water"!) and is fractured (broken into little pieces) by air and clouds.

"Investigate why the fixed stars and the various planets,
both those below the horizon and those situated elsewhere,
may reflect to us, or to other places on earth,
rays of their own light not merely from the heaven itself
but also from *the air, clouds, water,* mountains, and similar bodies.

Observe, too, the many fracturing of the heavenly rays
in the *air, the clouds, and the water,* and you will be driven
to wonder at and to praise the infinite goodness and wisdom of God."

(Dee, in Schumaker, p. 145, italics mine, for emphasis)

Schumaker cites a rainbow as an example of the "**many fracturings of the heavenly rays**." Another example might be the dramatic rays of light that pierce through a forest canopy on a sunny, but still misty morning. (Schumaker, p. 222-5.)

It's clear that Dee was not just aware of natural lighting effects, but quite moved ("**driven to wonder**") by their beauty.

Aphorism 52 deals with catoptrics (the art of using concave and convex mirrors) and its popularity with the ancients. Curiously, Dee also connects this optical science with inferior astronomy or alchemy, which he has already (by 1558) been studying intently.

The corollary seems to be explaining that images produced mirrors can appear quite real.

"If you were skilled in catoptrics, (Dee writes Katoptrikes in Greek)
you would be able by art,
to imprint the rays of any star much more strongly
upon any matter subjected to it than nature itself does.

This, indeed, was by far the largest part
of the natural philosophy of the ancient wise men.

And this secret is not of much less dignity
than the very august astronomy of the philosophers,
called inferior, whose symbols, enclosed in a certain Monad
and taken from my theories, I send to you along with this treatise.

Corollary

By this means obscure, weak, and, as it were, hidden virtues of things,
when strengthened by the catoptric art, may become quite manifest to our senses.

The industrious investigator of secrets has great help offered to him from this source
in testing the peculiar powers not merely of stars but also of other things,
which they work upon through their sensible rays."
(Dee, in Schumaker, p. 147)

Dee further discusses catoptrics in his 1570 *Preface to Euclid* under the Arte of Perspective, which is "the manner and properties of all Radiations Direct, Broken [Refracted], and Reflected."

At the beginning of the *Propaedeumata Aphoristica,* Dee claims that he has written 5 books on *Burning Glasses* (lenses), 2 books on *Perspective in Painting*, and 3 more books on the *Refraction of Rays*. None of these works have survived, but it's clear Dee was well-versed in options.

To conclude, the ideas of shimmering reflections of "fiery water" or the swirling vision of smoky air inside an empty dark room would have been among manifestations of the elements that would have fascinated Dee and he knew would fascinate others. He writes:

"This art of Perspective, is of that excellency
eafily beleve without Actuall profe perceived."
(Dee, *Preface*, Perspective, p. bj verso)

In other words, Perspective tricks used by artists can "fool the eye."
Well, in the first and second floor rooms of the Tower,
Dee has devised creative works of "installation art" that will also "fool the eye."

John Dee and Horometry

The other aspect of his "conceptual art rooms" is that they are integrated with the movement of the sun, which we use to mark time." Dee was exceptionally skilled in Horometry, and heincluded it as one of the Mathematical Arts in his *Preface to Euclid*.

Dee tells us that in his youth he:

"Invented a way, How in any Horizontall, Murall, or AEquinoctiall Diall & c. At all hours (the Sun shining) the Signe and Degree ascendant, may be known."

(Dee, Preface, p. d.i.j.)

(Signe means astrological sign.
"Degree ascendant" is the angular height of the sun above the horizon.)

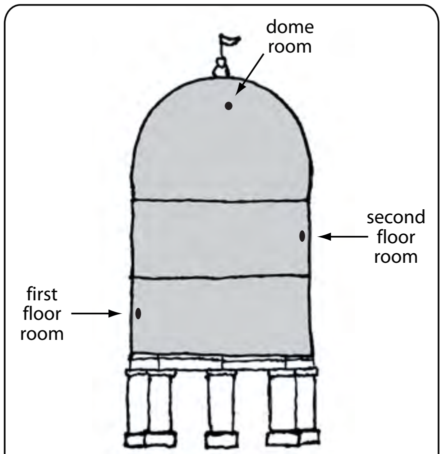
With a "Horizontal dial," the surface upon which the shadow (or solar disc) is projected is parallel to the earth's surface (like a floor). With a "Mural" dial, that surface is vertical, like a wall. With an "Equinoctial" dial, the surface is parallel to the plane of the ecliptic.

As I explained earlier, (but it's worth repeating here)
Dee hints about the camera obscura
at the end of his description of the Arte of Horometrie:

**"There remaineth (without parabolical meaning herein) among the Philosophers,
a more excellent, more commodious, and more marvelous way, then all these:
if having the motion of the Primovant (or first aequinociall motion),
by Nature and Art, Imitated:
which you shall (by further search in weightier studies)
hereafter, understand more of."**

(Dee, *Preface* p. d.i.j. verso.)

Not only is the Tower a compendium of Dee's interests, there are few people in Elizabethan times (and even today) that thought the way he did. The John Dee Tower is the John Dee Work of Art, or more succinctly John Dee's Brain.



3 camera obscura rooms

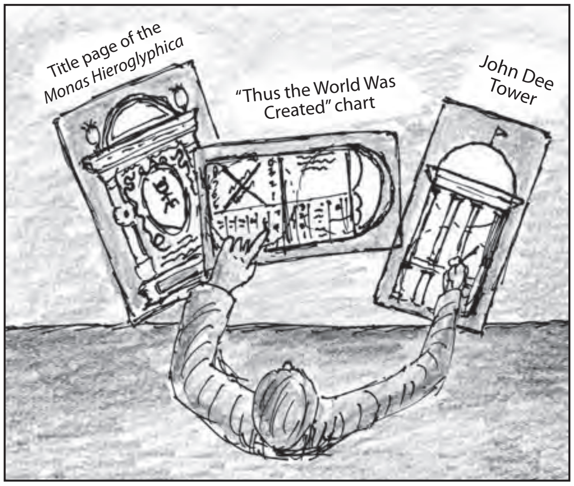
In summary,
I feel quite certain
that the Tower had a trio
of camera obscura rooms.

Sunrise (second floor room),
Noon (dome room),
and Sunset (first floor room)

Beginning,
middle,
and end.

In what year did Dee design the Tower?

The Tower was constructed in 1582, but Dee had designed it well before that. It is so well integrated with the Monas text and illustrations, its clear to me that Dee designed the Tower prior to 1564.



Bird's-eye view of Dee's desk, sometime around 1560

Dee had fully developed the mathematical cosmology and even designed the Monas symbol as its "summary" by 1558 when the *Propaedeumata Aphoristica* was published Then in the late 1550's and early 1560's the ideas of the Monas Hieroglyphica text and its architectural counterpart, the Tower, developed together, hand in hand.

That's why I refer to the *Monas Hieroglyphica* as a blueprint for the Tower. The *Monas* and the Tower were meant to be deciphered together. In other words, I think the Title page, the "Thus The World Was Created" chart and the John Dee Tower design plan were all on Dee's desk at the same time.

This is why my claim that the 2 "gaps" in the crescent-shaped reflections of the urns are a representation of the 2 flues in the tower is not as odd as it sounds. It's not that different than the similarities between the Tower's columns and the Title page's columns.

In fact, Dee probably felt his Tower design would be so famous there would be copies of it made throughout the world. (Perhaps someday that might happen.)

Nicholas Clulee states that the *Monas Hieroglyphica* was:

"for Dee a powerful symbol of cosmic unity and the unity of natural and divine knowledge."

(Clulee, *John Dee's Natural Philosophy*, p. 115)

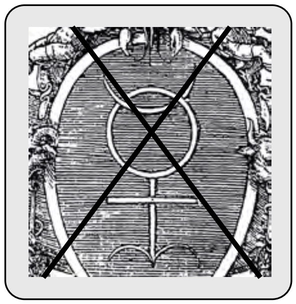
I suggest that Dee felt the same way about the Tower, the physical manifestation of the *Monas*. The ratios of its parts, the astronomical alignments, the camera obscuras, the special numbers it "infers," are simply "**Nature"** (artfully organized by Dee's mathematical mind.)

The Monas Hieroglyphica and the John Dee Tower were basically the same thing. One's a book, one's a building. They are both Dee's artistic expression of the "best of Nature."

Dee was obviously quite moved by his discovery of the integration of various mathematical and geometrical ideas. He wanted to share the "Laws of Nature" as he saw them, with the world.

Clues about the Tower's "sublimation and distillation" rooms on the Title page.

It's clear from Theorem 8 that Dee was fascinated by the idea that the Roman numeral for 10 was an "X," a graphic expression of "oppositeness." So let's rip a big X across the Title page. As the illustration is in the 4 to 3 ratio, the two diagonals do not intersect at right angles, but it's still quite obviously an X.

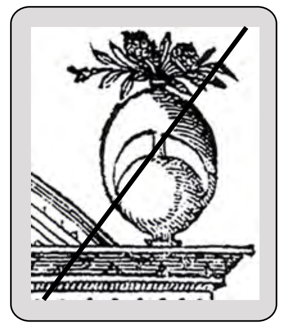


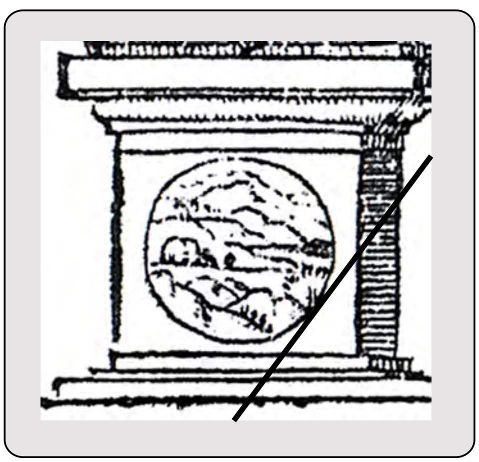
These two diagonals crisscross at the point in the center of the Sun circle.

Note that this is a depiction of the Title page "after restoration" (meaning that the emblem is centered in the theater).

Up above, the diagonals cut across those "gaps" in the reflections of the urns. This appears to be Dee's way of emphasizing the two flues in the John Dee Tower.

If this admittedly strange sounding assertion is true, we should expect to find a suggestion of "fiery water" down "below" on the Title page.

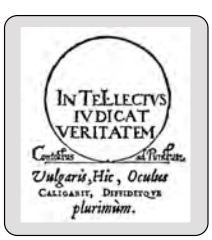




Let's zoom in for a close-up of the Water circle on the lower left pedestal. We've already seen that there appears to be a "thalamos," a one-room domed chamber on the left. This camera obscura room (althalmazat) is on the shore of a body of water (two bays or inlets are visible).

On the lower right edge of the circle are a group of radiating lines that suggest a bright reflection off of the water. It's a "fiery water" display that can be observed in the camera obscura room! It's depiction what takes place in the first floor room of the John DeeTower!

And to highlight it, Dee has placed this burst of light right where the Water circle is tangent to the diagonal line!



(This is another reason wht Dee included the "contact at a Point" emblem after Theorem 24.)

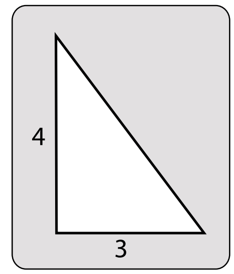
This means that Dee’s design for the tower was site specific, and this was something Deehad had determined prior to 1564. It had to have a western view that looked out over a body of water.

His house in Mortlake had just such a view. Even though Dee never crossed the ocean, he knew from Verrazano’s description and from Simon Fernandez’ reconnaissance mission that what is now Touro Park had a grand western view over water.

Geometric clues about the height of the Tower’s rooms on the Title page

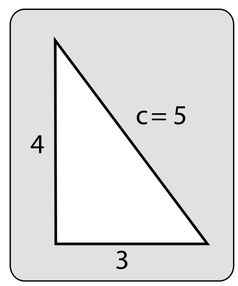
Each of the diagonal lines chops the Title page into two equal triangles. Let’s isolate one of these triangles for analysis.

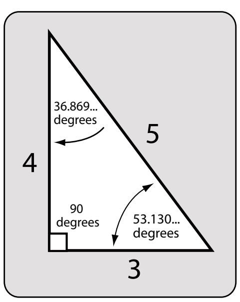
You no doubt remember how to solve this simple geomety question. If the two sides of this right triangle are 4 and 3, what is the length of the hypoteneuse?



$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = c^2$$
$$9 + 16 = c^2$$
$$25 = c^2$$
$$5 = c$$

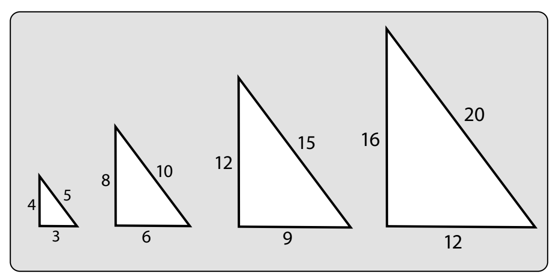
Pythagoras developed his famous Theorem to solve this one back in 500 BC.

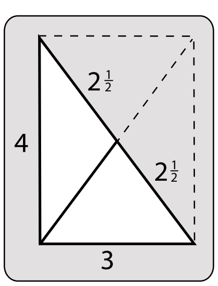




The sides of this 3-4-5 triangle are whole numbers. The 90 degree angle is of course a whole number, but the other two angles are not.

When the 3-4-5 triangle is doubled (to 6-8-10) or tripled (to 9-12-15)or quadrupled (to 12-16-20), all the angles remain the same.

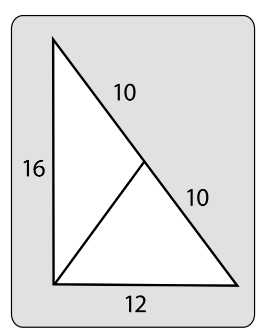




Next, let’s make two smaller triangles with a short line from the corner to the center. Although they might not look it, these two small triangles actually have the exact same area.

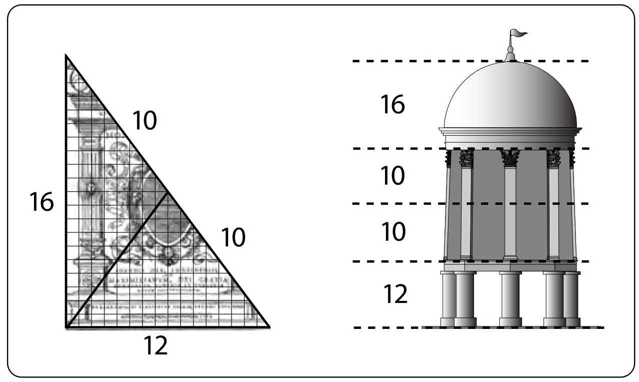
In the 12-16-20 triangle, splitting the hypotenuse in half makes two 10’s.

Do you see what we have now?

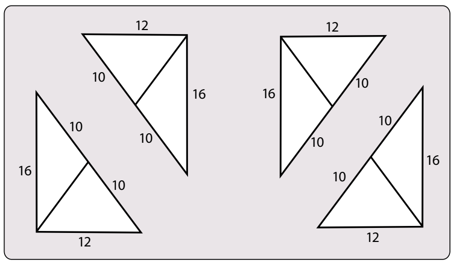


All the heights of the rooms in the John Dee Tower!

The 12 feet from ground level to the top of the pillar entablature.
The 10-foot-tall first floor room.
The 10-foot-tall second floor room.
And the 16-foot-tall dome room.



This hidden part of the blueprint is right in front of the reader’s face on the Title page. You just have need “geometry glasses to see it .



Indeed, these numbers (12, 10, 10, and 16) can be found in any of the large triangles created by the diagonals of the Title page.

Pythagoerean Triplets and some of Dee’s special numbers

Besides finding triangles which are multiples of the 3-4-5 triangle (like 12-16-20 or 36-48-60), the Pythagoreans found other right triangles whose sides are whole numbers.

In these “Pythagorean Triplets,” the hypotenuse is always “1 more” than one of the other sides. Examples include the (5-12-13) triangle and the (7-24-25) triangle. Do you recognize any thing special about these numbers?

Pythagorean Triplets

	x	$\frac{x^2-1}{2}$	$\frac{x^2+1}{2}$
x = 3	3	4	5
x = 5	5	12	13
x = 7	7	24	25
x = 9	9	40	41
x = 11	11	60	61
x = 13	13	84	85
x = 15	15	112	113
x = 17	17	144	145

Dee listed (12, 13) and (24, 25) in the “Thus the World Was Created” chart.

Besides relating to 3-D geometry (as in the cuboctahedron), they’re also important in 2-D geometry (right angle triangles).

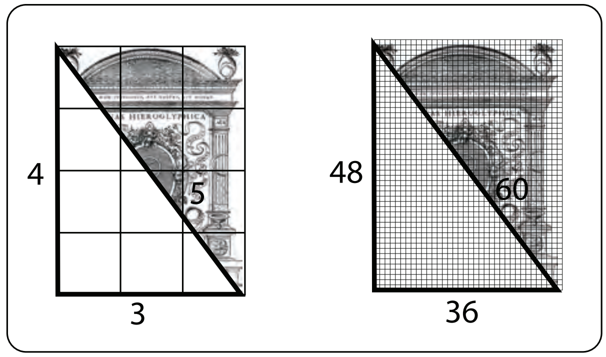
Though Pythagoras might have been aware of these Triplets, but they actually didn’t figure them out using an “unknown” like the “c” that I used in the earlier equation.

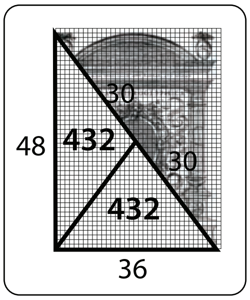
Diophantus of Alexandria (ca. 250 AD) was the first Greek to use an “unknown,” which was called an “arithmos” (literally “number”). Diophantus’ algebra was quite advanced compared to his contemporaries. (Dee owned a copy of Diophantus’ *Arithmetica*.)

(Roberts and Watson, Book 201, p. 214).

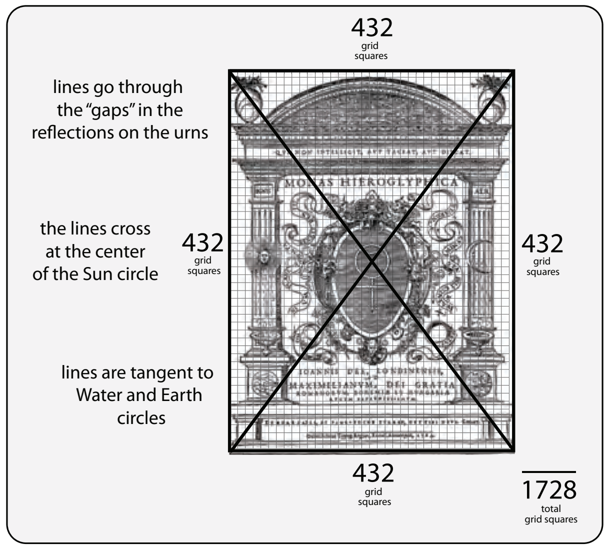
The diagonals and 432

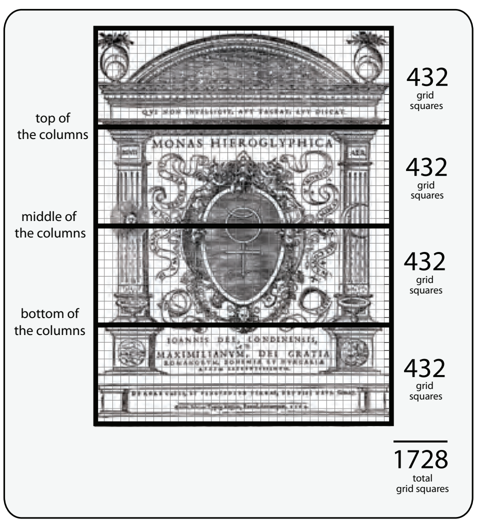
When the 3-4-5 triangle is scaled up 12 times, it becomes a 36-48-60 triangle. This is appears to be the main grid Dee used for the Title page.





As there are a total of 1728 grid squares, each of these four equal-area triangles contains 432 grid squares.





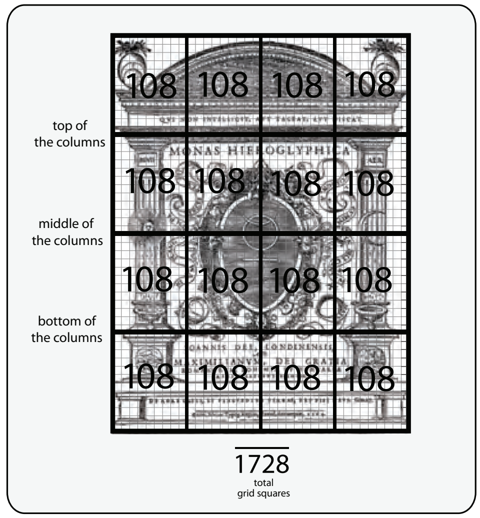
This number 432 is important to Dee. This can also be found by dividing up the Title page into 4 horizontal slices of 432 grid squares each. The horizontal lines fall at the bottom of the columns, the middle of the columns, and at the top of the columns.

Hindu Timekeeping
(in Years)

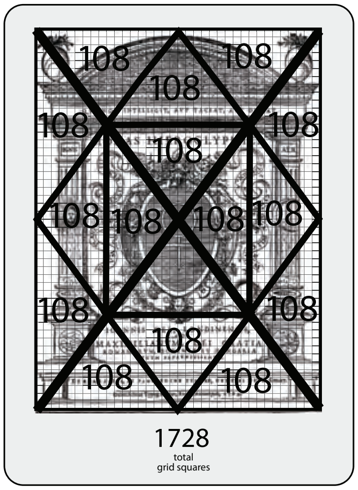
Kali Yuga	432,000
Dvapara Yuga	864,000
Treta Yuga	1,296,000
Krita Yuga	1,728,000

As we've seen, the number 432 was important to the ancient Hindu mathmeticians. A time period of 432 thousand years is a "Kali Yuga." The number 432 is special because it's 4 times the sacred Hindu number 108.

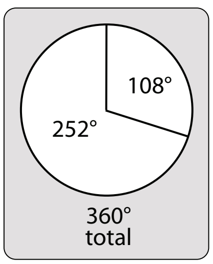
Each horizontal slice can easily be divided into 4 parts, making a 16 square-shaped boxes of 108 grid squares each.



But how can the triangles
(of 432 grid squares each)
be quartered ?

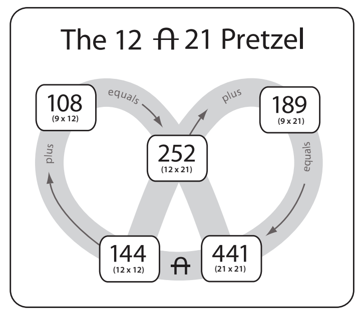


Easy! Simply connect the midpoints and 4 smaller, equal-sized triangles are formed. (This is yet another graphic depiction of Dee's expression "Quaternary rests in the Ternary.")



We've seen how 108 and 252 sum to Metamorphosis number 360.

We've also seen how 108 is integrated with Dee's Magistral number, 252, in the Syndex pretzel.



This chart shows a strange way that the multiples of 108 are related to every fourth multiple of 252.

Notice that the results are basically the same except for an **extra zero** in the hundreds column. (After 252 x 48 = 12096 this pattern changes, but its pretty amazing among these numbers).

As random as they seem to be, 108 and 252 are truly soulmates, cut from the same cloth.

A curious way that 108 and 252 are related

multiples of 108		various multiples of 252
1 x 108 = **108**	⟷	**1008** = 4 x 252
2 x 108 = **216**	⟷	**2016** = 8 x 252
3 x 108 = **324**	⟷	**3024** = 12 x 252
4 x 108 = **432**	⟷	**4032** = 16 x 252
5 x 108 = **540**	⟷	**5040** = 20 x 252
6 x 108 = **648**	⟷	**6048** = 24 x 252
7 x 108 = **756**	⟷	**7056** = 28 x 252
8 x 108 = **864**	⟷	**8064** = 32 x 252
9 x 108 = **972**	⟷	**9072** = 36 x 252
10 x 108 = **1080**	⟷	**10080** = 40 x 252

The Propaedeumata Aphoristica and the Monas Hieroglyphica express the same mathematical cosmology

As we shall see, there are so many cross-references (ideas, numbers, and code letters) between these two sister texts that it is evident that Dee had all of the main ideas of the Monas Hieroglyphica in his head in 1558, when the Propaedeumata Aphoristica was published. This should not be surprising for three reasons.

First, Propaedeumata Aphoristica means "Preparatory Aphorisms", so Dee obviously was "preparing" the reader for something yet to come.

Second, Dee mentions the Monas symbol in Aphorism 52 of the 1558 Propaedeumata Aphoristica:

> "**And this Secret is not of much less dignity then the most august, so-called, Inferior Astronomy, whose Symbols, which are enclosed in a certain MONAD based on our Theories, I send along with this treatise**."

1558
Propaedeumata Aphoristica
Title page

Third, he tells us in his 1564 Monas that he has been "pregnant" with the book for seven years. That means he had these ideas in 1557, a year before the 1558 Propaedeumata Aphoristica went to press.

The original 1558 Title page paled in comparison to the more finely crafted *Monas* Title page, so Dee replaced it in the 1568 version with the final page emblem from the *Monas*, which prominently features the Monas symbol. (Remember, this is the embem which can be folded upon itself, making a figure 8.)

In the 1568 edition, Dee made a number of minor wording revisions, but the only substantial alterations to the text are in Aphorism 18, Aphorism 73 and Aphorism 77.

1568
Propaedeumata Aphoristica
Title page

Aphorism 18

We've already seen how Aphorism 18 is a complete summary of how the energy of "zero-retrocity-one" creates 2, 3, and 4, and their interrelationships 1/2, 2/3, and 3/4. We might expect the "sentence additions" to Aphorisms 73 and 77 will express the cosmology embedded in the Monas Hieroglyphica as well.

Aphorism 73

Aphorism 73 explains that "inferior" (earthly) things imitate "celestial" things, not only in their "movement", but in other "properties and qualities."

In the 1568 edition, Dee added "Consectarium 1" and "Consectarium 2." A consectarium is an "inference", "something that follows logically." The prefix "con-" means "follow", so a "consectarium" is similar to a "conclusion" or "consequence."

"Inference 1" explains how a "diligent magus" will discover "a very great harmony" by seeing "stellar" things in the earthly "microcosm". The second and final sentence is a potent one.

Quae enim Uni	That which is One
Tertio convenient,	is an assemblage of Thirds,
& inter se convenientiam	so they must all be
habere necesse est.	in agreement with each other.

("Convenio" means "to assemble, join, unite, harmonize.)

To me this is a "loud and clear" (though cryptic) explanation of **the three parts of "zero-retrocity-one"** (which is also ȮȦṠ, or "circle-point-line").

Dee elucidates on these "Thirds" in "Inference 2":

"When any of two of these three have been noted,
what the other Third is can be deduced.
The anatomies of each of them
are peculiar to themselves, separately,
but they are also in the other two."

Zero, retrocity, and one certainly do each have their own "anatomy", but they are also intrinsically involved with each other; each is a part of the other two.

This can be seen in the Yin-Yang symbol. Even as the black swirl and the white swirl are "dancing their oppositeness dance", each contains within it a small circle of its opposite.

YIN YAN
HEREEEEE

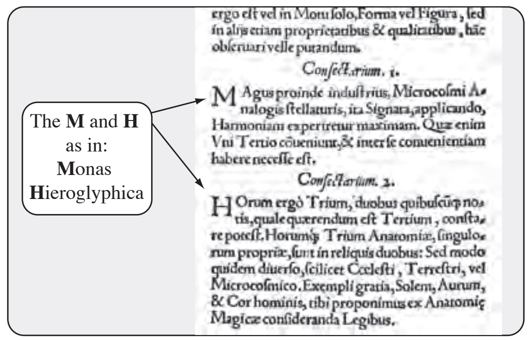
(In the Monas, Dee expresses the interrelationships of "pairs" of the members of "circle-point-line" in creative ways. The point and circle can be seen as the Sun circle, with its "visible center". The point and line can be seen as the " Yod with a Chireck on top" or the "letter i with it's dot on top". The line and circle can be seen in the number 10, which Dee notes that the "oldest Latin philosophers" honored with the letter X, a classic expression of oppositeness.)

Dee concludes with an example that obscures the first sentence of this Inference because it contains some very colorful concepts. But, note that Dee says that this example is "a different way" of looking at what the three things are.

> "**In a different way, it's evident that they are Celestial, Terrestrial, and Microcosmic. For example, I propose that the Sun, Gold, and the Heart of man are things to be considered by means of the laws of Anatomical Magic**." (Aphorism 73)

In short, the three "Thirds" of "the One" seem to relate to the "zero-retrocity-one", which is the OAS of Aphorism 18, which is also at a major theme hidden in the Monas Hieroglyphica.

Dee drops a very obvious supporting clue that he wants the reader to come to this conclusion. The large "drop cap" letter that begins Inference 1 is an "M", and for Inference 2, its an "H". If that doesn't shout out Monas Hieroglyphica, I don't know what does!



Aphorism 77

Aphorism 77 consisted of two rather obscure sentences in the 1558 text. Dee added 2 more sentences in the 1568 edition that quite clearly refer to his Monas mathematical cosmology.

Basically he explains how sometimes a weaker "agent" can appear to have a greater effect than a stronger "agent." Its unclear as to exactly what he is referring to here, but he adds "This is best known to those who have paid their respects to the threshold of the Holy Art."

Dee refers to the "Artis Sanctae" in Theorem 22 of the Monas where he reveals to King Maximilian an illustration containing the "vessels of the Sacred Art." (It had the letters that assemble to make the word LUX, and the Roman numerals that add up to 2520.)

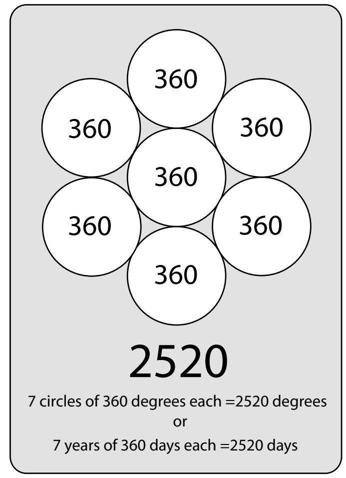
But the 2 new sentences he added in 1568 are even more revealing:

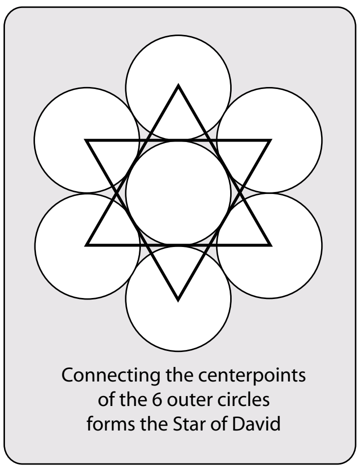
"What is Seven times Separated and
Seven times Cojoined in completing
the famous Earthly marriage?
This is, I affirm (God willing),
the Sabbatizat of David
which we express as Duality."

("Sabbatizat" is my translation of Dee's Hebrew letters
(Shin – Bet –Ayin Tav –Yod_Samech;
or Sh-B-A-T-Y-S),
as this is the word Dee uses in the *Monas*.)

When seven circles are joined together in closest -packing, the centers of the outer six circles form the Star of David.

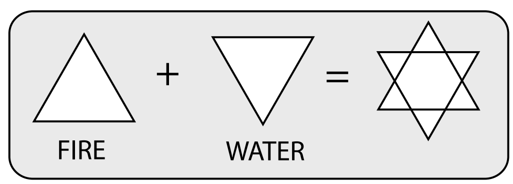
These circles can be seen as 360 degrees each, so the full assembly expresses 2520 degrees. Timewise, if each circle is a 360 day year, the seven circles make Sabbatizat, a 7-year period (the amount of time Dee says his mind was "pregnant with the *Monas Hieroglyphica*).

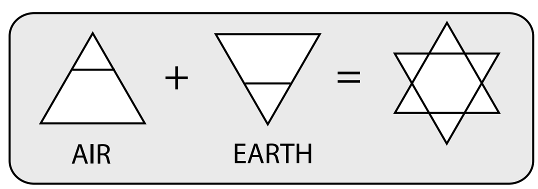




Dee is affirming that he sees the 2520 of his "arithmetical Metamorphosis sequence" ***and*** the 2520 of the "geometrical 7-circle arrangement" that forms the Star of David ***and*** David's "seven times" (a seven year period) in the Bible ***as all being the same thing.***

When Dee says "…which we express as Duality", he's referring to the alchemical arrangement of the fire-triangle joining the water-triangle or the air-triangle joining the earth-triangle.





Dee disguises his real arithmetical and geometrical intentions in theological and alchemical language quite thoroughly. To someone who doesn't catch his adrift, the whole thing sounds quite obscure. But it's really quite simple, and exquisite.

The hidden clues in the numbering Dee's Aphorisms

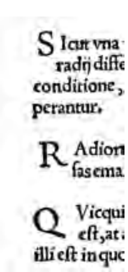
The third major change Dee made in his 1568 second edition was his method of enumerating the Aphorisms. There are some interesting things about those 26 Aphorisms which he chose to identify with Arabic numerals (versus the other 94 which he left in Roman numerals).

Most of them are either in the front or to the back of the book, with none in the middle. Prominent among them are the very first Aphorism (1) and the very last Aphorism (120) .

Notice that there are 3 large groupings. One is a cluster of 8, and the 2 other are clusters of 5 each. Finally, there are 2 "pairs" (8, 9 and 89, 90) and 4 "singles" (28, 92, 95, 120). The whole arrangement reeks of "hidden clues" to me.

1	XXV	LIX	LXXIII	XCVII
2	XXVI	L	LXXIIII	XCVIII
3	XXVII	LI	LXXV	XCIX
4	**28**	LII	LXXVI	C
5	XIX	LIII	LXXVIII	CI
VI	XXX	LIIII	LXXVIIII	CII
VII	XXXI	LV	LXXIX	**103**
8	XXXII	LVI	LXXX	**104**
9	XXXIII	LVII	LXXXI	**105**
X	XXXIIII	LVIII	LXXXII	**106**
XI	XXXV	LIX	LXXXIII	**107**
XII	XXXVI	LX	LXXXIIII	CVIII
XIII	XXXVII	LXI	LXXXV	CIX
XIIII	XXXVIII	LXII	LXXXVI	CX
15	XXXIX	LXIII	LXXXVII	CXI
16	XL	LXIIII	LXXXVIII	CXII
17	XLI	LXV	**89**	CXIII
18	XLII	LXVI	**90**	CXIIII
19	XLIII	LXVII	XCI	CXV
20	XLIIII	LXVIII	**92**	CXVI
21	XLV	LXIX	XCIII	CXVII
22	XLVI	LXX	XCIIII	CXVIII
XXIII	XLVII	LXXI	**95**	CXIX
XXIIII	XLVIII	LXXII	XCVI	**120**

The 26 Aphorisms which Dee identified with Arabic Numerals



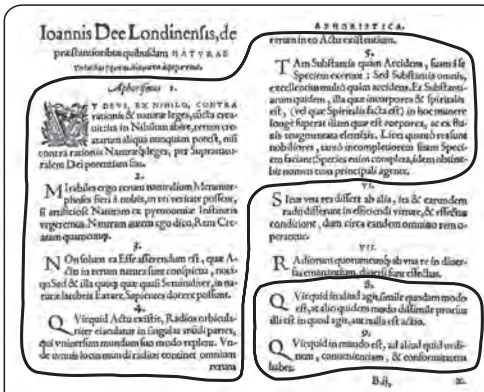
The other prominent graphic aspects of the text are the large "drop caps" at the start of each Aphorism. (The second letters of each of the Aphorisms are also slightly enlarged, but if they were included in the code there would be 240 letters, which seems far too unwieldy.)

All of the "first letters" of all of the Aphorisms are the same in the 1568 version as they were in the 1558 version. (One slight difference is that in the 1558 version, not all of them were enlarged with "drop caps.")

V	**1**	D	XXV	Q	XLIX	E	LXXIII	N	XCVII
M	**2**	S	XXVI	V	L	I	LXXIIII	E	XCVIII
N	**3**	T	XXVII	A	LI	Q	LXXV	D	XCIX
Q	**4**	**P**	**28**	K	LII	U	LXXVI	P	C
T	**5**	Q	XIX	S	LIII	A	LXXVIII	V	CI
S	VI	M	XXX	Q	LIIII	N	LXXVIIII	V	CII
R	VII	D	XXXI	Q	LV	S	LXXIX	**L**	**103**
Q	**8**	Q	XXXII	E	LVI	Q	LXXX	**V**	**104**
Q	**9**	S	XXXIII	M	LVII	E	LXXXI	**L**	**105**
Q	X	R	XXXIIII	O	LVIII	P	LXXXII	**S**	**106**
M	XI	A	XXXV	Q	LIX	E	LXXXIII	**A**	**107**
S	XII	O	XXXVI	Q	LX	L	LXXXIIII	V	CVIII
S	XIII	O	XXXVII	P	LXI	P	LXXXV	C	CIX
S	XIIII	O	XXXVIII	A	LXII	E	LXXXVI	A	CX
N	**15**	P	XXXIX	C	LXIII	Q	LXXXVII	I	CXI
Q	**16**	A	XL	P	LXIIII	P	LXXXVIII	S	CXII
P	**17**	Q	XLI	D	LXV	**P**	**89**	O	CXIII
I	**18**	E	XLII	A	LXVI	**Q**	**90**	O	CXIIII
S	**19**	E	XLIII	A	LXVII	N	XCI	E	CXV
E	**20**	Q	XLIIII	L	LXVIII	**D**	**92**	Q	CXVI
S	**21**	H	XLV	P	LXIX	L	XCIII	P	CXVII
S	**22**	O	XLVI	V	LXX	S	XCIIII	C	CXVIII
O	XXIII	O	XLVII	V	LXXI	**V**	**95**	X	CXIX
I	XXIIII	S	XLVIII	V	LXXII	I	XCVI	**I**	**120**

The first letters of 26 Aphorisms
which Dee identified with Arabic Numerals

This chart shows all of the 120 “starting letters”. (For emphasis, I have put the ones that begin the 26 Aphorisms identified with Arabic numerals in bold type.)



Aphorisms 1-5 (and also 8 and 9)
are identified with Arabic numerals

Let’s investigate them, one
“grouping” at a time.

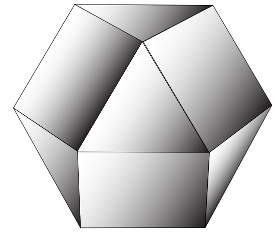
Do you know what
V M N Q T
of ***Aphorisms 1, 2, 3, 4, 5***,
might stand for?

V	**1**
M	**2**
N	**3**
Q	**4**
T	**5**
S	VI

They speak volumes in Dee’s language. As 1 2 3 4 is more important to Dee than 1 2 3 4 5, lets put the Aphorism 5’s letter” T” aside for a moment,and look at just V M N Q.

Recall that the “Q and V” are the first letters of the “Letter to Maximilian” and the “Letter to Gulielmo Silvius” respectively. There, they are hidden-code for the word **QVality** (or **QUality**, which is a little easier on the modern eye).

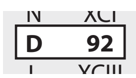
Can you figure out what the “**Quality of M and N**” refers to?



It’s simple. Dee’s letter/number code for the letters M and N are the numbers **12 and 13**.

Dee features this pair of numbers in both of his summarizing charts in the Monas. The “Quality of 12 and 13” is how they describe the 12 around 1= 13 “closest packing of spheres” arrangement, with its cuboctahedral shape.

This decipherment helps bind together two aspects of Dee’s cosmology:
the “arithmetical” 1, 2, 3, 4, (with their 3 harmonies 1/2, 2/3, 3/4) and the “geometrical” cuboctahedron. (In the Monas, Dee binds them together in his Artificial Quaternary, where the “essence of “1, 2, 3, 4” ,which is “1, 2, 3, 2”, multiplies to 12)



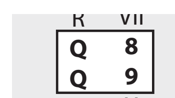
A confirming clue that Dee is referring to the “closest packing of spheres” here is that he uses an Arabic numeral for Aphorism 92. There are exactly 92 spheres in the third layer of “closest packing of spheres.”
Using Euler’s formula [10 times the layer number, +2], the number of spheres per layer goes 12, 42, 92, 162, 252, 362, 492…. You can see that only three of these are low enough to have been chosen by Dee, as he only wrote 120 Aphorisms.

That letter T, which starts Aphorism 5, is also closely assiciated with the letter M in the *Monas*. The letters T and M are at the heart of the “36 Boxes” diagram of Theorem 22. They were important to Dee as a grammatical link between GEO**M**E**T**RY and ARI**T**H**M**ETIC, two expression of the same thing in the art of MA**T**HE**M**A**T**ICS.

Thus, Dee seems to be also expressing the” QVality of M and T, as well as the “QVality of M and N”. Dee saw the cuboctahedron as an expression of the beauty of geometry as well as of arithmetic (12 vectors, 24 edges, etc).

Aphorisms 8 and 9 both begin with a Q

The fact that Aphorisms 8 and 9 both start with the letter Q (Dee’s “Quality”) seems like a pretty clear (yet still cryptic) reference to the “octave, null nine” of Consummata, or the Lunar Mercury Planets number (8) and the Solar Mercury Planets number (9) of the *Monas*.

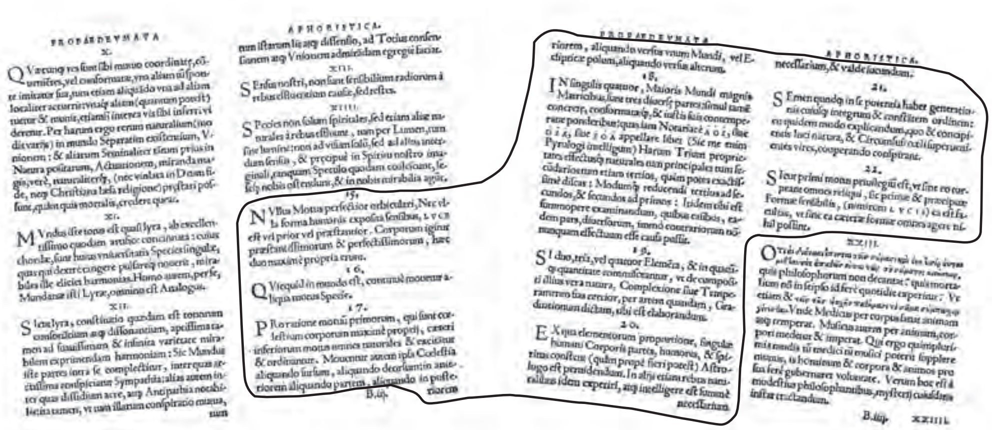


The idea that the 8 and 9 (each honored with a Q for quality) refers to the “octave, null nine” relates with the idea that Dee hid such an important summarizing clue in **Aphorism 18.** The number 18 if the null number which follows the “second octave of number”in Consummata, (10-17, null 18).

A super-confirming clue in the "first letters" of Aphorisms 15 – 22

The largest "grouping"of Aphorisms idenified with Arabic numerals includes the 8 Aphorisms from 15–22.

This group is ***especially important*** besauce it includes the very revealing ***Aphorism 18***!



The 8 Aphorisms identified with Arabic numerals
(from Aphorism 15 through Aphorism 22)

Here is the "octave "of their "first letters: **NQPISESS**

The" three S's" were discouraging. Not many words have three S's. Undaunted, I noticed that the last three letters were ESS, and the first letter was N, making NESS, quite possibly the suffix "-ness."

Rearranging the letters, I got: **QPIS -NESS**

The letters QPIS didn't look very hopeful. As any Scrabble player knows, the letter Q is virtually useless without its pal, U. It occurred to me that Q might be an O in disguise. But, alas, there was no A, to possibly make OAS. Hoverer, there was an S, Dee's hidden code-letter for "line."

S

And, there was an I, the letter that seems appears Dee should have chosen rather than S to represent "line" (if it wasn't so obvious). Could Dee be making some statement about how he considers "S" to simply be a sinuous "I"?

I

That would leave Q and P. Hey, we know what he means by that! **Prime Quality**, the solution to the puzzle of the large decorative letters that begin the three parts of the Monas (QVP). (A confirming clue is that when the second letters of Aphorisms 16 and 17 are included, they reveal: **Qu** and **Pr,** the first pairs of letters in Prime Quality.

PQ

I said the words aloud:

"Prime Quality, S, I, -NESS."

I added a few small words to make it clearer:

"The Prime Quality of (the letter) S is (the letter) I-NESS."

In other words:

"The Prime Quality (of the curvy line) of an S
is (the straight line) I-NESS"

Furthermore, it doesn't take much to see "(the letter) I –NESS" as "**oneness**". The letter "I" is essentially a vertical line. Our Arabic numeral "1" is essentially a vertical line. Even the Roman numeral "I" is a vertical line. (Indeed, "I" is the only numeralok,ol whose symbol is common to both numbering systems.)

Dee seemed to be expressing the idea that:

"The Prime Quality of "Line" is Oneness"

I was reminded of Martin Mull's wedding vows in the, Marin County new-age marriage of the 1980 movie Serial which began: "You-ness…..Me-ness…..One-ness." Wondering how old the word "oneness" was I hit the dictionaries.

The suffix "-ness" has been used to denote a "quality, state, or condition of being" starting way back in Old English (ca. 450 AD–ca. 1150 AD). Words like "goodness", "darkness" and "kindness" have been used for well over 1000 years.

More specifically, the word "oneness" was used throughout the 1200's, but became obscure from around 1300 to around 1500. But in the 1500's, it came back strong. Nicholas Harpsfield in his 1555 "A treatise on the pretended divorce between Henry VIII and Catherine of Aragon" writes: "For the oneness & conformity of mind that both were in, touching this matter." (OED, oneness, page 97)

Thus, it's reasonable to conclude the Dee used the word oneness here to mean the "quality" of "one", represented by a line, disguised as the letter S, in the cryptic trio OAS.

Dee apparently inserted this jumbled-letter code here to help those "non-Pyrologians" trying to decipher what the heck AOS, OSA, and SOA meant. (Or as a confirming clue to those who have already solved the puzzle).

Given this (cryptic) declaration of one of the three "Prime Qualities" is the "line" helps confirm that the other two are the "circle" and the "point" (as per Theorems 1 and 2).

The idea that the letter O is a "circle" is a no-brainer.
This means that the pointy-tipped A is indeed the "point".

So, if a line is "oneness", what would circle and point be?

If a line is the Arabic numeral 1,
then the circle is most likely the Arabic numeral 0 (or zero-ness).

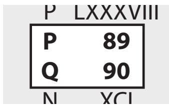
Then what can "point" possibly be?

As Dee says in Theorem 2, the circle and line would not even exist if it weren't for the point. The point certainly isn't "equality" or the "equals sign" because a line certainly doesn't really "equal" a circle. Likewise, having the point symbolize the "multiplication sign", the "addition sign", or the "greater than" sign doesn't make much sense either.

"Oppositeness" fits perfectly.

Like the Sun and Moon of the Monas, like the Hot and Cold or the Wet and Dry of Dee's Art of Graduation, like the two Mercuries on the Title Page, "zero and one" form a perfect pair of "opposites." ***Oppositeness*** is the "third thing."

Aphorisms 89 and 90 are easy clues to decipher



Next, the first letters of Aphorisms 89 and 90 are P and Q, respectively. This seems to be a confirming clue for the idea that the “P Q” in the Aphorism 15–22 octave means “Prime Quality”. And also that the Q in Aphorisms 4, 8 and 9, refers to “Quality.”

Aphorisms 103 -107 begin with letters that are meaningful to Dee

The 5 Arabic-numeraled Aphorisms 103-107 are L,V, L, S, and A, recpectively.

This seems to be shorthand for Dee’s discussion on the Cross of the Elements being seen as two “L’s” or two”V’s” (in Theorem 16). The “S, meaning line”,and the “A meaning point” could be a hint at his discussion of the Cross being Ternary (2 lines and a point) of Quaternary (4 lines), as per Theorems 4 and 20)
There is only one “V” in this group, but neighboring on both sides of the grouping of 5 Arabic-numeraled Aphorisms are two more V’s (Aphorisms 102 and 108).

In short, L V L S A appears to be a synopsis of several theorems of the *Monas*:
Dee wants us to investigate the “lines” (S) of the Cross of the Elements and the “intersection “point” of those lines (A) and also, to separate that X into L’s or V’s, because “then a LIGHT will appear.”

This analysis of the first letters of Aphorisms 103–107 corresponds with another graphic feature of the text. Just as the very first letter of the 3 main parts of the *Monas* are the greatly-enlarged and decoratived letters “P, Q and V”, the 1558 version of the *Propaedeumata Aphoristica* had 2 parts, the Letter to Gerard Mercator and the 120 Aphorisms, both of which started with a large decorative letter “V”.

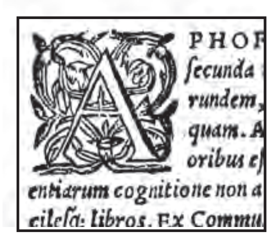
Beginning of the Letter to Gerard Mercator

Beginning of the very first Aphorism

These are the two V’s that join to make of Cross of the Elements in Theorem 16 (and discusses in depth in Theorem 20) where the only difference between the Cross being Ternary and Quaternary is that central point.

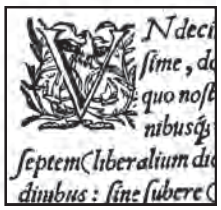
*"To the reader
who is studious
in the purer philosophy,
John Dee of London
sends hearty greetings."*

Well, in the 1568 version of the Monas, Dee added a third "section" (between the existing two sections) entitled:



The first letter of his short, one-page greeting is an extra-large, decorative letter "A."

Dee has added the idea of the "***point***", (**A**), to the two **V**'s, in an expression of the Cross of the Elements being Ternary or Quaternary.



Beginning of the Letter to Gerard Mercator

Beginning of Letter to the Reader

Beginning of the very first Aphorism

Dee seems to be making a reference to all these "letter clues" in his Letter to the Reader, when he explains that:

"the Syntagma is filled with marvelous and honorable ornamentation."

Dee writes "Syntagma" in Greek. It means "that which is put together in order"like "an arrangement" or a "collection"; from it we get our modern words syntax and syntagm (yes, its still a word).

There is no other "ornamentation" in the book besides the 3 decorative "first letters" and the 120 "drop caps" that begin the Aphorisms. Dee's Latin word "ornamentum", which comes from "ornare" (adorn), means a "ornament" in the sense of "decoration or embellishment".

Finally, let's look at the "Aphorisms identified with Arabic numerals" that are "singles" (meaning *not* in a group): 28, 92, 95, and 120.

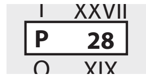
N XCI
D 92
I XCIII

We've seen how 92 is the number of spheres in the third layer of closest packing of spheres.

S XCIIII
V 95
I XCVI

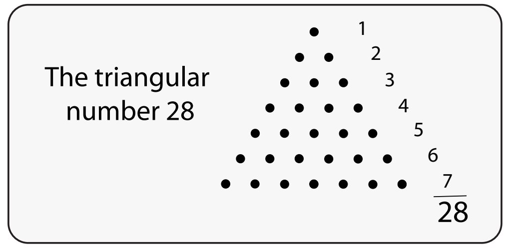
I'm not sure how the number 95, which is 5 x 19, fits in to the scheme of things. Perhaps it's an adjustment number for a calculation which we will see in the next chapter

The numbers 28 and 120 are much more exciting!

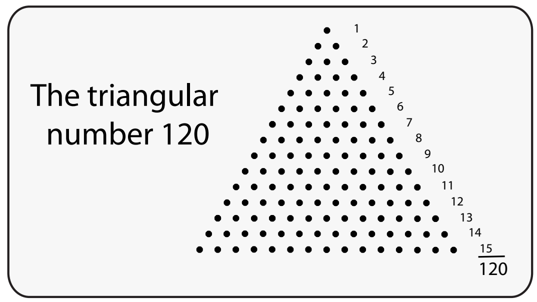


The Greeks, Neo-Platonists, and Boethius all celebrated 28 as a "perfect number", because its divisors (1,2,4,7, and 14) add to 28. (They adored reason "perfect number 6" for the same reason).

But Dee seems to be highlighting it here for a different reason--***because it's a triangular number***. Its the sum of the digits 1 + 2 + 3 + 4 + 5+ 6 + 7. Let's picture it as a triangle of dots, like Pythagoras' tetraktys.



The number 120 is important not only because it is the total number of Aphorisms, but because its also is a triangular number. Its the sum of (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15) is 120.



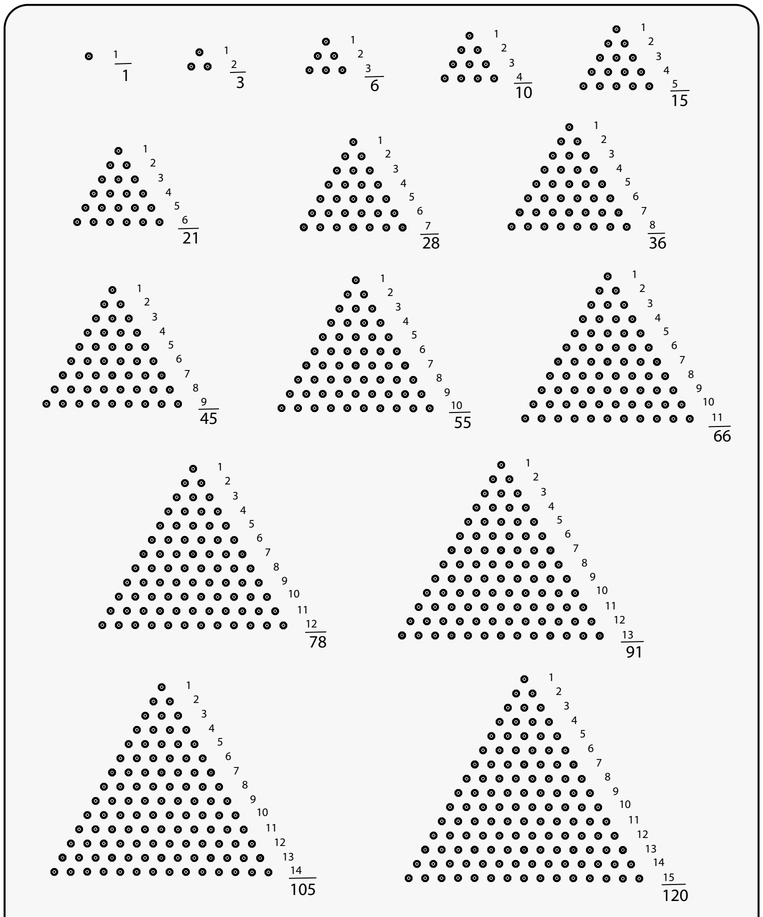
The first 20 triangular numbers

the sum of these "ranges"...	...makes these triangular numbers
1	1
1-2	3
1-3	6
1-4	10
1-5	15
1-6	21
1-7	28
1-8	36
1-9	45
1-10	55
1-11	66
1-12	78
1-13	91
1-14	105
1-15	120

This is a clear hint that Dee wants us to explore the triangular numbers. Seeing them as a list isn't very exciting. But they become more alive when seen in graphic form, as their anatomies and relative sizes become more apparent.

Nicomachus,in his famous texts on arithmetic, says that triangular numbers, "when expressed graphically", are "at once triangular and equilateral."
(Nichomachus, *Intro.to Arithmetic*, Book 2,Chapter8)

Boethius explains that the first trianular number is a triangle "in power, but not in act and operation." But, he adds that its "natural potency"creates the triangular number 3, "the first triangle in operation and act." Here again, "one" isn't seen as a number, but as the source of number.
(Boethius, *Arithmetica*, Book 2, Chapter7)

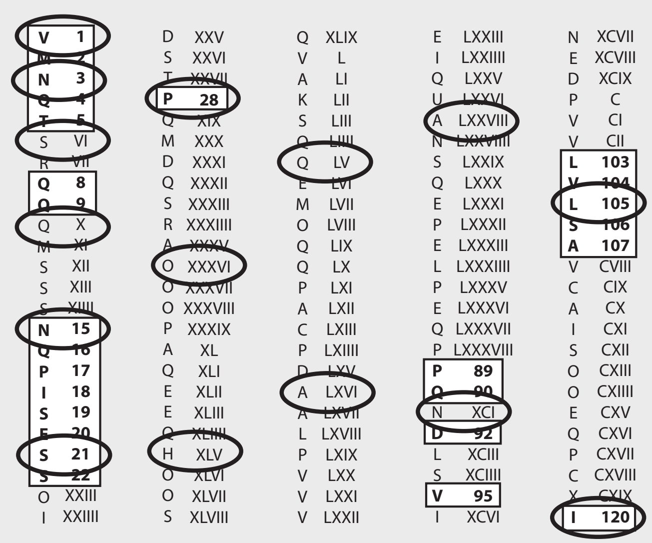


The 15 Triangular Numbers up to 120

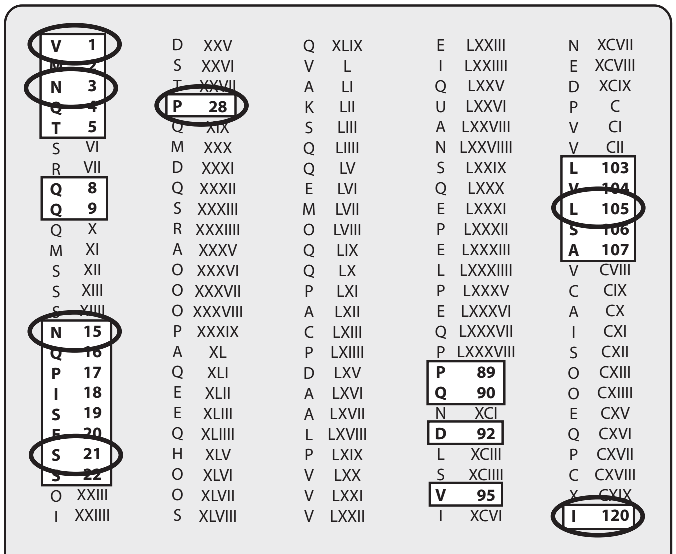
“when expressed
graphically”
trianlular numbers are
“at once triangular
and equilateral.”
(Nichomachus)

On the chart of Dee’s 120 Aphorisms
I’ve encircled all of the triangular
numbers.
Curiously, 7 of these triangular
numbers are also the numbers of
Aphorisms which Dee identified with
Arabic numerals!

(The 15 triangular numbers account
for ***about 12%*** of all the numbers
up to 120, yet Dee has highlighted
almost 50% of them.)



The first 15 triangular numbers

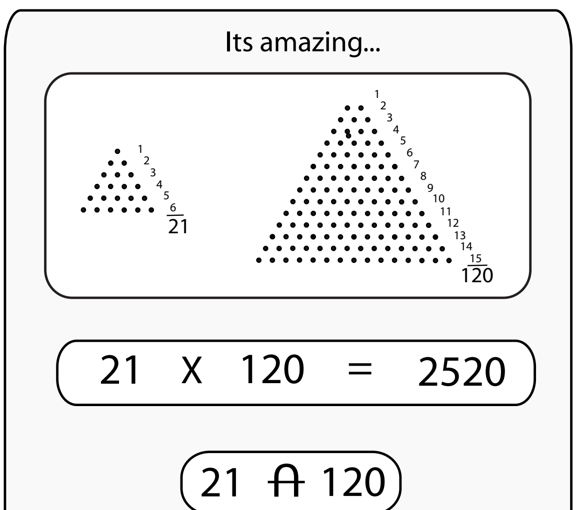


Dee calls attention to triangular numbers by identifying 7 of the first 15 of them with Arabic numerals

Dee seems to be imploring the reader explore the triangular numbers, especially the ones he has highlighted.

The observant reader will shortly come upon that gem of a number 21 (which Dee praised in Theorem 8 as being the "Ternary times the Septenary").

He will realize that 21 and 120 have three important "interconnections." Not only are they both ***triangular numbers***, but they ***multiply to 2520***, and they are ***reflective mates***!



21 and 210 are not only both triangular numbers, they're reflective mates!

This will incite the eager reader to explore how triangular numbers relate to ***all*** of the divisors of 2520.

The number 2520, famous for being the lowest number divisible by all the single-digits, has many double-and triple-digit numbers as divisors. Interestingly, if 1 and 2520 are included,it has ***exacly 48 divisors, or 24 "pairs "of divisors.***

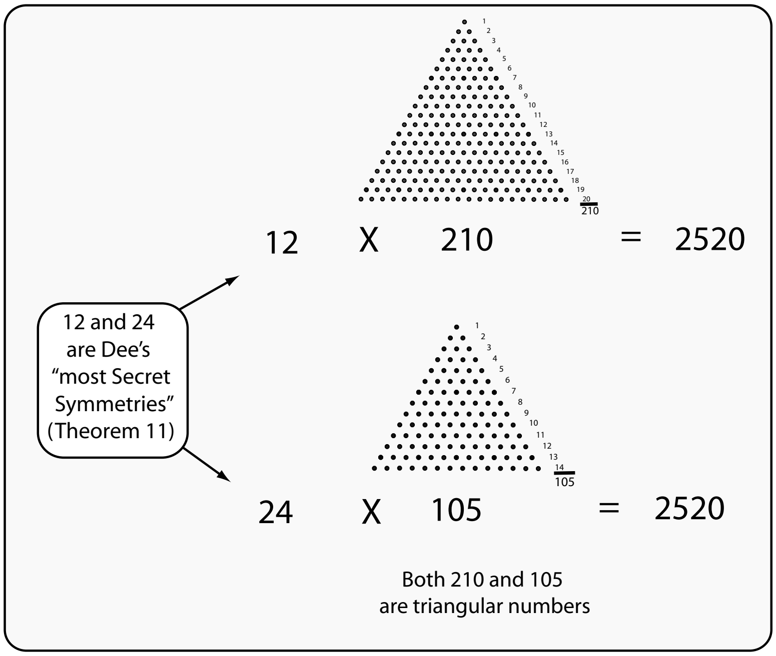
Notice that in this compilation, the first nine triangular numbers, as well as 105 and 120, are also divisors of 2520. There are some clusters (55, 66, 78, 91) and (136, 153, 171, 190) which are not divisors of 2520. I've also included triangular numbers to 210 and 630 (but for simplicity's sake I have not listed all the triangular numbers in between them which are not dividers of 2520).

triangular numbers	divisors of 2520
1	1
	2
3	3
	4
	5
6	6
	7
	8
	9
10	10
	12
	14
15	15
	18
	20
21	21
	24
28	28
	30
	35
36	36
	40
	42
45	45
	56
(55)	60
(66)	63
(78)	70
(91)	72
	84
105	105
120	120
(136)	126
(153)	140
(171)	168
(190)	180
210	210
	252
	280
	315
	360
	420
	504
630	630
	840
	1260
	2520

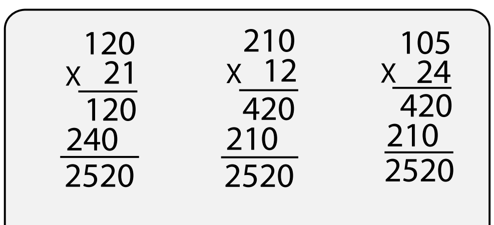
To see the "pairs" of divisors which multiply to 2520, let's fold this list over on itself. This makes that spectacular relationship between 21 and 120 graphically evident.

triangular numbers	divisors of 2520		more divisors of 2520	more triangular numbers
1	1	X	2520	
	2	X	1260	
3	3	X	840	
	4	X	630	630
	5	X	504	
6	6	X	420	
	7	X	360	
	8	X	315	
	9	X	280	
10	10	X	252	
	12	X	210	210
	14	X	180	(190)
15	15	X	168	(171)
	18	X	140	(153)
	20	X	126	(136)
21	21	X	120	120
	24	X	105	105
28	28	X	90	
	30	X	84	(91)
	35	X	72	(78)
36	36	X	70	(66)
	40	X	63	(55)
	42	X	60	
45	45	X	56	

Exploring this chart further, the reader will soon discover something special regarding two of Dee's "most secret symmetries", the numbers 12 and 24. Though not triangular numbers themselves, they multiply by triangular numbers to arrive at 2520. The first Metamorphosis number, 12, (that highly composite "docena") multiplies by triangular number 210 to make 2520. The second Metamorphosis number, 24, (the number of hours in a day) multiplies by 105 to make 2520.

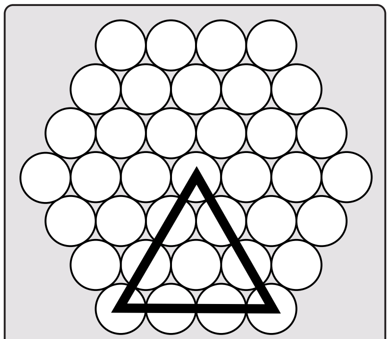


Both 210 and 105
are triangular numbers



"Long multiplcation"
reveals more
interweaving relationships

Looking at the "long multiplication" (not the short-cut, hand-calculator method) provides a glimpse "under the hood" of how these things multiply to 2520. In the "inner workings" here, eliminate the zeros and there's really nothing but 12, 21, 24, or 42.



Triangular numbers
are related
to closest packing of circles

In summary, Dee is showing us another way in which arith**met**ic and geo**met**ry are two sides of the same coin. When triangular numbers are seen as circles instead of dots, it's clear that they are intimately related to the closest packing of circles (and remember the first layer of closest packing of circles tells that wonderful story about 7 X 360= 2520). Dee signed his name with a triangle, and Dee loved numbers, so it its logical that Dee would have loved triangular numbers.

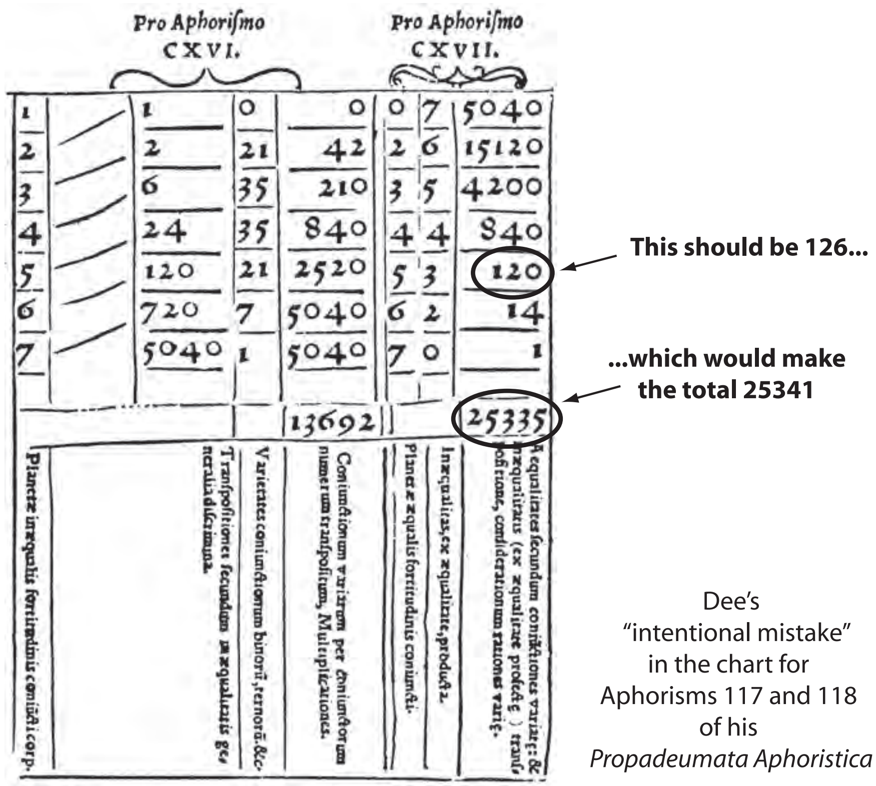
Dee has cryptically hidden all this mathematics "just under the surface" of Propaedeumata Aphoristica's framework. He didn't highlight ***all*** the triangular numbers by using Arabic numbers for their Aphorisms–but he highlighted ***just enough*** important ones to provide sufficient clues to his intent.

The case of the well-concealed 1234

On top of these letter and number code secrets seen so far, Dee has concealed yet another big secret in the Arabic numerals.

The chart in Aphorism 118 (the only complex chart in the whole text) shows the number of "conjunctions" that the 7 planets can be in with each other. A "conjunction" means when two planets are either 0 degrees, 30 degrees, 60 degrees, 120 degrees or 180 degrees from each other.

Without going into all the math, there are basically 2 parts of this chart. The left half explains a number of possible conjunctions when 2 "planetary powers" are the same (data explained in Aphorism 117).



1	/	1	0	0	0	7	5040
2	/	2	21	42	2	6	15120
3	/	6	35	210	3	5	4200
4	/	24	35	840	4	4	840
5	/	120	21	2520	5	3	120
6	/	720	7	5040	6	2	14
7	/	5040	1	5040	7	0	1
				13692			25335

Dee's "intentional mistake" in the chart for Aphorisms 117 and 118 of his *Propadeumata Aphoristica*

The right half explains the number of possible "conjunctions" when the "powers of the 2 planets are either equal or unequal." (data explained in Aphorism 118).

J.L. Heilbron notes that Italian Renaissance mathmeticians Luca Pacioli (1446-1517) and Girolamo Cardano (1501-1576) worked with problems of permutations. So did Rabbi ben Ezra, way back around 1140, "from whom Dee's analysis probably descends." (Shumaker and Heilbron, p. 91)

The final column adds up to **25,335**, which doesn't appear related to any of Dee's special numbers. But the keen-eyed Wayne Shumaker noticed that Dee "slipped in computing the number of conjunctions when five and only five bodies are equal." Schumacher writes, "the five can be chosen in 7! 5!/2! = 21 ways and can be ordered among the remaining two in 3!= 6 ways, making **126** possibilities **[not 120]**."

While Shumaker sees this as a computational error, there are for several good reasons why it should be recognized as another of Dee's "hidden" clues:

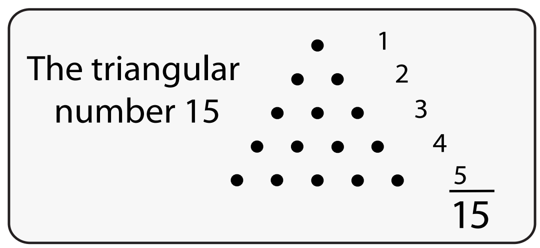
First, Dee made a similar "intentional error" with the awkward looking "Engraved 2" in the "Thus the World was Created" chart of the Monas.

Second, the chart appears the same way in 1568 as it did in 1558. Dee made a quite a few word changes in the second edition and hard to believe neither that he nor any of his readers wouldn't have spotted this error over a 10-year period.

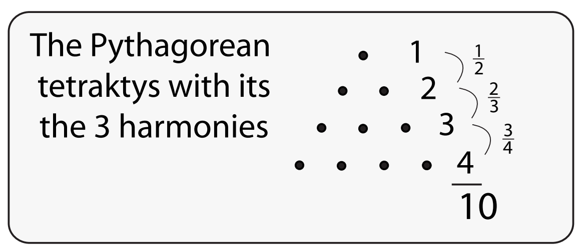
Third, Dee was meticulous about details especially when it came to numbers.

Fourth, the grand total amount of all of Dee's calculations actually should be 25,341, which is quite a peculiar number. It includes all of the digits 1, 2, 3, 4, and 5. Certainly there are many other numbers that include these 5 digits, but I get the sense that Dee felt that there was something special about this being a result of his calculations which start with what he elsewhere calls the "remarkable septenary." Dee felt his chart, as a whole, was special as it includes several key Metamorphosis numbers (12, 24, and 2520), as well as many "close relatives" of Metamorphosis numbers, like 840 (2520 divided by 3) and 5040 (2520 times 2).

Allow me to "dissect" 25,341 a little bit. It includes the digits 2, 5, 3, 4 and 1,which add up to 15. Rearranging things a little, we might even envision this as a "triangular number" in the fashion that Pythagoras drew his tetraktys.



This is a pretty picture, but certainly not as pretty as Pythagoras' tetraktys, with its 3 harmonious parts.

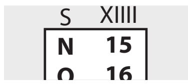


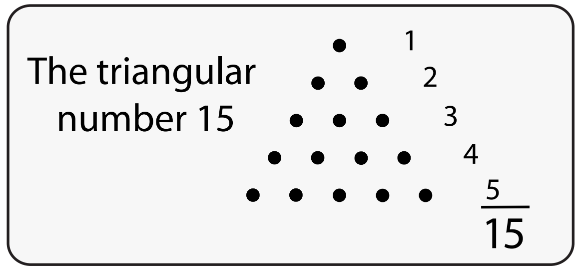
If Dee's result involved ***just*** the numbers 1, 2, 3, and 4, I'd say we had a really good clue on our hands. But, alas, it doesn't. The result 25,341 involves that extra number, 5. But, its sigificant that the digits in 25,341 add up to 15.

$$2+5+3+4+1=15$$

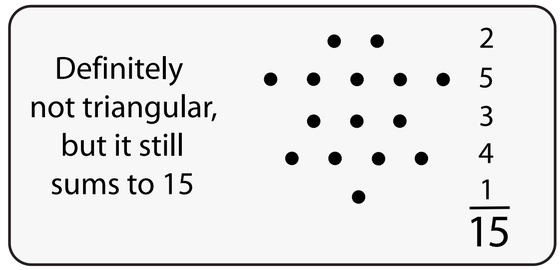
It seemed like Dee was trying to say something about "15-ness". This is the fourth time I had bumped into the number 15.

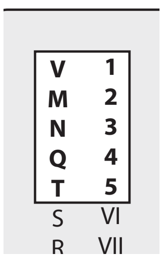
The first instance is Dee's choosing to identify Aphorism 15 with an Arabic numeral. As we've seen, he highlighted it because 15 is a triangular number.



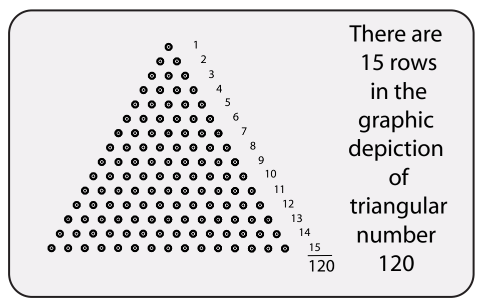


In a similar fashion, we might picture 25,341 adding to 15 this way:

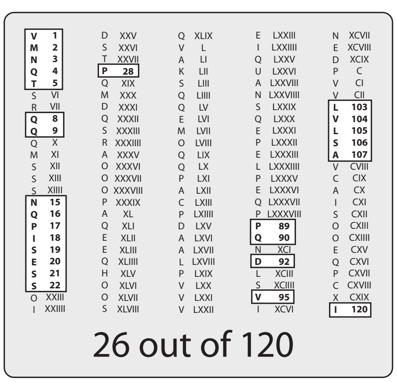




The second instance is Dee's highlighting of Aphorisms 1, 2, 3, 4, and 5. Again these numbers sum up to 15.

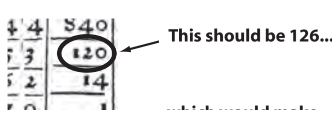


The third instance is Dee's highlighting of Aphorism 120. This number of the final Aphorism is the triangular number which is the sum of "1 and all the numbers up to 15."



Dee planted another clue that seemed to indicate that there was something special about 25,341. Remember that his "intentional mistake" was to use the number 120 instead of 126.

Well, out of the 120 Aphorisms, there are exactly 26 which he identified with Arabic numerals. Admittedly 26 and 126 are not exactly the same number, but 120 (his "intentional mistake") is the same as 120 (Aphorisms).



Sensing that I was on the trail of Dee's intent, I grabbed a hand calculator and added up all the "Aphorisms identified by Arabic numerals."

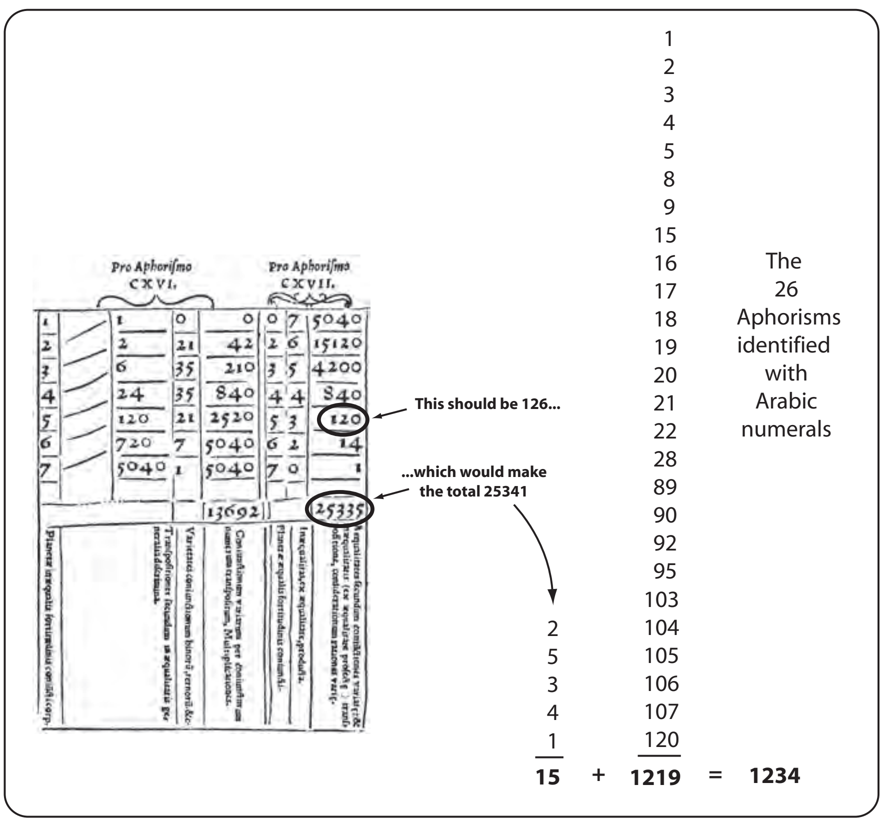
Unfortunately, was turned out to be fairly unspectacular number 1219.

I decided to check my addition. This second time, I accidentally input the number 15 twice, so I didn't get the correct result. But I was stunned by the number which appeared on my calculator: **1234**. Here was a number which not only contained the digits of the tetraktys, but they were all in their "natural order" as well.

Why didn't Dee sum to this stunning number rather than the more prosaic 1219?

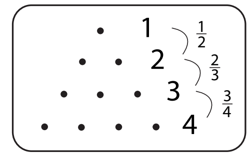
After some contemplation, it occurred to me that if Dee had made them all numeraled sum to 1234, the clue would have been ***way too obvious***.

Once the astute reader "restored" Dee's chart and got the correct 25,341, and saw that attention was being called to "Aphorisms 1, 2, 3, 4, and 5," as well as to "Aphorism 15" and "Aphorism 120" he would notice that Dee is trying to say something with the number 15. He wants us to see 1219 +15 = 1234, as summarized here:

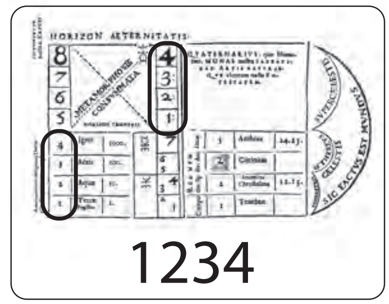


The number 1234 is not "directly" involved in the main numbers of Consummata and Metamorphosis. I thik that to Dee it was more of a "symbolic number" which beautifully expressed the "4 great Wombs is of the Larger World" (1, 2, 3, and 4) in consecutive order.

It's like a symbol of the Pythagorean tetraktys, and the 3 harmonies which it contains.

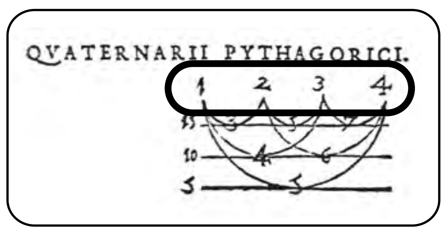


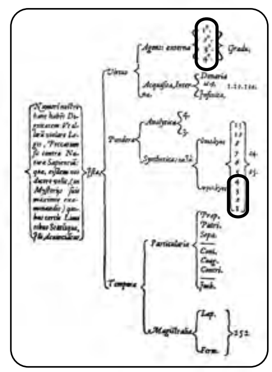
Dee's whole "25341 /Arabic numeral / triangular number" math-puzzle is simply a giant hint toreader grasp the meaning of Aphorism18. All things come from 1, 2, 3, and 4.



We might even see this number, 1234, in two places in the "Thus the World was Created" chart: as part of the octave in the "Above" half of the chart (where Dee adds three semi-colons as a hint to 1/2, 2/3, 3/4) and also in the "Quaternary" of the "first four digits" in the "Below" part of the chart.

We might also see it in the Pythagorean Quaternary and in two places in the Artificial Quaternary chart.





To summarize, in order to "find" 25,341, the astute reader must first find the "**intentional mistake**" in the chart of various conjunctions. (Dee played a similar game with the "Engraved 2" of the "Thus the World Was Created" chart in the *Monas*).

The reader then has to realize that Dee is expressing the triangular number **15**.

When this **15** is added to **1219**, the total of all the "Aphorisms identified with Arabic numbers," the reward is the even more thrilling **1234**.

1234 encapsulates the very beginning of things. The "1" really means the three things "zero–retrocity–one" from whence gushes the glorious 2, 3, and 4.

1234 also expresses the three harmonies 1/2, 2/3, and 3/4.

Dee's books are like **Japanese Puzzle Boxes;** they require several interrelated manipulations in order to open them. I unlocked the box with what seemed to be a fortuitous addition error, but it was my thinking about all this 15-ness which made me make the error in the first place. Dee knew that someone fiddling around with the "puzzle-box" long enough would eventually hit upon the solution. Inside the box are the numerals 1, 2, 3, and 4. The John Dee Tower is simply a very big puzzle box.